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THE USE OF COLOR IN THE OUTPUT ANALYSIS
OF STATISTICAL SIMULATIONS, AND ANALYSIS
OF ESTIMATORS OF SERIAL CORRELATION

by

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The Use of Color in the Output Analysis of Statistical Simulations, and Analysis of Estimators of Serial Correlation

by

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ABSTRACT

The use of color in the organization and analysis of the output of multifactor statistical simulations is investigated with the computer package SIMTBED (A Simulation Test Bed). Updating of this system to the current technology of color line printers is performed. It is shown how color can be used to code some factors in a multifactor simulation, compacting the output and enhancing analysis. An application to the analysis of the lag one serial correlation of normal and non-normal time series using four estimators (moment, maximum likelihood, robust regression, and the Cressie estimator) is provided as a demonstration of the uses of SIMTBED in statistical simulations. These estimators are examined for robustness and asymptotic bias, as well as relative behavior with various sample sizes. It is shown that for some time series the robust estimators of serial correlation are not acceptable due to bias and other considerations.

THESIS DISCLAIMER

The reader is cautioned that computer programs developed in this research may not have been exercised for all cases of interest. While every effort has been made, within the time available, to ensure that the programs are free of computational and logic errors, they cannot be considered validated. Any application of these programs without additional verification is at the risk of the user.

TABLE OF CONTENTS

LIST OF FIGURES

I. INTRODUCTION

This thesis consists of two parts: the analysis and updating of an existing computer simulation package to include the use of color in output analysis, and an application using the simulation methods in the analysis of robust estimators of serial correlation for time series. The technology of computers, especially the personal computer (PC), and the ability of the academic (as well as the user) community to perform computer-assisted simulations is rapidly changing, and this work represents an effort to use more modern methods and equipment in computer simulations, such as color printers and the faster personal computers. This allows more compact output analysis of the results of multi-factor simulations, through the use of color to code some of the parameters of the simulation experiment. Through the use of color, a direct graphic comparison of the behavior of different estimators in a statistical simulation can be made, enhancing the output analysis.

The research community has provided several methods for assessing autocorrelation that warrant further research, and the application of this simulation package to some of these methods seems appropriate. Specifically, some autocorrelation estimators purported to be robust are evaluated when applied to time series with various marginal distributions, including the Normal and other non-Normal distributions, and their behavior is compared. These results should prove useful to any student or researcher interested in either performing statistical simulations, or the estimation of autocorrelation parameters, for example in the examination of oceanographic data, or positions of shells on successive firings of an artillery piece. Serial correlation is of interest in the study of learning phenomena, such as accuracy of a rifleman in subsequent shots at a target. It is also used to study the independence of the individual members of a sample, to support assumptions of independence. In these situations the estimator of serial correlation used may be important.

The remaining chapters of this thesis discuss multi-factor statistical simulations, the FORTRAN simulation package SIMTBED (SIMulation Test BED) and its updating, and application to the estimation of serial correlation. With the pace of technology and research today, there is always a need for more effort to move existing systems into the current and future technology. As our abilities to conduct these types of experiments

improve, so will our understanding of the theoretical processes involved, and their application to the real world through research.

II. MULTI-FACTOR STATISTICAL SIMULATIONS

In their text on simulation methodology, Lewis and Orav [Ref. 1, p. I-2] describe a simulation as a controlled statistical sampling technique, and a controlled statistical experiment. This is to be contrasted with real-world statistical sampling, in which the statistician has no control over the population being sampled.

A. PURPOSE.

The purpose of a statistical simulation is therefore to assess some property of a statistical estimator of a parameter in an i.i.d. sample or in a random process (time series). These simulations are generally applied to situations where the processes being considered have no good analytical form to study. When there are multiple estimators, simulation can facilitate choice among them, given some criteria by which a comparison can be made.

B. EXAMPLES.

Examples of statistical simulations include:

- A study of the Student's t statistic applied to non-Normal random samples. Here, there is no good analytical result for the behavior of the statistic. A comparison can be made with the behavior of the t-statistic as applied to Normal samples.
- A study of various trimmed means (say, 5%, 7%, 10%), and their behavior compared to the arithmetic mean, for different distributions and sample sizes.
- A study of sampling, with replacement, from a generated population, and the study of the sampling distribution of a statistic (say, the median), for different distributions.
- Comparison of the Maximum Likelihood Estimator (MLE) with the Moment estimator for various statistics.
- Large sample studies of the asymptotic behavior of some statistic (i.e., higher moments, like skewness or kurtosis).
- A study to analyze the variability of some statistic, when there is no good analytical result (such as the variance of the coefficient of variation) for different distributions.

C. CONSIDERATIONS.

In all of the examples and applications of statistical simulations the following considerations are implied:

1. Factors.

Several factors are involved in the simulation. These can be the different estimators under study, the type of distribution involved in the study, some parameter variation on the distribution, the sample sizes involved, or any combination of these. The simulation experiment is to be repeated for each of these factors, and the output must be analyzed in an organized manner to examine the effect of varying these factors.

2. Replications.

There is to be some repetition of the statistical simulation to assess variability information. Random processes are involved, and repetition is required to observe behavior with any degree of assurance. Sometimes the amount of repetition required to gain an acceptable variability in the results is unknown at the start of the simulation.

3. Resources.

These experiments can be complex to conduct. Combined with the factors involved, a significant computing resource can be involved. A mainframe can provide computing power in terms of speed, but at a cost. Personal computers, while possessing much less speed, can accomplish similar tasks, with more time required, but the cost can be much less, especially considering the availability of the personal computer versus the mainframe. In any case, the user must often write some part of the routines that produce the simulation. This can include the driver routine, which generates the random process and calculates the values under study, or it could include the entire simulation, including the graphics and other output routines. The best approach is to have available some package which can take user driver routines and then aggregate the output from them into a compactly presented output, so the user can focus on the aspects of the simulation (i.e., the process and the results). This is the approach of SIMTBED.

D. OUTPUT ANALYSIS.

Consider the following example. We wish to study the behavior of four estimators of serial correlation (autocorrelation), specifically applying these estimators to samples from first order autoregressive processes whose marginal distribution is the standard Normal distribution, and with four other first order autoregressive processes with non-Normal marginal distributions. We wish to study, say, eight sample sizes, from 20 to 5000, and we want to know how these estimators behave when the true values of the serial correlation take values of -0.9, 0.9, and 0.0. Since we know the correlations of the random samples we generate, we can compare these estimators to each other, and also to the known values of the correlation. The number of simulations required to provide

only one realization of our estimators for each consideration is clearly the product of the number of each of the factors, or 480. This would be an intractable process without some organized approach to perform some of these runs as a batch. Further, in order to assess the variability in these results we wish to repeat the entire overall simulation a number of times. Not only does this represent a significant computing requirement, but some type of compact organization of the output is required for analyses and comparisons to be possible.

1. Graphics.

Graphical displays can portray a great deal of information clearly, quickly, and in a compact manner. We can use graphics and graphical symbols to combine the location and variation measures, like a boxplot, and we can plot related data close together for quick comparison. The repetitions performed to introduce stability (precision) into the simulation can be combined into the boxplot-type graph, with tables attached for reference. This represents only one way to approach output analysis, and the exact methods used depend on the simulations being conducted. For example, if we are comparing an estimator's behavior under different distributions, we would like a graphical representation which depicts each distribution's results together, so differences will be apparent. This graphical approach, combining several of the factors of the simulation in one plot, is the approach taken to output analysis with SIMTBED.

2. Regression and Asymptotic Behavior.

There are many statistical estimators in the literature with claimed asymptotic behavior with respect to sample size. Indeed, Cramer proves that any central moment of a sample of any population is asymptotically normally distributed, and he gives the mean and variance as functions of n, the sample size [Ref. 2, p. 365]. If we are interested in demonstrating asymptotic performance (say, for different distributions), we would want the estimator plots arranged by sample size to show trends. Of interest might be the sample size at which the aymptotic normal behavior of our estimator is apparent, which might allow all types of analysis possible for normally distributed data (confidence intervals, etc.). More likely, though, we are interested in the value of our estimator, suitably unbiased, which is approached as the sample size gets very large. In this situation, regression can be applied. Lewis and Orav present the results of the geometric expansion for the mean and variance of central moment estimators (the delta method), and further states that these results also apply well to estimators that are not functions of central moments [Ref. 1, p. VI-35]. In equation form, these results are:

$$E(\theta_m) = \theta + \left(\frac{a_1}{m}\right) + \left(\frac{a_2}{m^2}\right) + \left(\frac{a_3}{m^3}\right) + \ldots \tag{2.1}$$

$$Var(\theta_m) = \left(\frac{b_1}{m}\right) + \left(\frac{b_2}{m^{1.5}}\right) + \left(\frac{b_3}{m^2}\right) + \left(\frac{b_4}{m^{2.5}}\right) + \ldots \tag{2.2}$$

where a's$_i$ and b's$_i$ are constants, m is the sample size, and θ is the value of the estimator. Notice in equation (2.1) that the first term is the asymptotic value of the estimator θ itself. So, if we perform a regression using these equations and the appropriate powers of the sample size, we can attain a value and variance for the estimator as it behaves asymptotically. This regression is the type implemented in SIMTBED. With repetitions of our simulations we can observe the variability of the coefficients of the regression. Observe that more sample sizes in the simulation design allow an increased degree of regression, where by degree we mean the number of terms of equations (2.1) and (2.2) used, which typically have to be truncated to, say, four or six terms.

E. HUMAN FACTORS AND INFORMATION CODING.

When referring to information coding, we mean that the original information has been converted to a new form, and displayed symbolically, as discussed in the human factors industry [Ref. 3, p. 50]. In our output analysis, there are two forms of information coding.

1. Graphics.

The measures of location and spread for the statistical estimator have been converted to a graphical representation, the widely used boxplot. This allows rapid comparison of different runs (estimators, sample sizes, etc.) based on location and spread. Tables are also used, for detailed reference, if needed.

2. Simulation Factors.

In the update to SIMTBED as part of this thesis, the different estimators will have their identity coded by color, so that when plotted together, two estimators that otherwise have identical simulation factors (sample size, distribution, and parameters) will be distinguished by color for comparison. Further, as a consequence of the simulation processing, these estimator plots will also be coded by their position in the graph, which is not a good form of coding. Sanders and McCormick compare these coding dimensions (color and position) and others, and conclude that color is the best (in terms of correct responses), and position is the worst (configuration is the term used by these authors for position coding) [Ref. 3, p. 98]. Nonetheless, this makes it possible to use

these comparisons on any personal computer with a dot matrix printer, which SIMTBED uses for output, as well as the newer color dot matrix printers. This direct comparison of estimators in a simulation is an addition to the capabilities of SIMTBED.

The stage is now set to discuss the statistical simulation package, SIMTBED, as updated by this thesis, its capabilities, limitations, and method of use. Before doing this we note that the philosophy on the design of the original SIMTBED was to use line printer graphics for portability. This goal can still be achieved if color is included in the output graphics and the widely available and cheap dot-matrix printers are used for SIMTBED output.

III. SIMTBED.

This chapter discusses the package SIMTBED, in currently available versions, and the enhanced version created in this thesis (version 13). We discuss the SIMTBED approach to statistical simulation, the output organization it uses, and the updating to the use of color as a SIMTBED enhancement. For a detailed description of the details required to actually run SIMTBED, see Appendix A. Appendix A is designed to function as a user's guide for this version of SIMTBED.

A. DESCRIPTION.

SIMTBED is a collection of FORTRAN subroutines that perform the detailed output analysis for statistical simulations. The user writes a driver program that provides the necessary parameters of the simulation, and calls the SIMTBED subroutine. The user also provides the routines that generate the random process and calculate the statistics of interest. SIMTBED performs a sectioning of the sample sizes the user specifies and determines the number of calls required to each generating routine to produce the desired result. It then prepares all the plots and output statistics from the data supplied by the generating routines. In this way, the user focuses on the process involved in his simulation, and on the output he receives, and is relieved from the organization and generation of the output itself. This present version (13) of SIMTBED is designed to run on the IBM PC/AT family of personal computers, when compiled by any of the currently popular FORTRAN compilers. The update to older versions of SIMTBED that produced this version (13) includes the capability to produce combined plots of the user-supplied estimators, in color, and an increase in the number of estimators SIMTBED can process. The older versions could only process three estimators. The graphics used by SIMTBED, called line printer graphics, uses the dot matrix printer and its characters, in compressed-type mode, to produce the plots.

B. HISTORY.

SIMTBED originated with a mainframe version written by Lewis, Orav and Uribe in 1981. This was ported to and improved upon in a PC version in a Naval Postgraduate School Thesis by Hans-Walter Drueg [Ref. 4]. In this work, Drueg produced the first PC version of SIMTBED, with applications to statistics. His work was based on the regression methods of Lewis and Heidelberger [Ref. 5] and the graphical approach used

by Linnebur in the program RAGE [Ref. 6]. Since then, numerous updates have been incorporated by the Naval Postgraduate School simulation community, most notably by L. Uribe, and P. A. W. Lewis. Updates to SIMTBED include:

- Addition of percentile and quantile plots for each estimator.
- Computation of Mean Squared Error (MSE), when known values are input.
- Bivariate histograms to show relationships between pairs of estimators with respect to sample size.
- Super-replications to provide precision/variability information on all the SIMTBED statistics.
- A restart capability for long simulations, allowing a simulation to be performed as a series of super-replications, with each series building on the ones before.
- Most recently, the use of color printers to provide combined plots of estimators (the work of this thesis).
- Even more recently, a mainframe version utilizing laser printer technology, with even more capability (restricted to mainframe use).

Appendix D, the SIMTBED source listing, has comments at the beginning which also portray the historical development of the program.

SIMTBED has been incorporated into the Advanced Statistical Package, a published software package from P. A. W. Lewis (as SMTBPC). The manual from that package has been used in courses in simulation at the Naval Postgraduate School [Ref. 7].

C. THE APPROACH OF SIMTBED.

SIMTBED views each estimator (represented by a generating subroutine passed to SIMTBED with all other parameters) as a separate simulation run, and performs all the analysis on each estimator in turn (see Figure 1.). The capability added with this thesis consists of the color combined plots at the end of the SIMTBED processing. (Refer to Appendix B for a discussion of the programming aspects of color printing with SIMTBED.) If combined color-coded plots of up to five estimators are requested at the completion of the simulation experiment, the data needed from each estimator's simulation is kept until the end for preparation of the combined plots. Otherwise, each estimator is completely processed in sequence. Up to five estimators can be processed in one run of SIMTBED (each represented by a generating subroutine). This is an improved capability in that more estimators can be studied, and the results can be compared at the finish, graphically, in color.

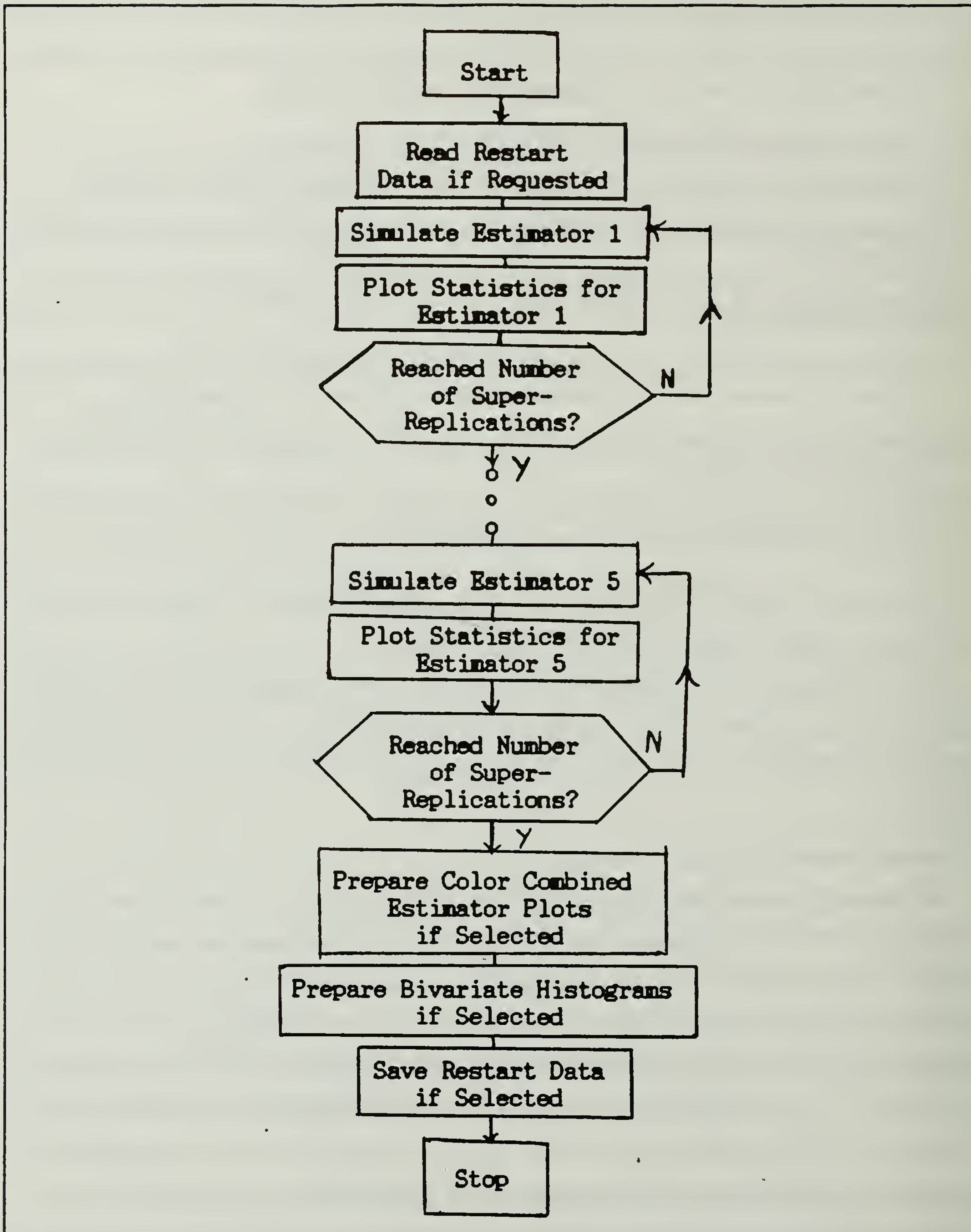

Figure 1. SIMTBED Processing Flowchart

1. Sectioning to Conduct Simulations.

a. Major parameters description.

In addition to routine parameters (number of estimators, subroutine names, etc.), SIMTBED must have information on the sample sizes that are to be used for the estimators, i.e., the sample sizes for which the properties of the estimator are to be examined. These are referred to as "sub-sample sizes," because the "sample size" refers to the global sample size from which each of the sub-samples is sectioned. We use "N" to represent this global sample size, and "NE(*)" to represent the sub-sample sizes, since they are passed in an array (from 1 to 8 sub-samples are possible). Further, a number of replications of each sub-sample size may be performed, in order to observe the variability in the regression performed on the values of the estimators (see Chapter 2, Section D.2). This is denoted by "M". These are the three major parameters affecting the simulation. The global sample size N is a function of the available memory of the machine being used.

b. Performing a simulation by sectioning.

Consider this example. Given a sample size, N, of 5000, a replication value, M, of 20, and a sub-sample size, NE(1), of 20, SIMTBED will proceed as follows. The sample space for the 5000 values of data from which the estimators can be computed will be divided up into groups of size 20, so there will be (5000/20) = 250 evaluations of the estimator, each based on a separate, independent sample of size 20. This will be repeated M times, so there will be 5000 evaluations of the estimator, based on a sample of size 20. This is the way SIMTBED sections to produce the necessary number of evaluations based on the desired sample size. Machine memory storage must be available for $(N \times M)/NE(1)$ variables in an array.

This process is independent of the estimator subroutine. For example, if our estimator were the coefficient of variation of an exponential(λ) distribution, SIMTBED would call that routine 250 separate times, and pass a new sample size of 20 from which to compute the estimate of coefficient of variation. The data generator routine would generate the random variables and compute the coefficient of variation. This process would repeat for each of the other (as many as eight total) sub-sample sizes, and if requested, a regression would be performed on the means of each of the sub-sample realizations of the estimator. This regression process is replicated M times to estimate the variability of the regression coefficients. See Chapter 2, Section D.2, and Appendix A for more on the regression performed by SIMTBED.

2. Super-Replications.

As each estimator is processed it turn, it is possible to perform the entire sectioning process for that estimator a specified number of times. These are referred to as super-replications, and allow the experimenter to evaluate variability and precision information about all of the results presented for one iteration of the simulation process. For example, one replication will yield a single value of the asymptotic expected value of the estimator being simulated (from Equation 2.1). The number of replications specified in the simulation allow estimating the spread of these coefficients of the regression, but still only one value is obtained. To obtain more realizations of this expected value, super-replications can be performed, and then tests for normality (i.e., quantile plots, etc.) can be performed for validity. These super-replications, being independent, should be normally distributed about the true estimator value, from the central limit theorem. SIMTBED performs these super-replications as specified by parameters when SIMTBED is called. All super-replications for one estimator are performed before the next estimator in the chain is processed. It is possible to obtain repeated output graphs for the first three of these super-replications, to check the simulation; thereafter, SIMTBED generates a summary table and optionally a normalized quantile plot (see section D).

3. Combined Plots in Color.

At the completion of simulation for all estimators, a plot of some of the information from each estimator in the chain that was processed can be generated. This amounts to plotting together some of the boxplots describing each estimator, with each a different color, aiding comparison of the behavior of the estimators relative to each other. While not a substitute for comparing tabulated values (such as mean squared errors, standard deviations, and means), this type of graphical comparison can provide meaningful information that could otherwise be hidden in the tabled values of the output. Analysis of the large volume of output produced in these types of simulations is thus aided by these types of graphical comparison, which can provide a wealth of information at a glance.

4. Restart and Other File Operations.

Due to the lack of good analytical form for the behavior of the estimators in these types of simulations, the amount of repetition (super-replications) required to get a specified degree of precision (with respect to asymptotic performance) or a low enough variance (say, in the mean value) may not be known until it is reached. Rather than keep repeating the simulation with larger and larger parameters, SIMTBED will use data

files to store the results of the super-replications, and subsequent super-replications can use the results from previous runs as though they had just been obtained. This allows the user to 'build up' his simulation until the values obtained have the needed precision, or until the required convergence/divergence or bias is seen. It may also be useful to have these results for processing by another package, or for record-keeping. SIMTBED can also perform this function with an output file. Refer to Appendix A for the actual use of these features.

5. A Variance Reduction Technique.

One popular technique that can be used to reduce the variability in the results of a statistical simulation is to use the same random number stream for each estimator in the simulation [Ref. 1, p. VII-24]. Thus, each estimator operates on the same series of random numbers, instead of on separate streams of random numbers, and the variability between different estimators that would be introduced by differing random number streams is reduced to only the variability introduced by the behavior of the estimators, as they are applied to the samples through the sectioning and replication process. This is accompished in SIMTBED with the random number seeds used to begin the generation of the pseudo-random numbers (see Appendix A for a detailed description of the SIMTBED parameters).

D. SIMTBED OUTPUT AND STATISTICS.

1. The Basic SIMTBED Plot.

Figure 2 represents the basic SIMTBED plot, a series of boxplots for each sub-sample size, representing the simulation for one estimator.[1]
Included in the boxplot for each sub-sample size are (M x N)/NE evaluations of an estimator. If regression is performed, the regression asymptote is also plotted. To facilitate plotting the regression line, the sub-samples are placed according to their value (i.e., the axis is scaled based on the sub-sample values). The following statistics are listed in tabular form for each sub-sample size: the first four central moments, the standard error of the mean, and (optionally) the mean squared error. The coefficients of the regression equation are also listed. The estimator name appears at the bottom of the plot.

a. An Example of Generating the Estimators.

In the example of Figure 2, we have generated Normal, AR(1) (autoregressive) time series with mean zero and variance 4, and the successive values of the time series are

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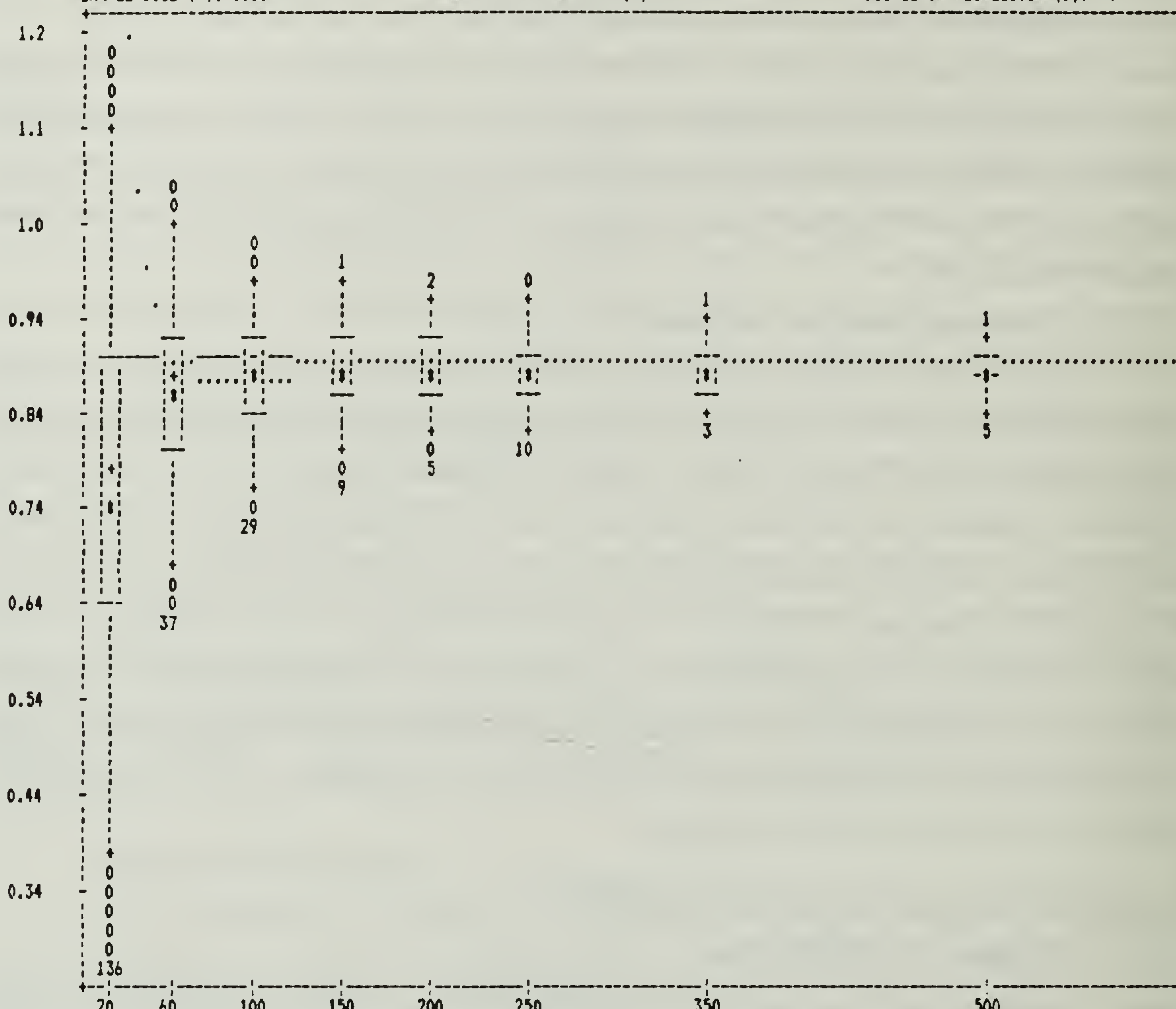
1 These plots have been reduced for enclosure in the thesis. They are normally full-sized.

SAMPLE SIZE (N): 6000   NO. OF REPLICATIONS (M): 20   DEGREE OF REGRESSION (D): 4



SUBSAMPLE SIZE	20	60	100	150	200	250	350	500
MEAN	0.7597	0.8696	0.8829	0.8936	0.8961	0.8958	0.8967	0.8979
STD	0.2038	0.9140E-01	0.6260E-01	0.4820E-01	0.3864E-01	0.3775E-01	0.2679E-01	0.2343E-01
STD MEAN	0.2631E-02	0.2044E-02	0.1807E-02	0.1704E-02	0.1578E-02	0.1723E-02	0.1453E-02	0.1512E-02
M.S.E	0.2474	.0.9631E-01	0.6488E-01	0.4862E-01	0.3885E-01	0.3798E-01	0.2699E-01	0.2352E-01
SKEWNESS	-0.9285	-0.7217	-0.7123	-0.5264	-0.2994	-0.7435	-0.2998	-0.2886
KURTOSIS	1.1272	0.7081	1.0511	0.7195	0.1484	1.1843	0.4549	0.5358

MEAN OF REGRESSION ON AVERAGES	0.8929	3.391	-721.9	0.3308E+05	0.1448E+05
VARIANCE OF REGRESSION	0.1987E-04	6.302	0.1472E+06	0.3663E+09	0.7319E+08
STD DEV OF REGRESSION	0.4457E-02	2.510	383.7	0.1914E+05	8555.
REGRESSION ON VARIANCE	-0.7434E-01	13.89	-195.7	1321.	-2875.

ESTIMATOR: **Rho of N(0,4) r.v.s, rho=0.9 using moment estimator
VERTICAL SCALE: YMIN = 0.2592
YMAX = 1.2417

Figure 2. Sample SIMTBED Simulation.

correlated with correlation coefficient $\rho = 0.9$. The estimator is the familiar moment estimator of serial correlation:

$$\hat{\rho} = \frac{\sum_{i=1}^{NE-1} (x_i - \bar{x})(x_{i+1} - \bar{x})}{\sum_{i=1}^{NE} (x_i - \bar{x})^2},$$

where $\bar{x}$ is the mean of the NE observations in the estimator.

Thus, we know the properties of the generated random number stream and we are observing the behavior of the estimator $\hat{\rho}$. Figure 2 shows that the simulation was performed for each of 8 subsample sizes, ranging from 20 to 500, and 6000 random variates were used to simulate each subsample size, and this was repeated 20 times to assess the precision of the regression coefficients. So, for example, for subsample size 100, the number of evaluations of $\hat{\rho}$ is given by:

$$\frac{6000 \times 20}{100} = 1200$$

These 1200 evaluations of $\hat{\rho}$ were obtained by proceeding as follows. The sample space of 6000 generated data values of the time series is sectioned into batches of size 100 (the subsample size in this example). For each section, one value of the estimator is computed. This sectioning is then replicated 20 times (M = 20), which yields $60 \times 20 = 1200$ evaluations of the estimator at subsample size 100. From Figure 2, the values of the moments computed below the column labeled '100' are thus based on 1200 evaluations of $\hat{\rho}$.

b. Subsample Statistics.

Continuing with subsample size 100 as an example from Figure 2, the mean, standard deviation, standard deviation of the mean, mean squared error, and the higher moments (coefficients of skewness and kurtosis using unbiased expressions) are computed, *using all 1200 evaluations of* $\hat{\rho}$. The theoretical standard error of the correlation estimate can be seen to follow the behavior given by:

$$\text{S.D. MEAN} = \frac{\text{sample S.D. of the estimates of the correlation}}{\sqrt{n}},$$

where here n is the total number of estimates of serial correlation used. For subsample size 100, the standard deviation of the average of the 1200 estimates of the serial correlation is estimated from the standard deviation of the 1200 computed values of serial correlation. This gives

$$\frac{.06260}{\sqrt{1200}} = 0.001807 .$$

Thus the mean value of the estimate of $\hat{\rho}$, namely 0.8829, is more that two estimated standard deviations from the known true value $\rho = 0.9$, indicating that the estimator is still biased at sample size 100.

The mean squared error (MSE) is used in this simulation since we know the value of the estimator under study for the sample we generated, i.e., we created the random number stream with a correlation of 0.9. We can thus measure how the estimator deviates from this value, still considering its inherent variability, by using the mean squared error given by[2]

$$\hat{MSE} = \sqrt{\hat{VAR}(\overline{\rho}) + (\overline{\rho} - \rho)^2} ,$$

where $\overline{\rho}$ is the average of all the estimates $\hat{\rho}$ of serial correlation (the estimator under study in this example), ρ is the known true value of the serial correlation of the time series, and $\hat{VAR}(\overline{\rho})$ is the sample variance of the estimates of serial correlation (there are 1200 of these for sub-sample size 100 in this example). This expression is realized by considering that the variance of the realized values of the estimators (serial correlation) is the second central moment about the expected value of the estimator ($E(\hat{\rho})$, usually estimated by $\overline{\rho}$), i.e.

$$VAR(\hat{\rho}) = E(\hat{\rho} - E(\hat{\rho}))^2 = \frac{1}{n} \sum_{l=1}^{n} (\hat{\rho}_l - \overline{\rho})^2 ,$$

where $\hat{\rho}$ represents the estimated value of the statistical estimator under study (i.e., serial correlation in the example), and $\overline{\rho}$ represents the average of all the observed values of $\hat{\rho}$ The expected value of the estimator, E($\hat{\rho}$), may in fact be biased, and not at all

2 Formula from SIMTBED source code.

approach the true value ρ. So, the Mean Squared Error is used, as the expectation of the values of the estimator, $\hat{\rho}$ about the true value ρ. Thus,

$$\mathrm{MSE}(\hat{\rho}) = \mathrm{E}(\hat{\rho} - \rho)^2 .$$

This Mean Squared Error can be thought of as consisting of two parts: the variation of the evaluated estimators about their average (their expected value), and the variation of this expected value about the true value ρ. Mean Squared Error then becomes

$$\mathrm{MSE}(\hat{\rho}) = \mathrm{VAR}(\hat{\rho}) + (\mathrm{bias})^2 ,$$

with the 'bias' term representing the difference between the expectation of the estimator, $\mathrm{E}(\hat{\rho})$ and the true value ρ, giving

$$\mathrm{MSE}(\hat{\rho}) = \mathrm{VAR}(\hat{\rho}) + (\mathrm{E}(\hat{\rho}) - \rho)^2 .$$

Now, using $\bar{\rho}$ to estimate the expected value of the estimator (serial correlation in the example), $\mathrm{E}(\hat{\rho})$, and computing the expected value from the evaluations of serial correlation gives the formula for computing the estimated Mean Squared Error,

$$\hat{\mathrm{MSE}}(\hat{\rho}) = \left[\frac{1}{n} \sum_{i=1}^{n} (\hat{\rho}_i - \bar{\rho})^2 + (\bar{\rho} - \rho)^2 \right]^{\frac{1}{2}}$$

This becomes

$$\hat{\mathrm{MSE}} = \sqrt{(\hat{\mathrm{VAR}}(\hat{\rho}) + (\bar{\rho} - \rho)^2)}$$

giving the formula used for mean squared error.[3]

If the known values of the estimator are not available for the simulation, MSE is not used (SIMTBED parameter entries).

*c. **Regression.***

Part of the tabled information of Figure 2 includes the regression coefficients, as discussed in Chapter 2. When replications are performed, the variance and standard deviation of the regression coefficients for the regression on the means of the estimator is supplied. In this example, the degree of regression (the number of terms of

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3 From analysis with thesis advisor.

equations 2.1 and 2.2 to be used) is 4, and the regression coefficients can be observed. Note that if the number of subsamples in the simulation is too small (eight is a maximum, not the required number), SIMTBED will not perform regression. The first value listed in the "mean of regression on averages" line in Figure 2 is the estimated asymptotic value of the estimator, given by this simulation. Since the estimate of this asymptote is 0.8929 and the true value is 0.9, the difference is 0.0071. The estimated standard deviation of the estimate of 0.044 (line labeled "std dev of regression"). Thus the estimate 0.8929 is within 2 (estimated) standard deviations of its true value.

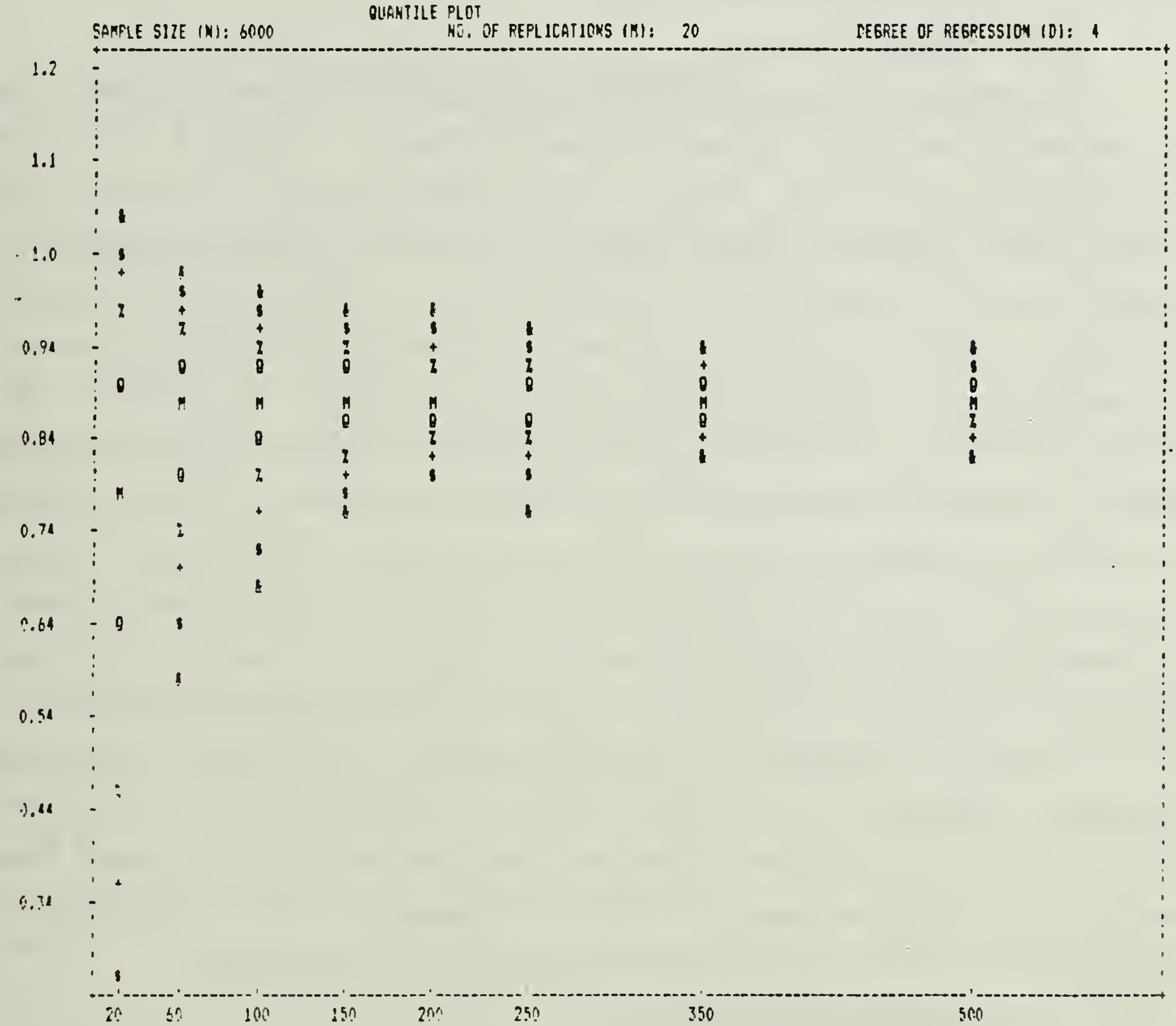
Note that if super-replications were performed we could obtain several evaluations of this asymptotic value, as well as its average over the number of super-replications performed.

d. Boxplot Graphics.

For each subsample size, a boxplot is provided which displays the location and spread behavior for all the evaluations of the estimator obtained for that subsample size. Continuing with the current example, for subsample size 100, then, 1200 values of $\hat{\rho}$ are incorporated into that particular boxplot. In Figure 2, the boxplots are constructed using reduced graphics, which means the extreme outliers are counted and displayed at the ends of the boxplot, rather than actually plotted, which would occupy valuable scaling space on the plot. This way, the majority of the space is devoted to the location measures (mean and median) and the spread measure (inter-quartile range) for the graph. The asymptote and regression lines are also included when regression is performed. This way, the behavior of the estimator can be observed with regard to sample size. In Figure 2, it can be seen that the moment estimator indeed has some bias at the lower sample sizes, but that the bias disappears as sample size increases. For further analysis the user can select the subsample sizes which will be plotted together for *all* the estimators in the simulation in the combined color plot at the end of the simulation run. Also of note is that this boxplot graph is only one realization of the data that is incorporated into super-replications. Up to three consecutive plots like Figure 2 can be prepared when super-replications are used, each an iteration itself.

2. Quantile Plots.

Figure 3 depicts the second SIMTBED plot produced as each estimator is processed. It consists of the empirical quantiles of the estimator at each subsample size, with each quantile represented by a symbol. A table below the plot lists the actual values. This plot can be seen to correspond exactly with the boxplots of Figure 2. It is



SUBSAMPLE SIZE QUANTILE	20	60	100	150	200	250	350	500
0.010	0.1657	0.5937	0.6999	0.7669	0.8015	0.7732	0.8248	0.8317
0.025	0.2728	0.6598	0.7400	0.7903	0.8163	0.8110	0.8415	0.8420
0.050	0.3692	0.7044	0.7697	0.8083	0.8252	0.8293	0.8532	0.8567
0.100	0.4778	0.7464	0.8016	0.8287	0.8430	0.8449	0.8621	0.8676
0.250	0.6479	0.8161	0.8500	0.8649	0.8735	0.8748	0.8792	0.8845
0.500	0.7947	0.8835	0.8835	0.8977	0.9002	0.9010	0.8979	0.8989
0.750	0.9070	0.9337	0.9252	0.9267	0.9236	0.9201	0.9151	0.9140
0.900	0.9832	0.9780	0.9565	0.9510	0.9403	0.9389	0.9284	0.9262
0.950	1.029	0.9997	0.9702	0.9664	0.9551	0.9525	0.9391	0.9331
0.975	1.058	1.014	0.9842	0.9794	0.9664	0.9600	0.9462	0.9390
0.990	1.089	1.034	1.008	0.9940	0.9814	0.9727	0.9586	0.9542
SER.COR.	0.0225	0.0435	-0.0007	0.0402	-0.0258	0.0192	-0.0751	-0.0316

ESTIMATOR: **Rho of N(0,1) r.v.s. rho=0.9 using moment estimator

Figure 3. Sample SIMTBED Quantile Plot.

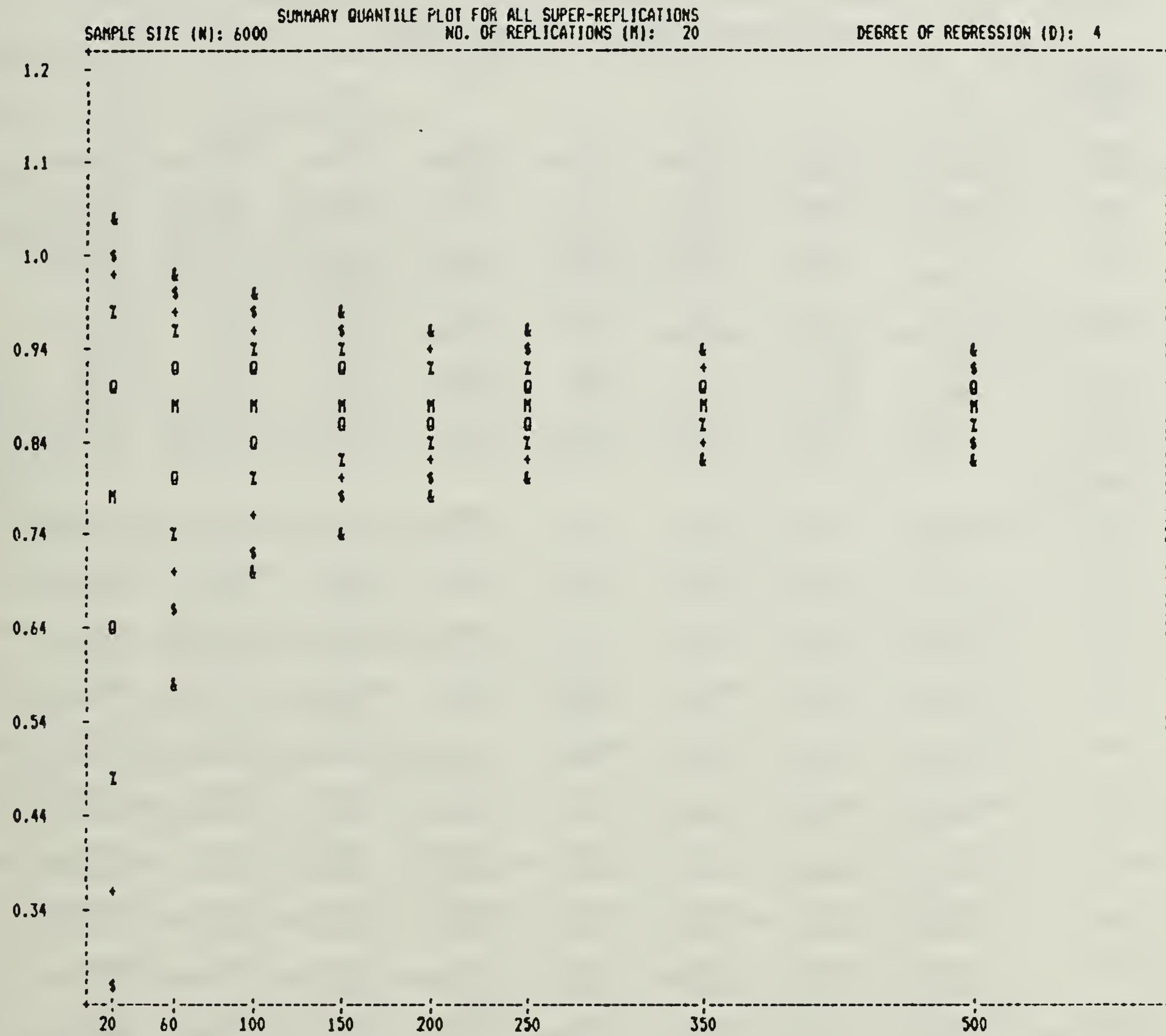
merely a more detailed representation of the data. Here, the tails of the estimator distribution can be examined for symmetry, etc.

3. Super-replications Output.

When super-replications are used, a summary quantile plot, like that in Figure 4, is prepared, along with a tabulated summary (Figure 5). Figure 4 is an example super-replication quantile plot. This plot is rather self-explanatory, beyond the fact that it consists of the quantiles of each iteration of the basic SIMTBED plot discussed above. In other words, in Figure 4, there are three values of the 0.5 quantile averaged and plotted with an 'M'; these quantiles are part of the table of summary statistics for all super-replications, an example of which appears in Figure 5. Note here that all the statistics included in the basic plot are present, and that each has an associated standard deviation, obtained by repeating the entire simulation experiment. This procedure can be carried on indefinitely by the use of the restart facility. When restart is used, the previous values are saved in a file, and all previous data is used to prepare the most recent super-replications summary table and plot. Refering to Figure 5, we can see the mean of regression on averages entry, which is a list of the regression coefficients, and we see the moment estimator asymptotically tending to 0.8991, which would indicate asymptotic unbiasedness for this estimator, given that the largest sample size used was 500, and the number of super-replications used here was 3. This would indicate convergance to the known value of 0.9 for this estimator. In the next chapter we will see the behavior as the super-replications are carried out much further.

4. Color Combined Estimator Plots.

After the simulation run has proceeded through all super-replications of as many as five statistical estimators, we may be interested in how the estimators compare to each other. This plot was incorporated into SIMTBED for that purpose. Figure 6 is an example of four estimators of serial correlation, all plotted together, for three of the eight subsample sizes of the simulation. Note that the group of boxplots (each containing four boxplots, one for each estimator) is positioned with respect to the second one in the group. Note also that the plots are coded dually, as discussed in Chapter 1, Section E. That is, they are coded by color and position. The names of the estimators appear at the bottom of the plot. Estimator 1, the robust least squares approach to serial correlation, is the first in each group for the three subsamples. All estimators are applied to the same distribution and their behavior can be directly compared. Referring to Figure 6, one can observe that for small sample sizes, the robust least squares estimator has the best behavior in terms of bias. Note that the samples come from the



SUBSAMPLE SIZE QUANTILE	20	60	100	150	200	250	350	500
0.010	0.1620	0.5978	0.7057	0.7475	0.7983	0.8052	0.8314	0.8403
0.025	0.2759	0.6624	0.7400	0.7849	0.8160	0.8247	0.8422	0.8522
0.050	0.3774	0.7061	0.7695	0.8074	0.8256	0.8376	0.8539	0.8609
0.100	0.4888	0.7507	0.8015	0.8292	0.8442	0.8520	0.8646	0.8686
0.250	0.6500	0.8190	0.8490	0.8634	0.8711	0.8752	0.8812	0.8852
0.500	0.7934	0.8829	0.8905	0.8949	0.8979	0.8991	0.8996	0.8985
0.750	0.9044	0.9334	0.9273	0.9229	0.9216	0.9186	0.9174	0.9130
0.900	0.9864	0.9754	0.9577	0.9472	0.9403	0.9364	0.9300	0.9244
0.950	1.025	0.9997	0.9748	0.9632	0.9537	0.9499	0.9394	0.9324
0.975	1.058	1.015	0.9884	0.9768	0.9666	0.9601	0.9461	0.9374
0.990	1.092	1.032	1.010	0.9909	0.9796	0.9698	0.9552	0.9507
SER.COR.	0.0091	0.0090	0.0127	0.0282	-0.0135	-0.0150	-0.0332	0.0070

ESTIMATOR: **Rho of N(0,4) r.v.s, rho=0.9 using moment estimator

Figure 4. Sample SIMTBED Super-replication Quantile Plot.

SUMMARY STATISTICS (MEAN/STD) 3 SUPER-REPLICATIONS

SUBSAMPLE SIZE	20	60	100	150	200	250	350	500
MEAN	0.7604 0.3710E-03	0.8704 0.4679E-03	0.8848 0.9913E-03	0.8910 0.1650E-02	0.8950 0.6414E-03	0.8964 0.3123E-03	0.8986 0.9754E-03	0.8979 0.2670E-03
STD	0.2006 0.1752E-02	0.9064E-01 0.3779E-03	0.6238E-01 0.6536E-03	0.4781E-01 0.8960E-03	0.3833E-01 0.4998E-03	0.3383E-01 0.2001E-02	0.2615E-01 0.8946E-03	0.2171E-01 0.8577E-03
M.S.E	0.2444 0.1629E-02	0.9534E-01 0.4905E-03	0.6420E-01 0.7706E-03	0.4870E-01 0.1069E-02	0.3867E-01 0.5250E-03	0.3402E-01 0.2014E-02	0.2623E-01 0.9125E-03	0.2181E-01 0.8533E-03
SKEWNESS	-0.9392 0.2681E-01	-0.8485 0.6518E-01	-0.6380 0.3722E-01	-0.5725 0.2559E-01	-0.2992 0.7093E-01	-0.3789 0.1943	-0.3030 0.1150	-0.2004 0.6242E-01
KURTOSIS	1.246 0.1107	1.481 0.3905	0.7619 0.1447	0.9390 0.1098	0.2828 0.8507E-01	0.5704 0.3086	0.1447 0.2563	0.1808 0.2592
SER.COR.	0.9131E-02 0.1472E-01	0.8963E-02 0.1940E-01	0.1268E-01 0.1598E-01	0.2818E-01 0.1293E-01	-0.1351E-01 0.1255E-01	-0.1500E-01 0.2025E-01	-0.3321E-01 0.4208E-01	0.7040E-02 0.2130E-01
QUANTILES								
0.010	0.1620 0.1123E-01	0.5978 0.1482E-02	0.7057 0.7951E-02	0.7475 0.1143E-01	0.7983 0.1652E-02	0.8052 0.1382E-01	0.8314 0.5055E-02	0.8403 0.4830E-02
0.025	0.2759 0.4175E-02	0.6624 0.1712E-02	0.7400 0.1839E-02	0.7849 0.3250E-02	0.8160 0.5319E-03	0.8247 0.6870E-02	0.8422 0.3525E-02	0.8522 0.5105E-02
0.050	0.3774 0.8658E-02	0.7061 0.1082E-02	0.7695 0.2315E-02	0.8074 0.3591E-02	0.8256 0.2214E-03	0.8376 0.4357E-02	0.8539 0.6690E-03	0.8609 0.2132E-02
0.100	0.4888 0.5519E-02	0.7507 0.2195E-02	0.8015 0.1647E-02	0.8292 0.2308E-02	0.8442 0.7577E-03	0.8520 0.3610E-02	0.8646 0.1393E-02	0.8686 0.4752E-03
0.250	0.6500 0.1236E-02	0.8190 0.1571E-02	0.8490 0.7112E-03	0.8634 0.2002E-02	0.8711 0.1605E-02	0.8752 0.6079E-03	0.8812 0.1184E-02	0.8852 0.4015E-03
0.500	0.7934 0.2088E-02	0.8829 0.6541E-03	0.8905 0.1049E-02	0.8949 0.1762E-02	0.8979 0.1166E-02	0.8991 0.1015E-02	0.8996 0.2060E-02	0.8985 0.5521E-03
0.750	0.9044 0.1447E-02	0.9334 0.1674E-03	0.9273 0.1045E-02	0.9229 0.1902E-02	0.9216 0.1295E-02	0.9186 0.7409E-03	0.9174 0.1311E-02	0.9130 0.5004E-03
0.900	0.9864 0.9247E-03	0.9754 0.1282E-02	0.9577 0.1376E-02	0.9472 0.1973E-02	0.9403 0.8830E-03	0.9364 0.1238E-02	0.9300 0.8169E-03	0.9244 0.1113E-02
0.950	1.025 0.2519E-02	0.9997 0.2632E-04	0.9748 0.2633E-02	0.9632 0.1801E-02	0.9537 0.2146E-02	0.9499 0.2391E-02	0.9394 0.9254E-03	0.9324 0.1328E-02
0.975	1.058 0.2093E-03	1.015 0.8745E-03	0.9884 0.2215E-02	0.9768 0.1489E-02	0.9666 0.1098E-02	0.9601 0.4257E-02	0.9461 0.1266E-02	0.9374 0.1666E-02
0.990	1.092 0.1359E-02	1.032 0.7543E-03	1.010 0.1090E-02	0.9909 0.2622E-02	0.9796 0.2078E-02	0.9698 0.4126E-02	0.9552 0.1751E-02	0.9507 0.2343E-02

MEAN OF REGRESSION ON AVERAGES	0.8991 0.3113E-02	0.2048 1.607	-267.8 228.7	0.1164E+05 0.1081E+05	5140. 4708.
STD DEV OF REGRESSION	0.4999E-02 0.3802E-03	2.809 0.2048	426.1 27.82	0.2111E+05 1325.	9405. 582.0
REGRESSION ON VARIANCE	0.8567E-01 0.1790	3.798 9.159	-20.98 148.3	180.5 936.7	-439.5 1967.

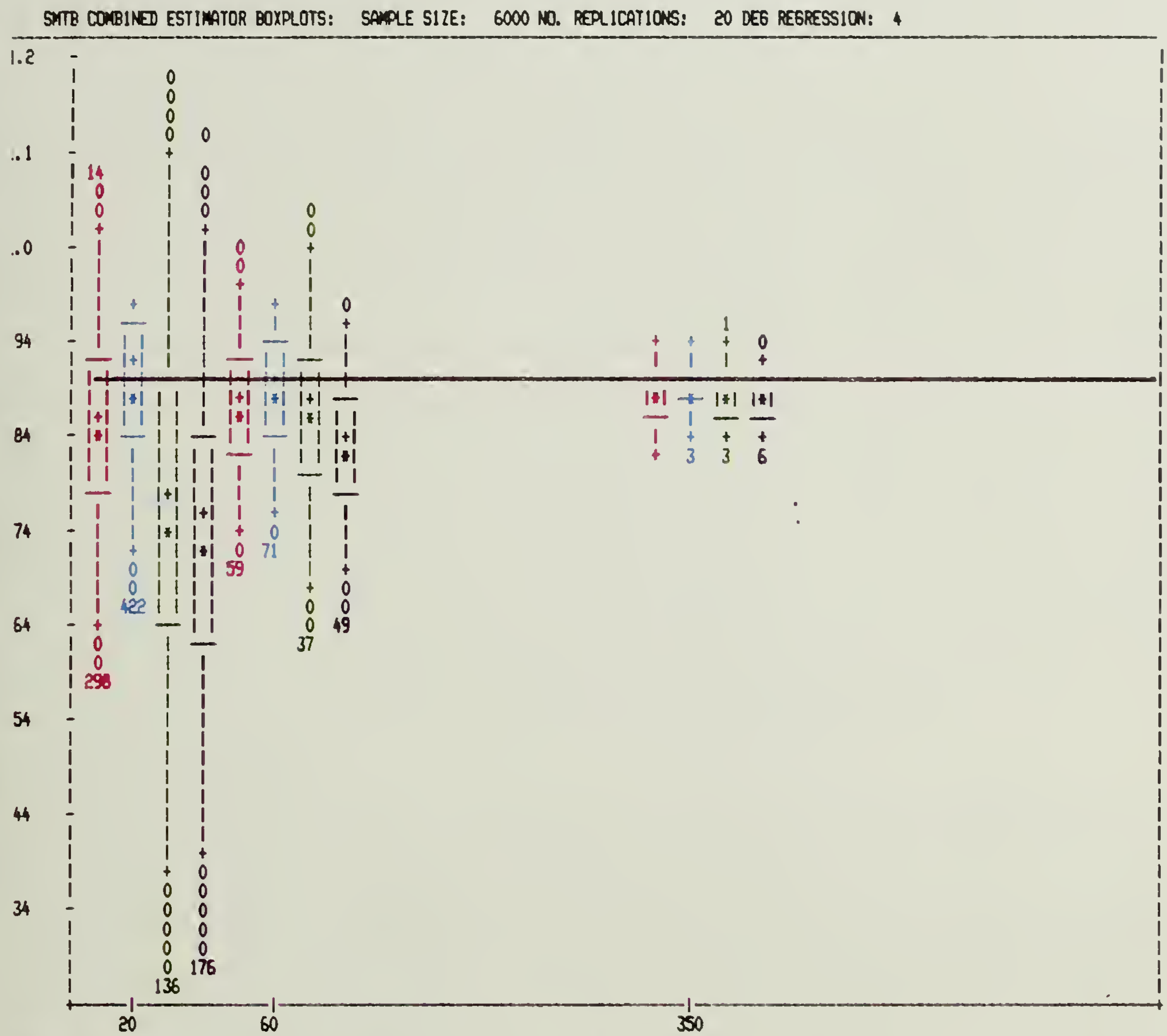
ESTIMATOR: !!Rho of N(0,4) r.v.s, rho=0.9 using moment estimator

Figure 5. Sample SIMTBED Super-replication Summary Statistics.

Normal AR(1) data. The next chapter will demonstrate that this good behavior of the robust estimator does not hold for other types of time series, but the point here is that with this combined estimator plot, the behavior is readily apparent, and though all the tabulated data from the entire simulation is available for review, this information is apparent from an immediate glance at this type of plot. Thus, the effort required to perform output analysis from the multi-factor statistical simulation has been significantly reduced through the use of color and graphical displays of the simulation results.

5. Multi-Factor Simulations With SIMTBED.

Chapter II discussed the multi-factor aspects of statistical simulations, and this section discusses how they can be addressed using SIMTBED. There is a direct application of the simulation factor sample size to the SIMTBED package. The SIMTBED subsample size becomes the sample size factor. With SIMTBED, there are five estimators possible, and there are three other factors involved in any common statistical simulation: estimator used, distribution of the samples, and parameters of that distribution. Thus, the estimator parameter of SIMTBED can be used to express any one of these remaining three factors, leaving two factors that will require multiple runs of the SIMTBED simulation. This still represents a great reduction in the amount of separate work required. For example, in the next section we will use four of the five positions of SIMTBED to represent the estimator used, and then SIMTBED will be run for differing combinations of distribution and parameter (e.g., Normal(0,1) $\hat{\rho}$ = 0.0, 0.9, and -0.9). When SIMTBED is run on the common personal computer, all one need do is execute SIMTBED in parallel on several available machines, and the multi-factor simulation is completed. Actually, we are using the personal computer itself to apply one factor of the simulation. That was the approach taken for the application presented in the next chapter.



		20	60	350
BSAMPLE SIZE:		20	60	350
TIMATOR 1:	MEAN:	0.8469	0.8740	0.8935
	STD:	0.1327	0.7156E-01	0.2711E-01
	STD MEAN:	0.1713E-02	0.1600E-02	0.1470E-02
BSAMPLE SIZE:		20	60	350
TIMATOR 2:	MEAN:	0.8827	0.8918	0.8980
	STD:	0.1352	0.7054E-01	0.2535E-01
	STD MEAN:	0.1745E-02	0.1577E-02	0.1375E-02
BSAMPLE SIZE:		20	60	350
TIMATOR 3:	MEAN:	0.7597	0.8696	0.8967
	STD:	0.2038	0.9140E-01	0.2679E-01
	STD MEAN:	0.2631E-02	0.2044E-02	0.1453E-02
BSAMPLE SIZE:		20	60	350
TIMATOR 4:	MEAN:	0.7277	0.8404	0.8891
	STD:	0.1849	0.7638E-01	0.2434E-01
	STD MEAN:	0.2388E-02	0.1708E-02	0.1320E-02

TIMATOR 1: **Rho of N(0,4) r.v.s, rho=0.9 using robust Least Squares estimator
TIMATOR 2: **Rho of N(0,4) r.v.s, rho=0.9 using Cressie estimator
TIMATOR 3: **Rho of N(0,4) r.v.s, rho=0.9 using moment estimator
TIMATOR 4: **Rho of N(0,4) r.v.s, rho=0.9 using Priestly Estimator

Figure 6. Sample SIMTBED Color Combined Estimator Plots.

IV. ANALYSIS OF LAG-1 SERIAL CORRELATION

The purpose of this chapter is twofold: to provide a demonstration of statistical simulation using SIMTBED, and also to provide some simulation results for some estimators of serial correlation gathered from the literature. Some of these estimators differ from the standard ones in that they have varying degrees of robustness claimed for them, and so we will run simulations to demonstrate this, or the lack of this robustness. By robustness, we mean the ability to produce nearly correct or unbiased estimates of the actual serial correlation when the underlying marginal distribution of the data is no longer the Normal distribution, and the time series are not linear AR(1) processes. With that in mind, the choice of distributions and time series to be used is made. We begin with a description of the estimators of lag-1 serial correlation that are compared, followed by a discussion of the distributions, parameters and processes chosen. The results of these simulations, run with SIMTBED on the PC are then discussed.

A. THE ESTIMATORS.

For this simulation experiment, we have available four estimators of the serial correlation of a random process (random number stream). These are:

- The moment estimator. This is the most widely known and used estimator of serial correlation in a time series, and so is included here. It is a non-parametric estimator.
- The Maximum Liklihood Estimator. Also called the Priestley estimator. Somewhat more involved computationally, and its derivation depends on the explicit assumption that the data comes from a Normal, linear autoregressive (AR(1)) time series.
- The Cressie estimator. Carrying the name of Noel Cressie, this estimator has the potential for robustness because of its choice of the median as the measure of location, rather that the more common mean.
- Robust Least Squares Regression. Also called Iterated Weighted Least Squares (IWLS), this method has appeal in its purported ability to reduce regression errors in successive passes, but the choice of the weighting function must be made, and it can be computationally intense.

Each of these estimators is discussed in greater detail below.

1. The Moment Estimator.

This estimator has a form that is analogous to that for the correlation coefficient for two samples given in any statistics textbook, except that the two samples are

actually the members of only one sample, but offset by one index, i.e., the data is obtained from a time series (discrete parameter stochastic process). This yields, from the Lewis and Orav text on simulation methodology [Ref. 1, p. VIII-57] the following estimator for the lag one serial correlation. The correlation coefficient of lag one is defined as

$$\rho(1) = \frac{\mathrm{Cov}\,(X_i, X_{i+1})}{\sqrt{\mathrm{Var}\,(X_i)\mathrm{Var}(X_{i+1})}} = \frac{\mathrm{E}[X_iX_{i+1}] - \mathrm{E}[X_i]\mathrm{E}[X_{i+1}]}{\sqrt{\mathrm{Var}\,(X_i)\mathrm{Var}(X_{i+1})}} .$$

For stationary time series, the correlation coefficient is independent of time (serial number), and the moment estimator becomes

$$\hat{\rho}_M = \frac{n\sum_{i=1}^{n-1}(X_i - \overline{X})(X_{i+1} - \overline{X})}{(n-1)\sum_{i=1}^{n}(X_i - \overline{X})^2} . \tag{4.1}$$

The term moment estimator is used for this estimator because the arithmetic mean $\bar{x}$ is used to estimate $\mathrm{E}[X]$ and the usual moment estimator is used for $\mathrm{E}[X_iX_{i+1}]$. In fact, though, the estimator used in this simulation is the form of equation (4.1) with the (n-1) bias term in the denominator replaced by (n), which yields the computational form referred to by Lewis and Orav as the Yule-Walker estimator [Ref. 1, p. VIII-56]:

$$\hat{\rho}_M = \frac{\sum_{i=1}^{n-1}(X_i - \overline{X})(X_{i+1} - \overline{X})}{\sum_{i=1}^{n}(X_i - \overline{X})^2} . \tag{4.2}$$

Again, this estimator is widely used for estimating serial correlation, and as will be seen, has good asymptotic properties as sample size increases (depending on the underlying distribution), but can have significant small-sample bias, especially with time series whose marginal distributions are markedly different from the Normal distribution.

2. The Priestley (conditional M.L.E.) Estimator.

This estimator is put forth as the conditional Maximum Liklihood Estimator (MLE), also called for convenience the Priestley estimator of lag-1 serial correlation [Ref. 8]. It also has good asymptotic properties, as well as the problem of small-sample bias, as the simulations will demonstrate. The estimator is

$$\hat{\rho}_{\mathrm{MLE}} = \frac{\sum_{i=1}^{n-1}(X_i - \hat{\mu})(X_{i+1} - \hat{\mu})}{\sum_{i=1}^{n-1}(X_i - \hat{\mu})^2} . \tag{4.3}$$

where

$$\hat{\mu} = \frac{(A\overline{X}_{(2)} - B\overline{X}_{(1)})}{[(A - B) + (n - 1)\overline{X}_{(1)}(\overline{X}_{(2)} - \overline{X}_{(1)})]} ,$$

and

$$\overline{X}_{(1)} = \begin{array}{c}\text{Average over first } (n-1) \text{ observations} \\ \text{in the time series of length } n\end{array}$$

$$\overline{X}_{(2)} = \begin{array}{c}\text{Average over last } (n-1) \text{ observations} \\ \text{in the time series of length } n\end{array}$$

and

$$A = \sum_{i=1}^{n-1} X_i^2 ,$$

and finally,

$$B = \sum_{i=1}^{n-1} X_i X_{i+1} .$$

3. The Cressie Estimator.

This interesting estimator of lag-1 serial correlation has vague sources. It was obtained by personal communication with Cressie by Professor Lewis. The estimator is

$$\hat{\rho}_{CR} = 1 - \frac{1}{2}\left[\frac{\sum_{i=1}^{N-1}(|X_i - X_{i+1}|)^{\frac{1}{2}}}{\sum_{i=1}^{N-1}(|X_i - \tilde{X}|)^{\frac{1}{2}}}\right]^4, \tag{4.4}$$

where $\tilde{X}$ is the median of the sample. One can only comment and speculate on the behavior of this estimator. Lacking analytical results, we resort to statistical simulation to observe its behavior.

B. ROBUST REGRESSION TO ESTIMATE SERIAL CORRELATION.

We start here by presenting the Normal autoregressive lag-1 model (AR(1)), and then show that normal regression yields the usual estimate for ρ discussed in section A.1 above. Then the Iterated Weighted Least Squares (IWLS) procedure is presented, with a short discussion concerning the literature.

In the Normal (Gaussian) AR(1) model, the observations x_{k+1} are determined from the previous x_k, the correlation coefficient ρ, and a random deviation that can be Normal or non-Normal (it is Normal for the Normal model).[4] Thus,

$$x_{i+1} = \rho x_i + \varepsilon_i\,. \tag{4.5}$$

This is referred to by Denby and Martin as the "Innovations Outlier Model (IO)"[Ref. 9, p. 140]. They also describe an "Additive Effects Outlier Model (AO)," but that will not be pursued here. The model (4.5) is the approach taken to generate correlated Normal random streams, as will be discussed presently. If one wishes to assume the ε in Equation (4.5) are normally distributed with mean zero, and standard deviation σ, then a zero-intercept regression model can be applied, and shown to yield the moment estimator above. It is when this assumption of normality is not applicable that iterating the least squares procedure with weighting functions can be interesting. Thus, using the matrix notation adopted in the study of linear regression, we can restate the IO model as:

---

4 This model was introduced to me by P.A.W. Lewis, but also appears in [Ref. 9 , p. 140].

$$Y = \rho X + \varepsilon \tag{4.6}$$

where Y is the vector of the last N-1 members of the random number stream, and X is the vector of the first N-1 members. Thus, we are regressing the random stream against itself, offset by one element. Using the matrix solution to the least squares regression procedure:[5]

$$\hat{\rho} = (X^T X)^{-1} X^T Y .$$

This yields, for the IO model above:

$$\hat{\rho} = \frac{\sum_{i=1}^{N-1} X_i X_{i+1}}{\sum_{i=1}^{N-1} X_i^2}$$

This formula actually uses the fact that we know in the simulation that the mean value of the $X_{i's}$ is zero. The sample mean would normally be subtracted from the data, as in Equation 4.2, but this is generally a second order effect. The above formula is actually the starting solution used in the IWLS procedure. From this estimate of ρ, the residuals are computed, from Equation 4.6, by substituting $\hat{\rho}$ for ρ and inverting the (linear) equation. A weighting function is then applied to the residuals, and the weights are multiplied onto the original data points. A data point which gave a small residual will produce a larger weight, and points which produced large residuals will yield smaller weights, even to the point of deletion from the regression for extreme outliers. The scaling constant helps determine these aspects of the weights. The regression is then repeated, yielding a better estimate of ρ, in terms of the residuals. This is briefly discussed by Weisberg [Ref. 10, p. 87], but is presented in more detail by Beaton and Tukey [Ref. 11, pp. 151-152]. These authors refer to this estimation procedure as "w-estimates," being multiple steps of "m-estimates." The common least squares weighting function, the "bi-square" function, given by

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5 From course notes, OA3103, Prof. Larson, Spring 1987.

$$w(u) = \begin{cases} (1 - u^2)^2 & \text{for } |u| \le 1 \\ 0 & \text{else} \end{cases} \qquad (4.7)$$

is used to determine the weight for the next step of the regression, based on the value of the residuals. The arguments for this function in the IWLS procedure are the residuals calculated from the previous regression step, using the last estimate for ρ. The median, denoted here by S, multiplied by a scaling constant, c (chosen to provide good performance of the IWLS method), is used to scale the residuals. Thus, the argument u for Equation 4.7 becomes

$$u = \frac{r_i}{cS} ,$$

with r_i denoting the residuals, S the median of the data, and c, the scaling constant, usually taking a value of 4.2 or 6. [Ref. 11, p. 151] In addition, Denby and Martin chose to divide the median by another factor of 0.6745, for reasons not stated. We ran a simulation with and without this factor, and chose the standard approach, without the additional scaling factor.

The procedure just described is also known as robust regression. Indeed, in the International Mathematics and Statistics Library of FORTRAN subroutines (IMSL) implemented on the mainframe computer at the Naval Postgraduate School, there is a subroutine which accomplishes this regression.[6] Since the source code is available for these routines, it was incorporated into the SIMTBED simulation experiment carried out with this research. The weighting function used there is the bisquare weighting function just described.

Robust regression needs a stopping rule. A comparison of the latest estimated value of ρ with the last estimated value can be used. When there is only slight change the regression can be halted. Additionally, a maximum number of iterations can also be specified.

This robust regression approach to serial correlation is applied in this simulation to both Normal and non-Normal distributions. In their paper on robust estimation of serial correlation, Denby and Martin refer to "M-estimates" and "GM-estimates", or generalized M-estimates, depending on the choice and number of the weighting functions

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6 All IMSL routines have source code available through the LIBSOURCE utility of the VM/CMS mainframe operating system.

used [Ref. 9, p. 141]. This regression approach, which is applied to their "IO" model, is an "M-estimate", due to the choice of one weighting function, and the use of the "rescending Tukey bisquare influence function". According to Denby and Martin,

> As we show subsequently, ϕ_M is highly robust in terms of effiency for good choices of $\psi(x)$ when model IO holds, but has an asymptotic bias which can be as catastrophic as that of the least squares estimate for Model AO. [Ref. 9 , p. 141].

This simulation attempts to compare these biases, restricted to the IO model, in addition to the behavior of the other estimators described. The distributions and random processes used will be discussed below.

C. DISTRIBUTIONS USED.

Two different distributions were used in this simulation for marginal distributions of the processes. The first is the Normal distribution, with mean zero, and variance four, i.e. N(0,4), and the second is the l-Laplace distribution, with parameter l.. The processes with these marginal distributions are discussed below.

1. The Normal distribution.

Creating correlated random number streams that are marginally Normally distributed involves a simple application of the IO model described earlier. The only real consideration is the initial value to use, X_0 This process is referred to as the Normal (or Gaussian) Autoregressive process, AR(1). The IO model is

$$X_{i+1} = \rho X_i + \varepsilon_i ,$$

where ε is Normal$(0,\sigma^2)$. The value of ρ is known. For X_0, use $\varepsilon_0/(1 - \rho^2)^{\frac{1}{2}}$. If the variance of the ε random sample is σ^2, the variance of the X sample, after transformation, will be $\sigma^2/(1 - \rho^2)$.[7] So, we can thus construct a correlated stationary normal time series, and then apply all the estimators to it.

2. The l-Laplace Distribution.

A discussion of the l-Laplace family of probability distributions can begin with the widely known Laplace distribution. This distribution is known to be constructed as the difference of two Exponential (λ) random variables, and the standard Laplace random variable has mean zero, and λ equal to one. It is also known that the Exponential distribution is a member of the Gamma family of distributions, with shape parameter equal to one, and scale parameter equal λ (this discussion can be found in almost any

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7 From P.A.W. Lewis, thesis advisor.

introductory probability text) [Ref. 12, p. 274]. Thus, the standard Laplace distribution (the difference of two standard exponentials) can be obtained as the difference of two Gamma(1,1) random variable streams (the first parameter is taken as the shape, and the second as the scale). Dewald, et al, present the family of distributions defined as l-Laplace distributions as being the difference of two Gamma(l,1) random variables [Ref. 13, p. 4]. This distribution can take a variety of shapes, as shown in Figure 7.
Because of the various shapes possible by the choice of parameter l of this distribution, it was selected for the non-Normal simulation in this study.

The l-Laplace distribution is symmetric about the origin, and l can be chosen to have the shape resemble the Normal distribution (large l), or a very heavy-tailed distribution which is very non-Normal in nature (small l).

Figure 7 contains examples of empirical histograms for various generated l-Laplace samples (Closed forms for the densities are hard to compute). But, these samples are independent, and this simulation is concerned with serial correlation. We need a method to generate *correlated* l-Laplace streams.

Unfortunately, random number streams which are created by the linear first order autoregressive IO model (AR(1)) have what is called a "zero defect," meaning the value zero for the residual can occur with a positive probability. Moreover, if the parameter l is small, this effect can become large. Thus, direct application of the method described above for generating correlated Normal random streams (the "linear method") to l-Laplace time series can cause severe problems in a simulation such as this, where a lot of processing is done to the random stream. For this reason, DeWald, Lewis and McKenzie introduce the Square Root Beta Laplace transformation and time series, which is non-linear, and does not possess the "zero defect." [Ref. 13, p. 9]

We can take the expression for the Gaussian AR model (Equation 4.5), and treat the correlation parameter (ρ) as a random entity. DeWald, et al, do this, and define the l-Beta Laplace First-Order Autoregressive Process, l-BELAR(1), as

$$X_i(l) = \sqrt{\mathrm{A}_i(l\alpha, l\bar{\alpha})}\; X_{i-1}(l) + \sqrt{\mathrm{B}_i(l\bar{\alpha}, l\alpha)}\; \mathrm{L}_i(l)\,, \tag{4.8}$$

where A_i is an independent, identically distributed (i. i. d.) stream of Beta ($l\alpha$, $l\bar{\alpha}$) variables, and the B_i's are also Beta distributed, with parameters given above. L_i (l) is an i. i. d. stream of l-Laplace variables with parameter l, and $\bar{\alpha}$ equal to (1-α). [Ref. 13, p. 10]. The parameter α is chosen to fix the serial correlation. With this process, any positive correlation can be implemented, with any desired shape of the probability density. As

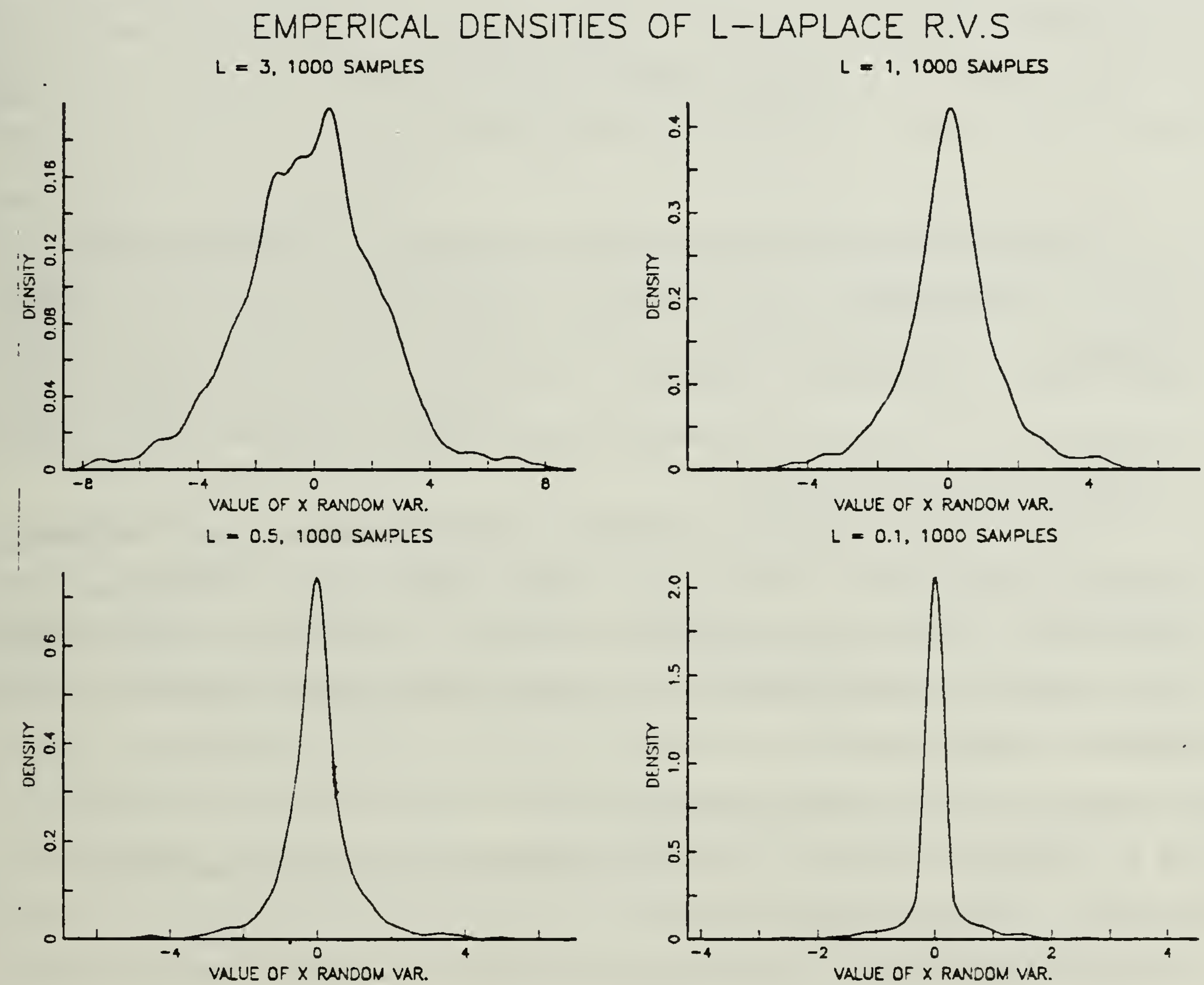


Figure 7. Various Shapes of the l-Laplace distribution

α goes from zero to one, the full range of positive correlations is covered. To see the relation between correlation ρ and α, and l, note that we can interpret this in the IO model discussed above, with the right-most term of Equation 4.8 as the innovation, and the coefficient of $X_{i-1}(l)$ as a randomly distributed correlation coefficient. This approach is taken by DeWald et al, and the correlation coefficient is given in terms of α and l, as the expected value of $\sqrt{A_i(l\alpha, l\bar{\alpha})}$ [Ref. 13 , p. 11]:

$$\rho(1) = \frac{\alpha\Gamma(l\alpha + \frac{1}{2})\Gamma(l + 1)}{\Gamma(l + \frac{1}{2})\Gamma(l\alpha + 1)} . \tag{4.9}$$

Thus, this distribution can be used to generate time series with various correlations, and varying degrees of departure from the familiar Normal-shaped distribution, allowing us to investigate the behavior of the statistical estimators of serial correlation under different distributional conditions. Note also that the $L_i(l)$ random stream in Equation 4.8 is an l-Laplace distributed variable, which is computed as the difference of two Gamma random variables.

Figure 8 shows the behavior of several correlated random number streams generated using Equation 4.9. Note that the streams cycle, or consist of long runs of very small values, followed by peak values, and that this correlation can be the same for widely different distributional shapes, given by l (α is then chosen to acquire the desired correlation) [Ref. 13, p. 12].

D. CONDUCT OF THE EXPERIMENT.

It was desired to study the estimators under three conditions of correlation:

- High positive correlation (ρ = 0.9).
- Low correlation/near independence (ρ = 0.0).
- Negative correlation (ρ = -0.9).

Due to the complex form of Equation 4.9, it is not possible to get an exact inverse relationship between α and ρ for the non-Normal distribution. Therefore, ρ approximating the above values was used.

We studied five marginal distributional shapes (including the Normal distributon):

- Normally distributed.
- l = 3, nearly Normal in shape.
- l = 1, departing from Normal behavior.

- l = .4, definitely non-Normal.
- l=.1, extremely non-Normal.

Further, for the Normal distribution study, we wished to investigate the convergence of the estimators, so a run consisting of up to 23 super-replications of the simulation was conducted to demonstrated this. The speed of Normal random number generators is such that this is possible in a reasonable time. With the Gamma and Beta generators, this type of extended simulation will take an extremely long time, depending on the choice of l and α (a few super-replications can easily take several days, so this type of run could take weeks on a PC).

The sample sizes for the experiment ranged from 20 to 500, with eight being used in all runs (see the results figures in the next chapter). Also, we performed a run with the robust regression estimator using eight sample sizes, the largest sample size being 5000, to observe the bias of this estimator.

Since each run of SIMTBED can accomodate eight sub-sample sizes, these can all be combined in a run. SIMTBED can also take up to five estimators. Four will be used in this experiment, one for each estimator. Thus one run of SIMTBED is required for each distribution and for each correlation under study, giving 15 runs of SIMTBED as a base. Of course, debugging and small-sample checkout of the simulations must be done. This number of runs is a good size project, but not unmanageable. It compares very differently with the example in Chapter 2, without using a simulation package such as SIMTBED. The actual number of runs needed to conduct the experiment is much more than 15 (about 30), since all the programming related factors must be debugged (random number generators, output files, array sizes, etc). The results of the experiment are discussed in the next chapter. The amount of output obtained in an experiment like this is extremely large, even using an analysis package like SIMTBED. Approximately 500 pages of graphs, examples of which follow, were generated by this experiment.

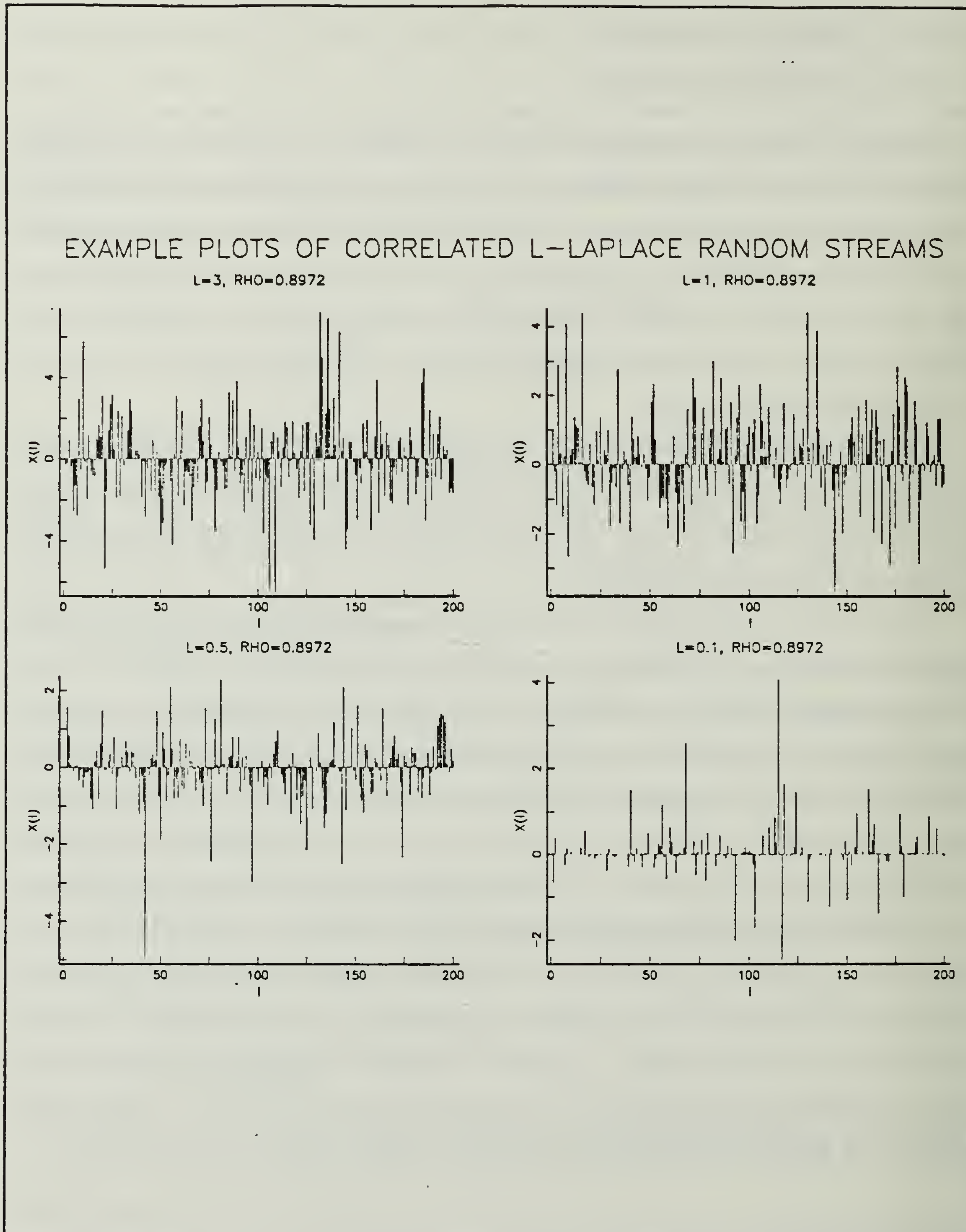


Figure 8. Example Sample Paths for the BELAR(1) Process

V. SIMULATION RESULTS

A. NORMALLY DISTRIBUTED SAMPLES.

Consider first the correlated Normal samples ($\rho = 0.9$). Figure 9 shows the color combined estimator plot for each estimator (for three selected subsample sizes), as is shown in the legend at the bottom of the graph. With this plot added to SIMTBED, immediate comparison of the estimators is possible, and from this graph many things about the behavior of these estimators are evident.

First, all four estimators seem to approach the same value, i.e. the true value $\rho = 0.9$. This is evident from the asymptote lines, which are all plotted at the same location. Further, the tabled data in the plot, as well as the other plots in the following figures, support this observation. So, these estimators can be seen to be asymptotically unbiased, for this sample distribution (Normal).

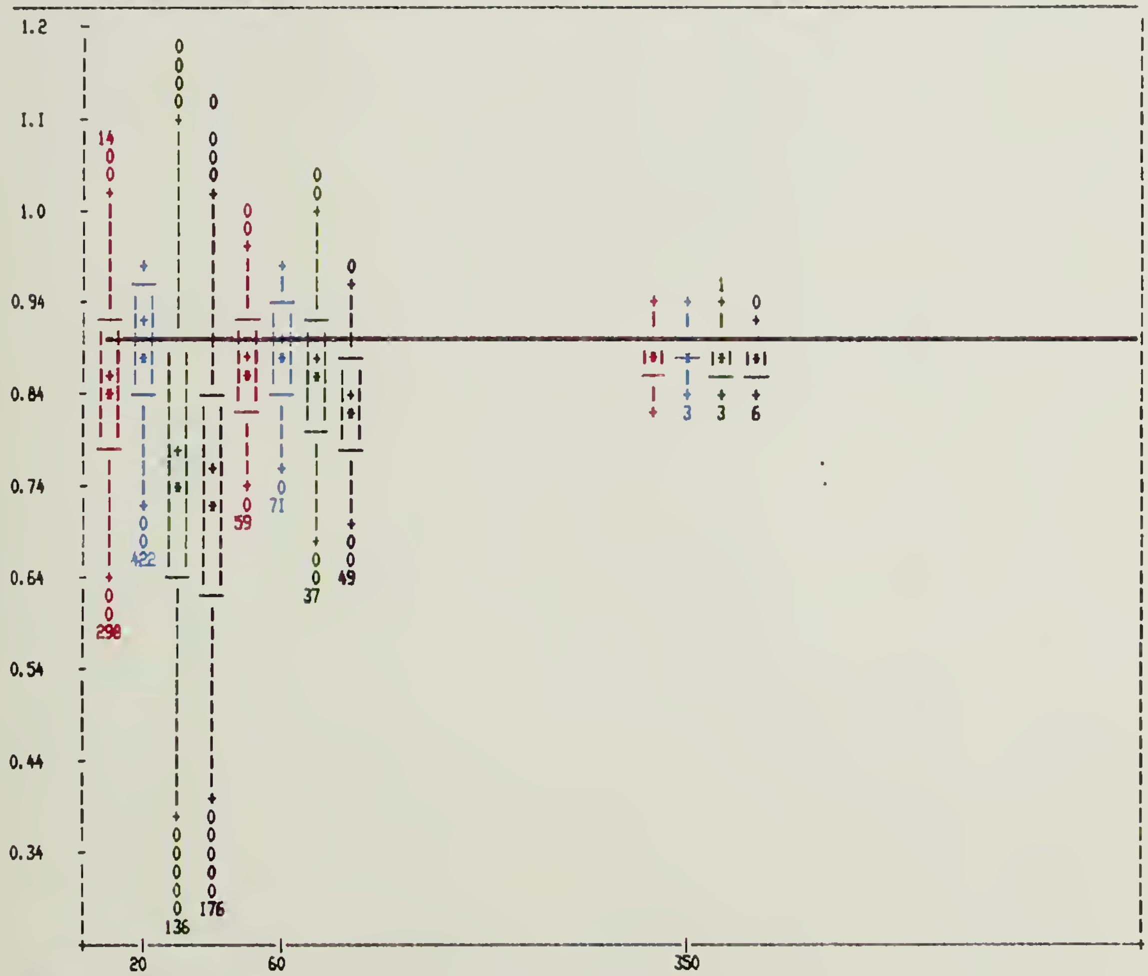
Next, consider the small-sample distribution of the estimators. It can be seen that the Cressie estimator appears to have the best behavior, in terms of bias at sample sizes 20 through 60, although there are more negative outliers in the plots for the Cressie estimator than for the others. As an example calculation, we can show how many evaluations of each estimator are represented by a particular boxplot. Take sample size 20 for example. The number of evaluations of each of the four estimators represented by the boxplot is:

$$\left(\frac{N \times M}{\text{sub-sample size}} = \frac{6000 \times 20}{20} \right) = 6000 \text{ evaluations}$$

Similar calculations can be done for any sub-sample size in any SIMTBED simulation.

Referring to the tabulated data in Figure 9, we can see the means of all the estimators approaching the known value of 0.9, for all four estimators. The standard deviations can all be seen to decrease as the convergance proceeds with increasing sample size. Note that this is only a brief reproduction of the tabulated data placed in the plots for the individual simulations, shown in the later figures. This plot is designed to accomodate a maximum of five estimators, so brevity is dictated here. Also note that each estimator in the "cluster" of boxplots for each sub-sample size has its description in matching color, but that they are doubly coded by position, so this plot could be used even if color printers were not available.

SMTB COMBINED ESTIMATOR BOXPLOTS: SAMPLE SIZE: 6000 NO. REPLICATIONS: 20 DEG REGRESSION: 4



		20	60	350
SUBSAMPLE SIZE:		20	60	350
ESTIMATOR 1:	MEAN:	0.8469	0.8740	0.8935
	STD:	0.1327	0.7156E-01	0.2711E-01
	STD MEAN:	0.1713E-02	0.1600E-02	0.1470E-02
SUBSAMPLE SIZE:		20	60	350
ESTIMATOR 2:	MEAN:	0.8827	0.8918	0.8980
	STD:	0.1352	0.7054E-01	0.2535E-01
	STD MEAN:	0.1745E-02	0.1577E-02	0.1375E-02
SUBSAMPLE SIZE:		20	60	350
ESTIMATOR 3:	MEAN:	0.7597	0.8696	0.8967
	STD:	0.2038	0.9140E-01	0.2679E-01
	STD MEAN:	0.2631E-02	0.2044E-02	0.1453E-02
SUBSAMPLE SIZE:		20	60	350
ESTIMATOR 4:	MEAN:	0.7277	0.8404	0.8891
	STD:	0.1849	0.7638E-01	0.2434E-01
	STD MEAN:	0.2388E-02	0.1708E-02	0.1320E-02

ESTIMATOR 1: **Rho of N(0,4) r.v.s, rho=0.9 using robust Least Squares estimator
ESTIMATOR 2: **Rho of N(0,4) r.v.s, rho=0.9 using Cressie estimator
ESTIMATOR 3: **Rho of N(0,4) r.v.s, rho=0.9 using moment estimator
ESTIMATOR 4: **Rho of N(0,4) r.v.s, rho=0.9 using Priestly Estimator

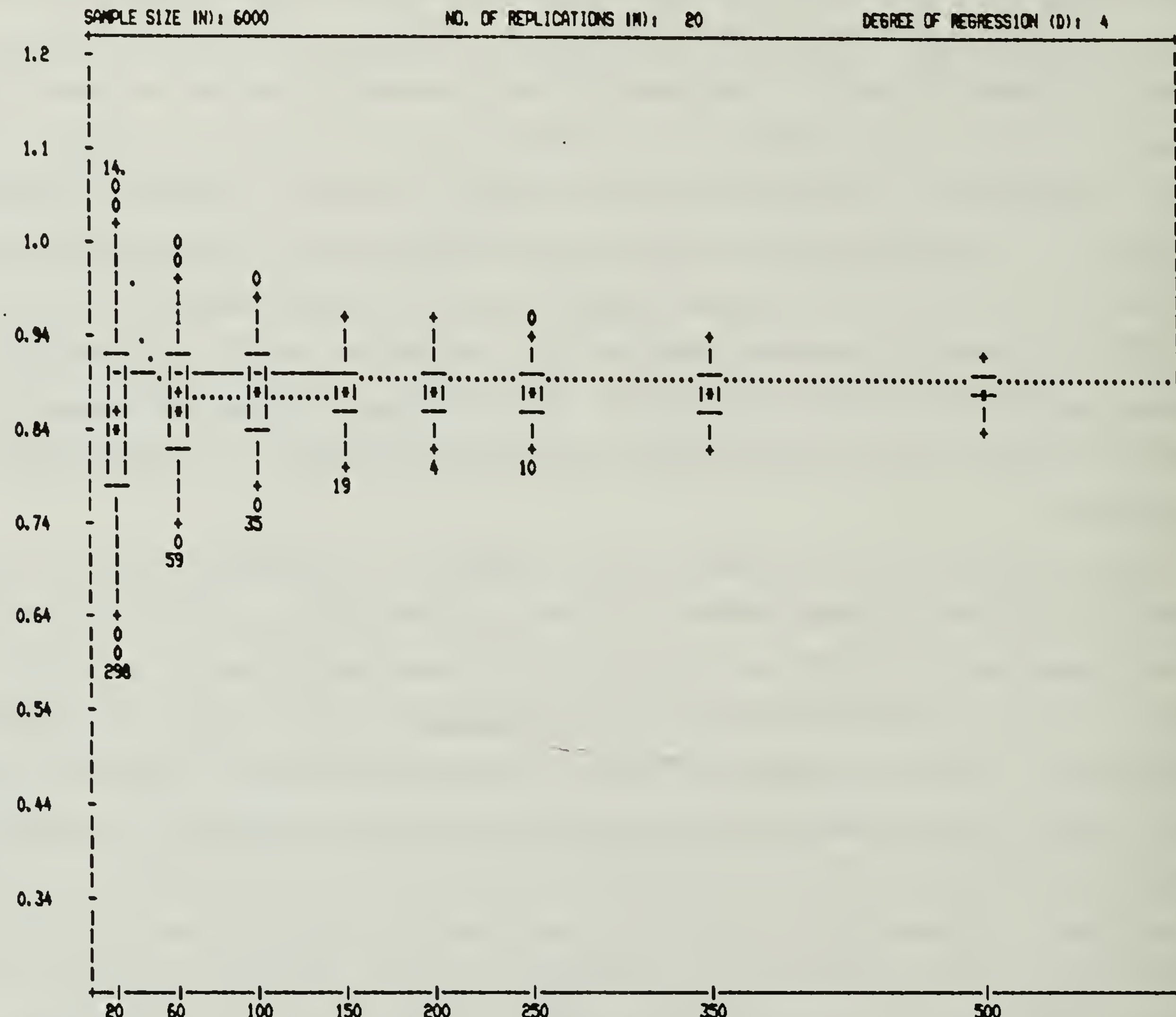
*** WIDEST Y VALUES FOUND: YMIN=-.7296 - YMAX= 1.362 *** SMTB ***
*** THIS WAS A RESTART RUN. NSR START/END= 1 8

Figure 9. Combined Estimator Plot, Normal, Correlated Samples

Figures 10 and 11 are examples of the individual plots produced as each estimator is processed by SIMTBED. These figures contain the same boxplots that are used to prepare the combined estimator plot of Figure 9. All eight sub-sample size plots are present in Figures 10 and 11. The means of each sub-sample size can be seen to approach the known true correlation of 0.9, and the regression asymptote is listed in the 'mean of regression on averages' listing of the regression equation, which was discussed in Chapter III. Finally, the mean squared error (MSE) is listed in these plots for each sub-sample size. This shows the small-sample bias, and as the sub-sample sizes increase, the MSE approaches the standard deviation of the estimators, which is evidence of decreasing bias from the known value of 0.9. There are plots produced for moment and Priestley estimators which are not included here; they are similar to the those which are included here.

Further insight into the behavior of the estimators is provided from the super-replications summary statistics tables, which are provided in Figures 12, 13, 14, and 15. In the simulation, SIMTBED has duplicated the process which generated the plots of Figures 10 and 11 a total of 23 times, and has 23 values of each statistic presented in the summary table. This was possible because of efficient generation of Normally distributed variables, and this degree of super-replication cannot be efficiently performed for complicated distributions, like the l-BELAR(1) process, due to time requirements, as will be seen later. In these figures, though, we can observe a very good degree of convergence to the known values, for all four estimators, as the sub-sample size increases. The MSE also approaches very closely the standard deviation for all estimators, and the variability of all the statistics in the summary table can be seen to be small. Of interest perhaps is the standard deviation of the fourth moment, the kurtosis, which is the largest of the moment statistics. This is indicative of the higher variability of the higher central moments. We can conclude from this part of the simulation that all four estimators are asymptotically unbiased with respect to sample size. We can further observe that the Cressie estimator appears to be the best behaved in terms of small-sample bias. These results apply only to Normally distributed samples, of course.

The results just discussed apply to only one aspect of the analysis for the Normally distributed samples: those with strong positive correlation. Figures 16 and 17 depict the color combined estimator plots for independent (ρ = 0.0) and negatively correlated (ρ = -0.9) sample distributions. It is interesting to observe that, in terms of small-sample bias, the uncorrelated Normal samples have the lowest bias, versus the strongly correlated samples. Due to the indicated asymptotic convergance (with respect to

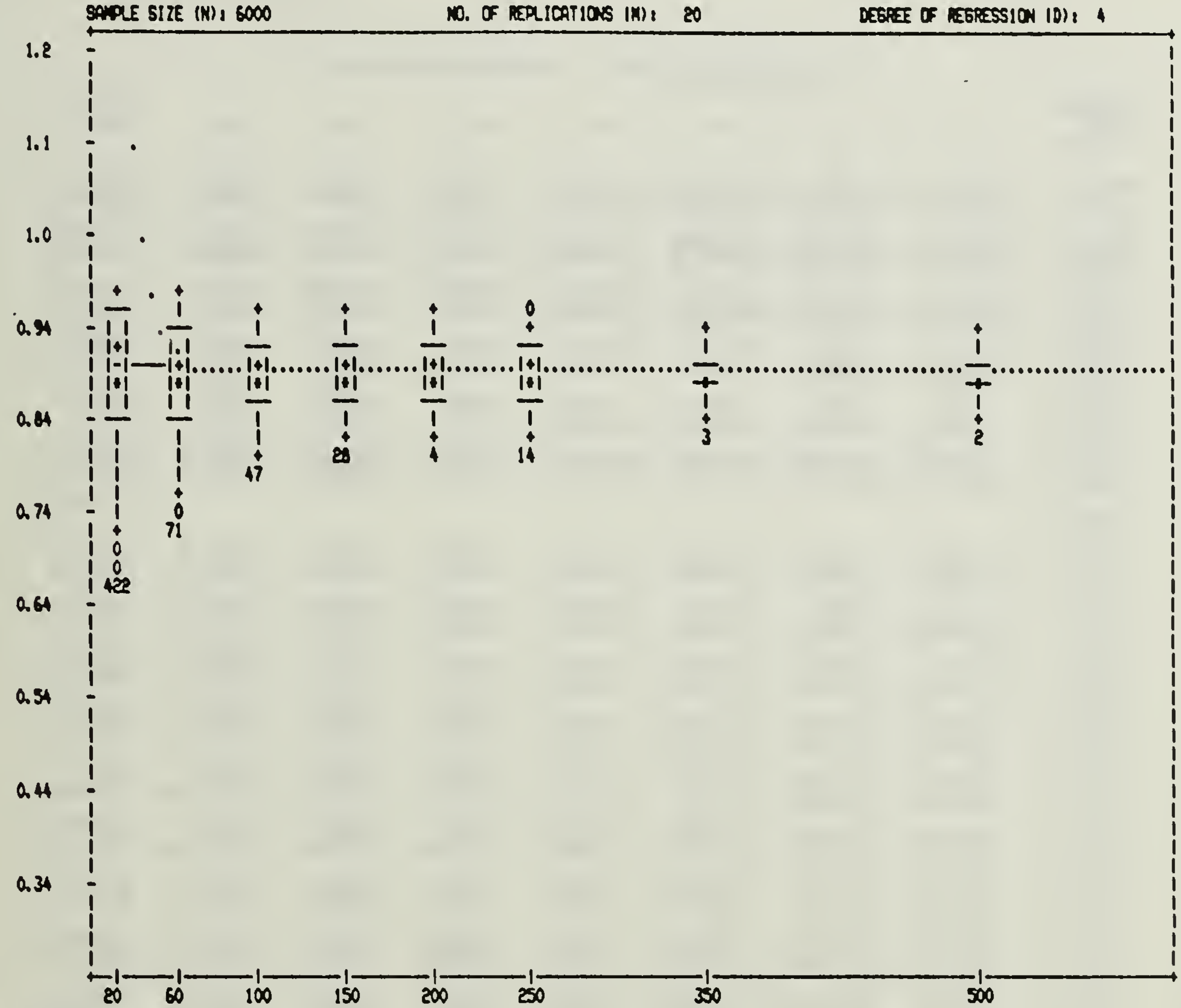


SUBSAMPLE SIZE	20	60	100	150	200	250	350	500
MEAN	0.8469	0.8740	0.8841	0.8906	0.8928	0.8938	0.8935	0.8958
STD	0.1327	0.7156E-01	0.5415E-01	0.4099E-01	0.3304E-01	0.3354E-01	0.2711E-01	0.2170E-01
STD MEAN	0.1713E-02	0.1600E-02	0.1563E-02	0.1449E-02	0.1349E-02	0.1531E-02	0.1470E-02	0.1401E-02
M.S.E	0.1429	0.7615E-01	0.5644E-01	0.4206E-01	0.3382E-01	0.3411E-01	0.2788E-01	0.2211E-01
SKEWNESS	-1.4704	-1.1124	-1.0700	-0.8397	-0.4841	-0.9106	-0.4450	-0.7112
KURTOSIS	3.4869	2.0289	2.2005	0.8986	-0.0849	1.8315	0.1889	0.4165

MEAN OF REGRESSION ON AVERAGES	0.8946	0.7556	-291.8	0.1291E+05	5333.
VARIANCE OF REGRESSION	0.1067E-04	2.998	0.7196E+05	0.1888E+09	0.3882E+08
STD DEV OF REGRESSION	0.3266E-02	1.731	268.3	0.1374E+05	6231.
REGRESSION ON VARIANCE	0.6054	-15.87	235.2	-1350.	2653.

ESTIMATOR: **Rho of N(0,4) r.v.s, rho=0.9 using robust Least Squares estimator
VERTICAL SCALE: YMIN = 0.2592
YMAX = 1.2417

Figure 10. Normal, Correlated Samples. Robust Least Squares Estimator.



SUBSAMPLE SIZE	20	60	100	150	200	250	350	500
MEAN	0.8827	0.8918	0.8954	0.9001	0.8996	0.8993	0.8980	0.8985
STD	0.1352	0.7054E-01	0.5330E-01	0.4027E-01	0.3317E-01	0.3457E-01	0.2535E-01	0.2246E-01
STD MEAN	0.1745E-02	0.1577E-02	0.1539E-02	0.1424E-02	0.1354E-02	0.1578E-02	0.1375E-02	0.1450E-02
M.S.E	0.1362	0.7102E-01	0.5350E-01	0.4027E-01	0.3317E-01	0.3458E-01	0.2542E-01	0.2251E-01
SKEWNESS	-2.5888	-1.5373	-1.4577	-1.1001	-0.6168	-1.2788	-0.4792	-0.6104
KURTOSIS	9.4694	3.3935	3.9951	1.5088	0.0244	2.7945	0.2913	0.3392

MEAN OF REGRESSION ON AVERAGES	0.8929	3.332	-499.0	0.2225E+05	9403.
VARIANCE OF REGRESSION	0.1213E-04	3.461	0.7670E+05	0.1858E+09	0.3686E+08
STD DEV OF REGRESSION	0.3483E-02	1.860	277.0	0.1363E+05	6071.
REGRESSION ON VARIANCE	0.5423	-12.80	185.5	-1046.	2038.

ESTIMATOR: **Rho of N(0,4) r.v.s, rho=0.9 using Cressie estimator
VERTICAL SCALE: YMIN = 0.2592
YMAX = 1.2417

Figure 11. Normal, Correlated Samples. Cressie Estimator.

SUMMARY STATISTICS (MEAN/STD)   23 SUPER-REPLICATIONS

SUBSAMPLE SIZE	20	60	100	150	200	250	350	500
MEAN	0.8485 0.3737E-03	0.8766 0.2952E-03	0.8847 0.2582E-03	0.8899 0.2668E-03	0.8918 0.2573E-03	0.8932 0.2220E-03	0.8947 0.2478E-03	0.8969 0.2715E-03
STD	0.1325 0.4926E-03	0.6979E-01 0.2828E-03	0.5150E-01 0.3262E-03	0.4112E-01 0.2576E-03	0.3551E-01 0.2351E-03	0.3133E-01 0.2755E-03	0.2655E-01 0.2263E-03	0.2165E-01 0.2091E-03
M.S.E	0.1422 0.5586E-03	0.7362E-01 0.3100E-03	0.5373E-01 0.3543E-03	0.4234E-01 0.2929E-03	0.3646E-01 0.2498E-03	0.3207E-01 0.2810E-03	0.2710E-01 0.2447E-03	0.2191E-01 0.2196E-03
SKEWNESS	-1.563 0.1539E-01	-1.220 0.2793E-01	-0.9856 0.1695E-01	-0.8151 0.2223E-01	-0.7345 0.3206E-01	-0.6298 0.3140E-01	-0.5457 0.3942E-01	-0.5173 0.4518E-01
KURTOSIS	4.377 0.1100	2.655 0.2179	1.792 0.1116	1.159 0.9746E-01	0.9843 0.1596	0.6610 0.1384	0.4992 0.1433	0.3921 0.1036
SER. COR.	0.1886E-03 0.3073E-02	0.9501E-02 0.4579E-02	-0.2508E-02 0.5061E-02	-0.3114E-02 0.7737E-02	-0.6912E-02 0.8181E-02	-0.1159E-01 0.9568E-02	-0.9112E-02 0.1283E-01	-0.5283E-02 0.1643E-01
QUANTILES								
0.010	0.3847 0.3962E-02	0.6515 0.2206E-02	0.7237 0.1716E-02	0.7681 0.1880E-02	0.7866 0.2059E-02	0.8037 0.1575E-02	0.8214 0.1620E-02	0.8351 0.1592E-02
0.025	0.5047 0.2300E-02	0.7063 0.1568E-02	0.7622 0.1420E-02	0.7931 0.1007E-02	0.8108 0.1105E-02	0.8224 0.9280E-03	0.8356 0.1143E-02	0.8467 0.1402E-02
0.050	0.5929 0.1494E-02	0.7461 0.7981E-03	0.7895 0.1202E-02	0.8141 0.7738E-03	0.8271 0.6115E-03	0.8363 0.8721E-03	0.8469 0.7937E-03	0.8574 0.8383E-03
0.100	0.6808 0.1067E-02	0.7852 0.6924E-03	0.8175 0.8685E-03	0.8358 0.6729E-03	0.8446 0.5690E-03	0.8511 0.6021E-03	0.8592 0.6336E-03	0.8685 0.5012E-03
0.250	0.7949 0.5102E-03	0.8419 0.4034E-03	0.8569 0.3602E-03	0.8669 0.4186E-03	0.8710 0.3476E-03	0.8747 0.3540E-03	0.8780 0.3983E-03	0.8836 0.4444E-03
0.500	0.8773 0.2679E-03	0.8888 0.2991E-03	0.8920 0.3570E-03	0.8952 0.2526E-03	0.8960 0.3148E-03	0.8963 0.3232E-03	0.8973 0.2958E-03	0.8986 0.3117E-03
0.750	0.9324 0.3309E-03	0.9248 0.3544E-03	0.9210 0.2822E-03	0.9190 0.2907E-03	0.9172 0.3548E-03	0.9155 0.3238E-03	0.9137 0.2747E-03	0.9123 0.3036E-03
0.900	0.9760 0.3695E-03	0.9512 0.4261E-03	0.9424 0.3347E-03	0.9373 0.3227E-03	0.9335 0.3755E-03	0.9307 0.3975E-03	0.9264 0.3454E-03	0.9233 0.3300E-03
0.950	1.005 0.4111E-03	0.9653 0.4240E-03	0.9539 0.3900E-03	0.9474 0.3688E-03	0.9419 0.3765E-03	0.9386 0.5317E-03	0.9339 0.4118E-03	0.9291 0.4015E-03
0.975	1.033 0.5911E-03	0.9777 0.5575E-03	0.9635 0.6122E-03	0.9552 0.4930E-03	0.9490 0.4896E-03	0.9451 0.5696E-03	0.9396 0.5684E-03	0.9345 0.5370E-03
0.990	1.070 0.8586E-03	0.9937 0.8516E-03	0.9750 0.8519E-03	0.9651 0.6636E-03	0.9581 0.6723E-03	0.9531 0.6145E-03	0.9470 0.8324E-03	0.9394 0.6976E-03

MEAN OF REGRESSION ON AVERAGES	0.9000 0.7927E-03	-1.795 0.3863	40.05 56.07	-1239. 2775.	-511.3 1240.
STD DEV OF REGRESSION	0.4070E-02 0.1430E-03	2.159 0.8386E-01	327.2 12.70	0.1643E+05 616.7	7372. 271.6
REGRESSION ON VARIANCE	0.1364 0.7970E-01	4.403 3.592	-66.08 55.39	447.9 343.8	-989.4 719.4

ESTIMATOR: **Rho of N(0,4) r.v.s, rho=0.9 using robust Least Squares estimator

Figure 12. Summary Statistics, Normal Samples. Robust Least Squares.

SUMMARY STATISTICS (MEAN/STD)   23 SUPER-REPLICATIONS

SUBSAMPLE SIZE	20	60	100	150	200	250	350	500
MEAN	0.7075 0.6741E-03	0.8481 0.3571E-03	0.8706 0.3819E-03	0.8816 0.3483E-03	0.8855 0.2629E-03	0.8889 0.2416E-03	0.8918 0.1969E-03	0.8948 0.2774E-03
STD	0.2337 0.8122E-03	0.8784E-01 0.3766E-03	0.5942E-01 0.3788E-03	0.4476E-01 0.2470E-03	0.3846E-01 0.2568E-03	0.3338E-01 0.3317E-03	0.2759E-01 0.2654E-03	0.2231E-01 0.2206E-03
M.S.E	0.3028 0.9572E-03	0.1020 0.4361E-03	0.6630E-01 0.4717E-03	0.4848E-01 0.3107E-03	0.4110E-01 0.2962E-03	0.3518E-01 0.3414E-03	0.2878E-01 0.2917E-03	0.2294E-01 0.2306E-03
SKEWNESS	-1.796 0.2126E-01	-1.444 0.3775E-01	-1.152 0.2434E-01	-0.9486 0.2057E-01	-0.8443 0.4650E-01	-0.7748 0.2858E-01	-0.6735 0.5017E-01	-0.5085 0.3638E-01
KURTOSIS	4.755 0.1801	3.560 0.3288	2.059 0.1371	1.364 0.9237E-01	1.235 0.3144	0.9245 0.1155	0.8587 0.2380	0.3037 0.9161E-01
SER. COR.	0.4448E-02 0.1850E-02	0.1426E-02 0.3949E-02	0.1616E-02 0.5818E-02	0.2127E-02 0.7332E-02	-0.5963E-02 0.9536E-02	-0.1448E-01 0.9541E-02	-0.2790E-01 0.1175E-01	-0.7374E-02 0.1604E-01
QUANTILES								
0.010	-0.1136 0.5099E-02	0.5593 0.2846E-02	0.6828 0.2713E-02	0.7467 0.1498E-02	0.7711 0.2253E-02	0.7885 0.2346E-02	0.8135 0.2014E-02	0.8323 0.1436E-02
0.025	0.9394E-01 0.3825E-02	0.6290 0.1992E-02	0.7247 0.1459E-02	0.7749 0.8285E-03	0.7966 0.1384E-02	0.8117 0.1659E-02	0.8299 0.1141E-02	0.8445 0.104[illegible]
0.050	0.2439 0.2422E-02	0.6784 0.1245E-02	0.7583 0.1367E-02	0.7974 0.1023E-02	0.8152 0.8948E-03	0.8269 0.9572E-03	0.8420 0.8520E-03	0.8549 0.7981E-03
0.100	0.3989 0.2008E-02	0.7314 0.9676E-03	0.7918 0.1157E-02	0.8216 0.8118E-03	0.8340 0.6494E-03	0.8445 0.7001E-03	0.8550 0.6210E-03	0.8653 0.5065E-03
0.250	0.6098 0.1098E-02	0.8054 0.6190E-03	0.8391 0.4525E-03	0.8563 0.5684E-03	0.8629 0.3704E-03	0.8697 0.3849E-03	0.8753 0.3695E-03	0.8811 0.3787E-03
0.500	0.7726 0.6972E-03	0.8670 0.4453E-03	0.8814 0.4044E-03	0.8888 0.4276E-03	0.8908 0.3605E-03	0.8930 0.4140E-03	0.8948 0.2471E-03	0.8966 0.4080E-03
0.750	0.8750 0.4881E-03	0.9115 0.3260E-03	0.9135 0.3423E-03	0.9139 0.3181E-03	0.9136 0.3831E-03	0.9129 0.3103E-03	0.9116 0.2522E-03	0.9108 0.3577E-03
0.900	0.9280 0.3754E-03	0.9396 0.2678E-03	0.9358 0.3655E-03	0.9322 0.3178E-03	0.9301 0.4014E-03	0.9281 0.3459E-03	0.9247 0.3197E-03	0.9221 0.3665E-03
0.950	0.9474 0.2724E-03	0.9522 0.2924E-03	0.9461 0.2923E-03	0.9415 0.4187E-03	0.9385 0.3698E-03	0.9359 0.3764E-03	0.9319 0.4278E-03	0.9286 0.4916E-03
0.975	0.9592 0.2574E-03	0.9612 0.3461E-03	0.9539 0.3930E-03	0.9493 0.5072E-03	0.9447 0.4342E-03	0.9421 0.4716E-03	0.9373 0.5166E-03	0.9332 0.6219E-03
0.990	0.9691 0.2916E-03	0.9693 0.3034E-03	0.9616 0.4732E-03	0.9559 0.4803E-03	0.9516 0.5536E-03	0.9490 0.4735E-03	0.9436 0.5155E-03	0.9383 0.6260E-03

MEAN OF REGRESSION ON AVERAGES	0.9004 0.8709E-03	-2.902 0.4460	12.75 68.41	-2092. 3449.	-999.9 1551.
STD DEV OF REGRESSION	0.4623E-02 0.1273E-03	2.536 0.6885E-01	388.4 11.10	0.1955E+05 579.9	8783. 264.2
REGRESSION ON VARIANCE	0.8652E-01 0.1243	6.971 5.474	-104.3 82.82	766.2 507.8	-1562. 1055.

ESTIMATOR: **Rho of N(0,4) r.v.s, rho=0.9 using Cressie estimator

Figure 13. Summary Statistics, Normal Samples. Cressie Estimator.

SUMMARY STATISTICS (MEAN/STD)   23 SUPER-REPLICATIONS

SUBSAMPLE SIZE	20	60	100	150	200	250	350	500
MEAN	0.7600 0.6871E-03	0.8699 0.3106E-03	0.8854 0.3441E-03	0.8924 0.3933E-03	0.8941 0.2910E-03	0.8965 0.2337E-03	0.8974 0.2850E-03	0.8988 0.2789E-03
STD	0.1996 0.4228E-03	0.9001E-01 0.3315E-03	0.6220E-01 0.2913E-03	0.4633E-01 0.2495E-03	0.3916E-01 0.2733E-03	0.3388E-01 0.3286E-03	0.2730E-01 0.2729E-03	0.2178E-01 0.2103E-03
M.S.E	0.2438 0.6195E-03	0.9492E-01 0.3765E-03	0.6391E-01 0.3196E-03	0.4698E-01 0.2835E-03	0.3962E-01 0.2848E-03	0.3408E-01 0.3376E-03	0.2746E-01 0.2770E-03	0.2186E-01 0.2102E-03
SKEWNESS	-0.9218 0.7841E-02	-0.7961 0.2101E-01	-0.6651 0.1706E-01	-0.4969 0.1604E-01	-0.4635 0.3171E-01	-0.4242 0.2945E-01	-0.3902 0.4017E-01	-0.3111 0.3096E-01
KURTOSIS	1.098 0.3169E-01	1.289 0.9518E-01	1.050 0.7533E-01	0.6774 0.4536E-01	0.6199 0.1295	0.5758 0.6320E-01	0.5366 0.1333	0.2615 0.8273E-01
SER.COR.	0.2222E-02 0.2858E-02	0.1256E-02 0.5024E-02	0.5333E-02 0.5907E-02	0.1104E-01 0.7012E-02	0.5055E-02 0.8896E-02	-0.1765E-01 0.8031E-02	-0.2087E-01 0.1109E-01	-0.2878E-02 0.1342E-01
QUANTILES								
0.010	0.1624 0.2223E-02	0.6041 0.2602E-02	0.7054 0.2544E-02	0.7645 0.2507E-02	0.7857 0.2015E-02	0.8019 0.2059E-02	0.8221 0.2088E-02	0.8388 0.1313E-02
0.025	0.2810 0.1778E-02	0.6630 0.1558E-02	0.7435 0.1295E-02	0.7896 0.1144E-02	0.8099 0.1186E-02	0.8212 0.1651E-02	0.8383 0.1047E-02	0.8514 0.1077E-02
0.050	0.3800 0.1899E-02	0.7065 0.1218E-02	0.7743 0.1164E-02	0.8107 0.8020E-03	0.8247 0.9220E-03	0.8362 0.1109E-02	0.8498 0.8209E-03	0.8609 0.6623E-03
0.100	0.4882 0.1457E-02	0.7513 0.7031E-03	0.8047 0.8239E-03	0.8322 0.7868E-03	0.8428 0.6183E-03	0.8523 0.7813E-03	0.8616 0.5963E-03	0.8703 0.5294E-03
0.250	0.6500 0.1007E-02	0.8191 0.5297E-03	0.8492 0.4225E-03	0.8646 0.5525E-03	0.8702 0.3573E-03	0.8761 0.3436E-03	0.8803 0.3240E-03	0.8847 0.4219E-03
0.500	0.7928 0.7792E-03	0.8810 0.4412E-03	0.8917 0.4505E-03	0.8964 0.3977E-03	0.8973 0.3869E-03	0.8991 0.2846E-03	0.8992 0.3455E-03	0.9000 0.3775E-03
0.750	0.9053 0.6534E-03	0.9325 0.2772E-03	0.9282 0.4748E-03	0.9236 0.3750E-03	0.9211 0.3584E-03	0.9192 0.2496E-03	0.9162 0.4372E-03	0.9139 0.3562E-03
0.900	0.9840 0.7913E-03	0.9745 0.4338E-03	0.9585 0.5678E-03	0.9475 0.5177E-03	0.9406 0.5084E-03	0.9369 0.3682E-03	0.9300 0.4689E-03	0.9257 0.4222E-03
0.950	1.023 0.6711E-03	0.9984 0.6454E-03	0.9763 0.6317E-03	0.9620 0.6130E-03	0.9533 0.7064E-03	0.9480 0.4785E-03	0.9395 0.5989E-03	0.9321 0.4985E-03
0.975	1.054 0.7002E-03	1.017 0.7544E-03	0.9912 0.6805E-03	0.9758 0.9218E-03	0.9645 0.7890E-03	0.9580 0.8209E-03	0.9463 0.6942E-03	0.9387 0.5863E-03
0.990	1.089 0.1124E-02	1.036 0.8574E-03	1.009 0.8872E-03	0.9904 0.1180E-02	0.9772 0.1176E-02	0.9701 0.1015E-02	0.9575 0.1284E-02	0.9473 0.1005E-02

MEAN OF REGRESSION ON AVERAGES	0.9004 0.8449E-03	-0.7042 0.4236	-96.85 62.88	2082. 3105.	677.9 1383.
STD DEV OF REGRESSION	0.4654E-02 0.1130E-03	2.578 0.6555E-01	398.9 11.09	0.2018E+05 594.0	9077. 273.3
REGRESSION ON VARIANCE	0.1029 0.1070	4.343 4.793	-42.67 73.48	360.5 455.3	-870.0 953.1

ESTIMATOR: **Rho of N(0,4) r.v.s, rho=0.9 using moment estimator

Figure 14. Summary Statistics, Normal Samples. Moment Estimator.

SUMMARY STATISTICS (MEAN/STD)   23 SUPER-REPLICATIONS

SUBSAMPLE SIZE	20	60	100	150	200	250	350	500
MEAN	0.7293 0.4724E-03	0.8403 0.2820E-03	0.8641 0.2848E-03	0.8764 0.3760E-03	0.8816 0.2601E-03	0.8856 0.2363E-03	0.8894 0.1998E-03	0.8932 0.2364E-03
STD	0.1832 0.3794E-03	0.7673E-01 0.3120E-03	0.5321E-01 0.2954E-03	0.4040E-01 0.2372E-03	0.3446E-01 0.2182E-03	0.3040E-01 0.2986E-03	0.2512E-01 0.2443E-03	0.2028E-01 0.1613E-03
M.S.E	0.2504 0.5201E-03	0.9720E-01 0.3522E-03	0.6419E-01 0.3606E-03	0.4681E-01 0.3588E-03	0.3907E-01 0.2857E-03	0.3367E-01 0.3210E-03	0.2726E-01 0.2600E-03	0.2141E-01 0.1864E-03
SKEWNESS	-1.044 0.1049E-01	-1.159 0.2239E-01	-1.039 0.2095E-01	-0.8300 0.1978E-01	-0.7417 0.3845E-01	-0.6748 0.2731E-01	-0.6237 0.4198E-01	-0.4724 0.3159E-01
KURTOSIS	1.543 0.4246E-01	2.201 0.1211	1.842 0.1311	1.036 0.7692E-01	0.8613 0.2004	0.6586 0.9390E-01	0.6491 0.1485	0.2946 0.1000
SER.COR.	0.9372E-02 0.1900E-02	0.5046E-02 0.3845E-02	0.3351E-02 0.5853E-02	0.1244E-02 0.8001E-02	-0.4149E-02 0.8783E-02	-0.1315E-01 0.1035E-01	-0.1979E-01 0.1266E-01	-0.9689E-03 0.1618E-01
QUANTILES								
0.010	0.1621 0.2553E-02	0.5973 0.2081E-02	0.6995 0.2287E-02	0.7570 0.1918E-02	0.7813 0.2053E-02	0.7966 0.2244E-02	0.8160 0.2127E-02	0.8359 0.1411E-02
0.025	0.2788 0.1552E-02	0.6535 0.1444E-02	0.7357 0.1410E-02	0.7819 0.1185E-02	0.8026 0.1222E-02	0.8155 0.1458E-02	0.8333 0.9667E-03	0.8481 0.9040E-03
0.050	0.3758 0.1625E-02	0.6953 0.1226E-02	0.7647 0.9900E-03	0.8012 0.8427E-03	0.8184 0.8374E-03	0.8306 0.9789E-03	0.8447 0.6976E-03	0.8570 0.7858E-03
0.100	0.4807 0.1220E-02	0.7382 0.6890E-03	0.7939 0.7962E-03	0.8224 0.7989E-03	0.8352 0.6563E-03	0.8452 0.7085E-03	0.8561 0.5156E-03	0.8666 0.4250E-03
0.250	0.6330 0.9068E-03	0.8001 0.5574E-03	0.8354 0.3866E-03	0.8530 0.5704E-03	0.8613 0.4621E-03	0.8675 0.4380E-03	0.8739 0.3225E-03	0.8805 0.3559E-03
0.500	0.7644 0.6709E-03	0.8546 0.4112E-03	0.8727 0.3324E-03	0.8820 0.4481E-03	0.8856 0.3265E-03	0.8890 0.3506E-03	0.8921 0.2729E-03	0.8946 0.3073E-03
0.750	0.8590 0.3670E-03	0.8952 0.3021E-03	0.9019 0.3290E-03	0.9054 0.2989E-03	0.9067 0.2539E-03	0.9075 0.2907E-03	0.9078 0.3516E-03	0.9076 0.2507E-03
0.900	0.9260 0.4399E-03	0.9235 0.3016E-03	0.9236 0.3668E-03	0.9229 0.3538E-03	0.9223 0.2332E-03	0.9217 0.3295E-03	0.9192 0.3634E-03	0.9182 0.3642E-03
0.950	0.9628 0.5385E-03	0.9378 0.3188E-03	0.9347 0.3796E-03	0.9321 0.3375E-03	0.9302 0.3599E-03	0.9294 0.4063E-03	0.9260 0.2938E-03	0.9241 0.3974E-03
0.975	0.9942 0.6697E-03	0.9484 0.3677E-03	0.9429 0.3718E-03	0.9394 0.4135E-03	0.9363 0.4239E-03	0.9354 0.5702E-03	0.9314 0.4405E-03	0.9285 0.5457E-03
0.990	1.033 0.1131E-02	0.9611 0.6075E-03	0.9518 0.4491E-03	0.9474 0.4173E-03	0.9434 0.4999E-03	0.9421 0.6430E-03	0.9375 0.6760E-03	0.9344 0.6135E-03

MEAN OF REGRESSION ON AVERAGES	0.9009 0.7216E-03	-4.089 0.3700	73.38 54.64	-3551. 2688.	-1612. 1196.
STD DEV OF REGRESSION	0.4068E-02 0.1257E-03	2.230 0.7074E-01	342.8 11.24	0.1728E+05 582.0	7764. 264.6
REGRESSION ON VARIANCE	0.1247 0.8608E-01	3.396 3.871	-49.59 59.37	401.4 367.2	-888.4 766.8

ESTIMATOR: **Rho of N(0,4) r.v.s, rho=0.9 using Priestly Estimator

*** WIDEST Y VALUES FOUND: YMIN=-1.130 - YMAX= 1.368 *** SMTB ***
*** THIS WAS A RESTART RUN. NSR START/END= 16 23

Figure 15. Summary Statistics, Normal Samples. Priestley Estimator.

sample size) of these samples indicated by the individual plots, the run of 23 super-replications was not repeated. Observe that once familiarity is gained with these plots, a great deal of data can be obtained by the analyst in a very short time. It does, however, take some time to build these simulations.

B. NON-NORMALLY DISTRIBUTED SAMPLES, CORRELATED.

1. Parameter l = 3.

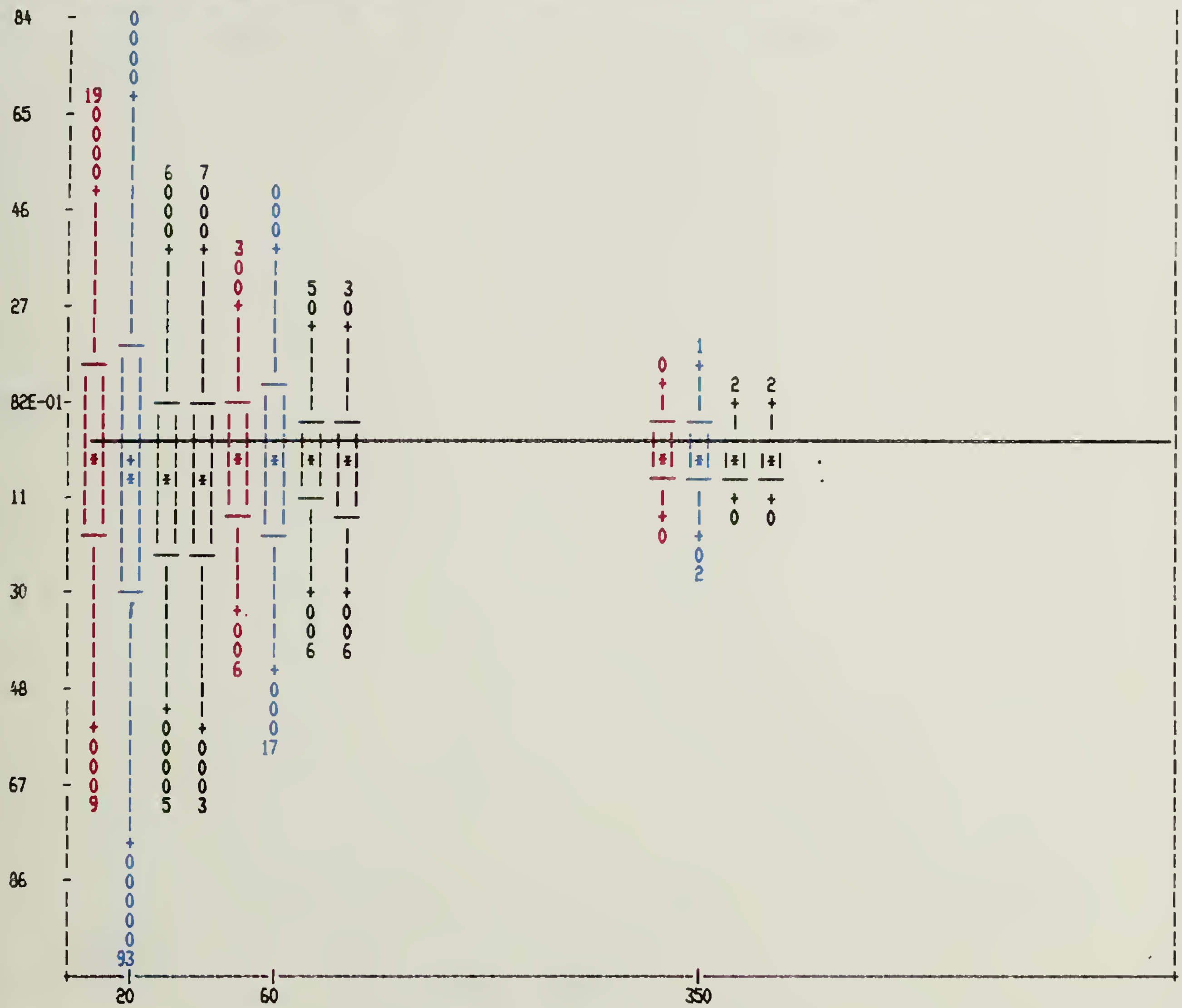
The next series of SIMTBED plots describes the behavior of the four estimators for strongly positively correlated *l*-Laplace variables, with ρ approximately 0.9. Due to the number of runs of this distribution, and the lower efficiency of the random number generators when generating Beta and Gamma distributed variables, only three super-replications were performed.

Figure 18 is a color combined estimator plot from SIMTBED of the four estimators, for sample distributed according to the BELAR(1) process, with *l* equal three. First, observe that now, all four estimators are not asymptotic to the same value. The sample distribution has begun to depart from Normality, and the estimators are beginning to exhibit corresponding changes. The moment and Priestley estimators appear to be approaching the true value of 0.8972, or very close, while the Cressie estimator has definite bias, and the robust regression estimator is showing slight asymptotic bias.

With respect to small-sample bias, the two estimators that seem to behave well asymptotically also seem to have the worst cases of small-sample bias, as indicated by the boxplots of sub-sample sizes 20 and 60.

Figure 19 is an individual estimator plot for the Cressie estimator. From this plot the asymptote of the regression can be seen to disagree with the true value, and the boxplots demonstrate the small-sample bias for this estimator. A comparison of the standard deviation with the mean squared error also shows the asymptotic bias. The standard deviation of the samples starts at 0.1839, at sub-sample size 20, and steadily decreases to 0.01520 as the sample size increases, but the MSE does not approach the standard deviation, indicating some inherent difference between the true serial correlation and the samples. The MSE decreases from .2322 to .03578 at sub-sample size 150, and then remains relatively constant with respect to sample size. This variability is thus attributed to the bias, not to the random variations of the estimator's calculations on the random samples. Further, from the regression asymptote and its standard deviation, we observe that the regression asymptote of 0.9327 is more than ten standard deviations (0.003479) removed from the true value of .8972; the standard deviation of the regression

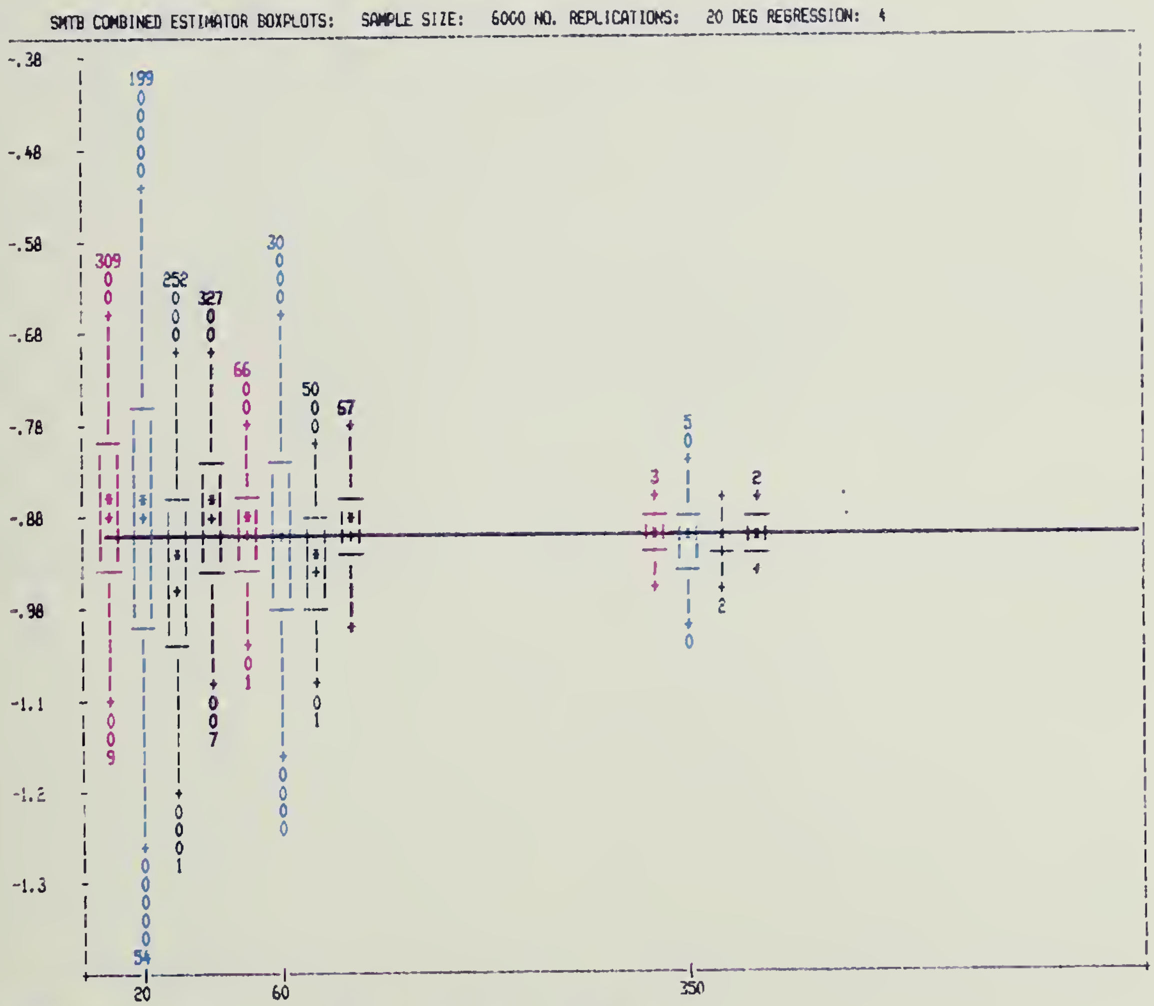
SMTB COMBINED ESTIMATOR BOXPLOTS: SAMPLE SIZE: 6000 NO. REPLICATIONS: 20 DEG REGRESSION: 4



		20	60	350
BSAMPLE SIZE:		20	60	350
TIMATOR 1:	MEAN:	-0.2815E-02	-0.4349E-02	0.1756E-02
	STD:	0.2530	0.1502	0.6348E-01
	STD MEAN:	0.3266E-02	0.3358E-02	0.3443E-02
BSAMPLE SIZE:		20	60	350
TIMATOR 2:	MEAN:	-0.5887E-01	-0.2345E-01	-0.3504E-02
	STD:	0.3686	0.2021	0.8613E-01
	STD MEAN:	0.4759E-02	0.4520E-02	0.4671E-02
BSAMPLE SIZE:		20	60	350
TIMATOR 3:	MEAN:	-0.5376E-01	-0.1855E-01	-0.1596E-02
	STD:	0.2227	0.1309	0.5304E-01
	STD MEAN:	0.2875E-02	0.2926E-02	0.2877E-02
BSAMPLE SIZE:		20	60	350
TIMATOR 4:	MEAN:	-0.5368E-01	-0.1857E-01	-0.1589E-02
	STD:	0.2234	0.1311	0.5300E-01
	STD MEAN:	0.2885E-02	0.2931E-02	0.2874E-02

TIMATOR 1: **Rho of N(0,4) r.v.s, rho=0.0 using robust Least Squares estimator
TIMATOR 2: **Rho of N(0,4) r.v.s, rho=0.0 using Cressie estimator
TIMATOR 3: **Rho of N(0,4) r.v.s, rho=0.0 using moment estimator
TIMATOR 4: **Rho of N(0,4) r.v.s, rho=0.0 using Priestly Estimator

Figure 16. Combined Plot, Normal Samples. Here, Rho = 0.0.



SUBSAMPLE SIZE:		20	60	350
ESTIMATOR 1:	MEAN:	-0.8489	-0.8781	-0.8954
	STD:	0.1337	0.6939E-01	0.2507E-01
	STD MEAN:	0.1726E-02	0.1552E-02	0.1359E-02

SUBSAMPLE SIZE:		20	60	350
ESTIMATOR 2:	MEAN:	-0.8617	-0.8840	-0.8966
	STD:	0.2268	0.1230	0.5040E-01
	STD MEAN:	0.2928E-02	0.2750E-02	0.2733E-02

SUBSAMPLE SIZE:		20	60	350
ESTIMATOR 3:	MEAN:	-0.9162	-0.9137	-0.9023
	STD:	0.1394	0.7322E-01	0.2453E-01
	STD MEAN:	0.1800E-02	0.1637E-02	0.1330E-02

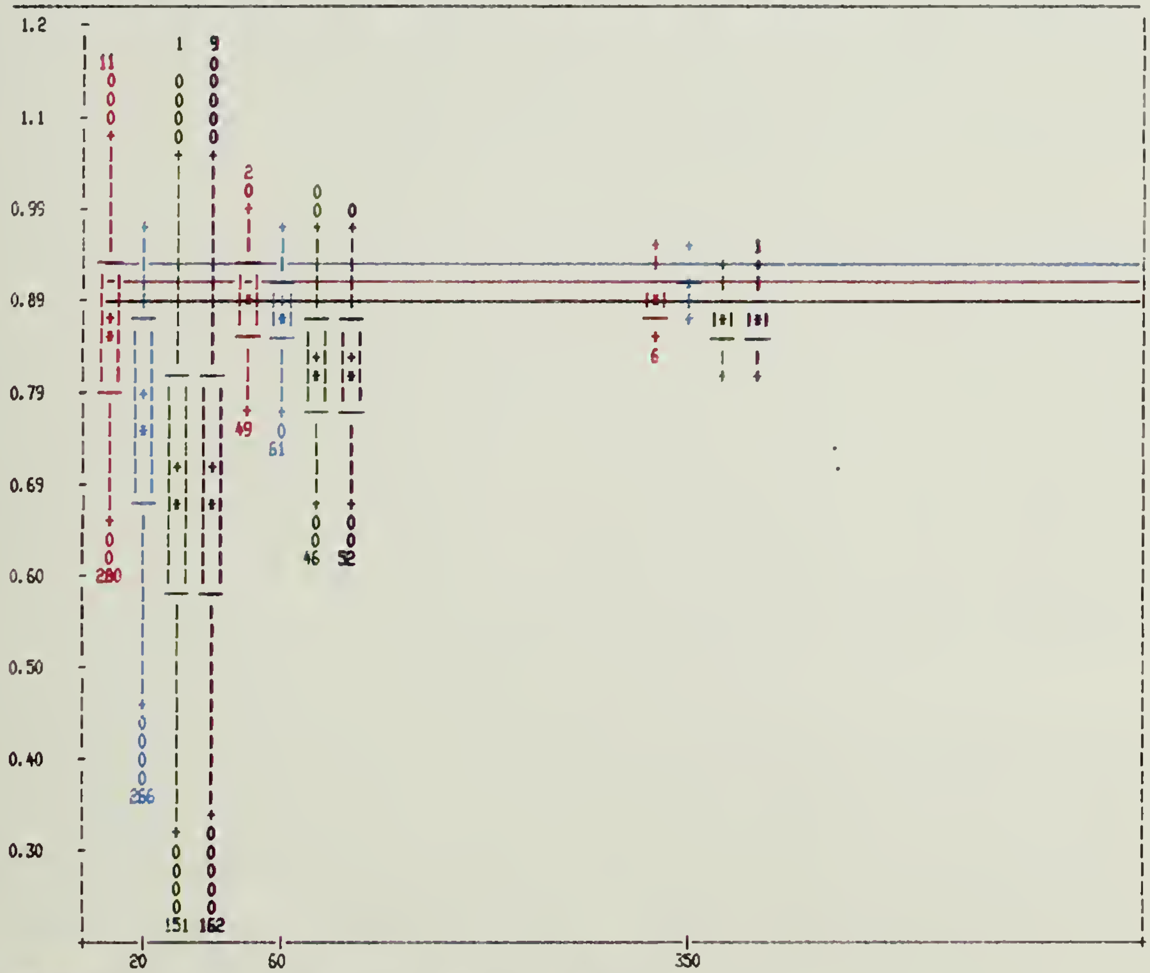
SUBSAMPLE SIZE:		20	60	350
ESTIMATOR 4:	MEAN:	-0.8544	-0.8802	-0.8945
	STD:	0.1167	0.6050E-01	0.2240E-01
	STD MEAN:	0.1506E-02	0.1353E-02	0.1215E-02

ESTIMATOR 1: **Rho of N(0,4) r.v.s, rho=-.9 using robust Least Squares estimator
ESTIMATOR 2: **Rho of N(0,4) r.v.s, rho=-.9 using Cressie estimator
ESTIMATOR 3: **Rho of N(0,4) r.v.s, rho=-.9 using moment estimator
ESTIMATOR 4: **Rho of N(0,4) r.v.s, rho=-.9 using Priestly Estimator

*** WIDEST Y VALUES FOUND: YMIN=-2.059 - YMAX=0.5137 *** SMTB ***
*** THIS WAS A RESTART RUN. NSR START/END= 1 8

Figure 17. Combined Plot, Normal Samples. Here, Rho = -0.9.

SMTB COMBINED ESTIMATOR BOXPLOTS: SAMPLE SIZE: 6000 NO. REPLICATIONS: 20 DEG REGRESSION: 4



		20	60	350
SUBSAMPLE SIZE: ESTIMATOR 1:	MEAN:	0.8639	0.8971	0.9082
	STD:	0.1340	0.5989E-01	0.2411E-01
	STD MEAN:	0.1730E-02	0.1339E-02	0.1307E-02
SUBSAMPLE SIZE: ESTIMATOR 2:	MEAN:	0.7555	0.8868	0.9270
	STD:	0.1839	0.6012E-01	0.1785E-01
	STD MEAN:	0.2374E-02	0.1344E-02	0.9679E-03
SUBSAMPLE SIZE: ESTIMATOR 3:	MEAN:	0.6857	0.8285	0.8829
	STD:	0.1944	0.8405E-01	0.2679E-01
	STD MEAN:	0.2510E-02	0.1879E-02	0.1453E-02
SUBSAMPLE SIZE: ESTIMATOR 4:	MEAN:	0.6913	0.8264	0.8844
	STD:	0.1950	0.8382E-01	0.2664E-01
	STD MEAN:	0.2518E-02	0.1874E-02	0.1445E-02

ESTIMATOR 1: +++Rho(.8972) of SqrtBeta L laplace AR r.v.s, A=.82, L= 3 Using Robust Least Squares estimator (C=4.2)
ESTIMATOR 2: +++Rho(.8972) of SqrtBeta L laplace AR r.vs., A=.82, L= 3 Using Cressie Estimator
ESTIMATOR 3: +++Rho(.8972) of SqrtBeta L laplace AR r.v.s, A=.82, L= 3 Using moment estimator
ESTIMATOR 4: +++Rho(.8972) of SqrtBeta L laplace AR r.vs., A=.82, L= 3 Using Priestly Estimator

+++ WIDEST Y VALUES FOUND: YMIN=-1.033 - YMAX= 1.769 +++ SMTB +++
+++ THIS WAS A RESTART RUN. NSR START/END= 1 3

Figure 18. Combined Plot, L-laplace samples. Here Rho = 0.8972.

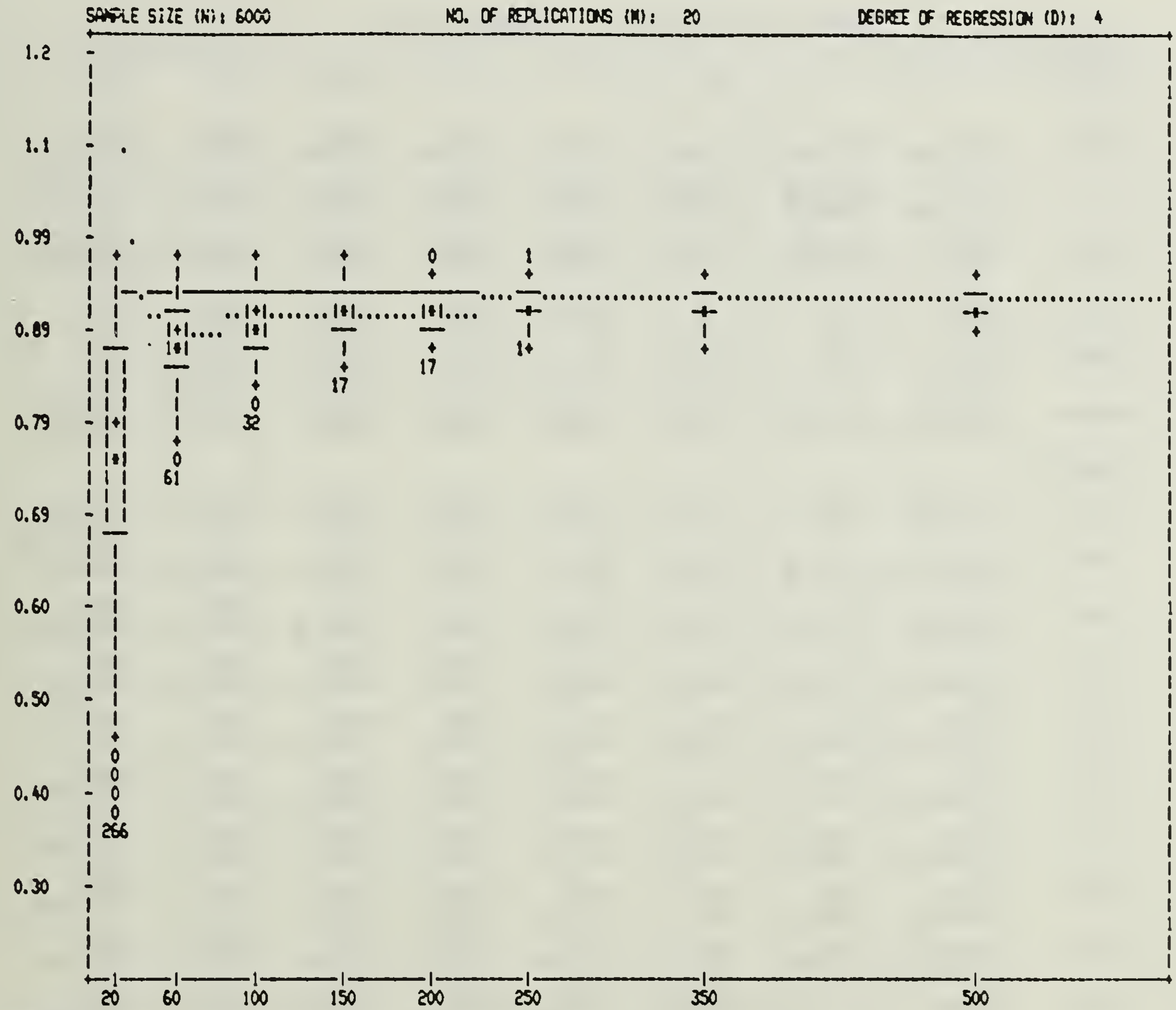
coefficients was determined from the 20 replications as part of the generation of this plot.

The summary table for all super-replications (of which there are three) involving the Cressie estimator is shown in Figure 20. With super-replications, the MSE is observed to actually increase, and does not approach the standard deviation at all. The bias of this estimator under conditions of only slightly non-Normal sample seems extreme.

Figures 21 and 22 show the similar behavior for the robust regression estimator. While not as extreme in the case of asymptotic bias (yet) as the Cressie estimator, the robust regression approach seems to approach the wrong value as sample size increases (bias). Again, the MSE decreases, then reaches some (relatively) constant value, reinforcing the case for asymptotic bias. It is noteworthy here to observe that the largest sub-sample size we use is 500, which seems like a reasonable sample size for which to get an acceptable value of serial correlation. Shortly we will observe the behavior of robust regression for even larger sub-sample sizes.

Figures 23 and 24 show the behavior of the moment estimator under these sample distribution conditions ($l = 3$, strong positive correlation). Here we see a large small-sample bias, indicated by the boxplot of sub-sample size 20 falling off from the asymptote line, as well as from the tabulated means, standard deviations, and mean squared errors. With these simulation parameters (overall sample size of 6000, and 20 replications), the regression asymptotic value of 0.8889 for the moment estimator is still accurate to two decimal places (with normal rounding). This, combined with the behavior of the MSE for this estimator, would indicate the estimator is asymptotically unbiased (although it may not approach the correct value as rapidly as would be the case under conditions of Normal samples). From Figure 24, we can see the regression asymptote after three super-replications and observe an even closer value to the true correlation of 0.8972. We conclude the moment estimator is unbiased for this sample distribution.

The simulation behavior of the Maximum Liklihood Estimator (MLE), i.e. the Priestley estimator, is depicted in figures 25 and 26, again, for the parameter l equal to three, and α chosen for strong positive correlation. By a similar analysis, we conclude this estimator is also unbiased, but has a large small-sample bias, just as did the moment estimator.



SUBSAMPLE SIZE	20	60	100	150	200	250	350	500
MEAN	0.7555	0.8868	0.9068	0.9169	0.9219	0.9259	0.9270	0.9296
STD	0.1839	0.6012E-01	0.4106E-01	0.2988E-01	0.2506E-01	0.2056E-01	0.1785E-01	0.1520E-01
STD MEAN	0.2374E-02	0.1344E-02	0.1185E-02	0.1056E-02	0.1023E-02	0.9383E-03	0.9679E-03	0.9813E-03
M.S.E	0.2322	0.6101E-01	0.4217E-01	0.3578E-01	0.3519E-01	0.3528E-01	0.3472E-01	0.3581E-01
SKEWNESS	-1.7524	-1.4788	-1.2877	-0.8721	-0.8715	-0.6517	-0.6053	-1.0158
KURTOSIS	4.4780	4.0808	2.7534	1.3120	1.0142	0.9154	0.5622	3.0215

MEAN OF REGRESSION ON AVERAGES	0.9327	-1.161	-241.5	0.1120E+05	5034.
VARIANCE OF REGRESSION	0.1211E-04	4.110	0.1120E+06	0.3051E+09	0.6306E+08
STD DEV OF REGRESSION	0.3479E-02	2.027	334.6	0.1747E+05	7941.
REGRESSION ON VARIANCE	0.4137	-14.02	216.4	-1286.	2781.

ESTIMATOR: ***Rho(.8972) of SqrtBeta L laplace AR r.vs., A=.82, L= 3 Using Cressie Estimator
VERTICAL SCALE: YMIN = 0.2176
YMAX = 1.1921

Figure 19. Individual Plot, L-Laplace samples. Cressie Estimator.

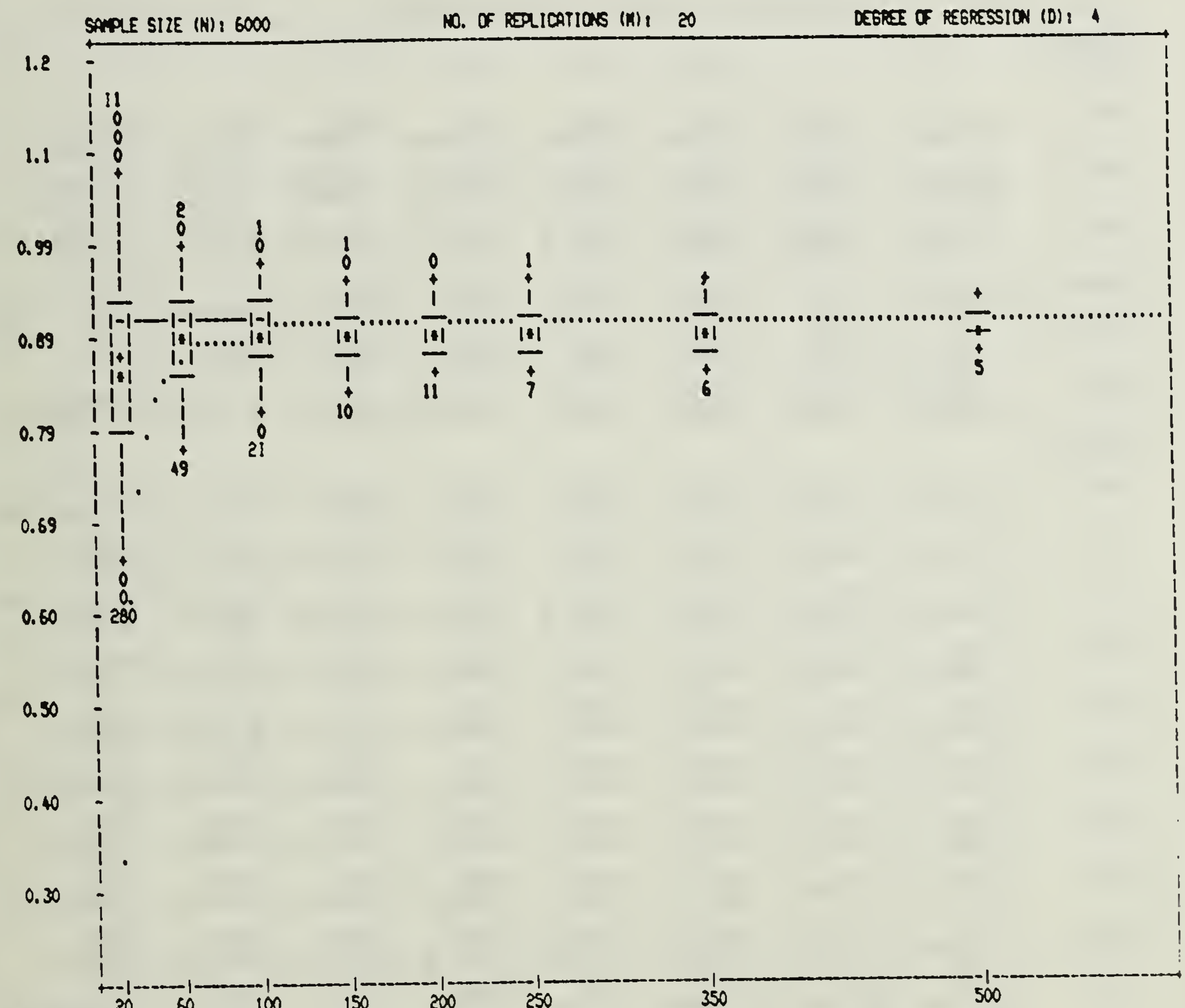
SUMMARY STATISTICS (MEAN/STD) 3 SUPER-REPLICATIONS

SUBSAMPLE SIZE	20	60	100	150	200	250	350	500
MEAN	0.7524 0.1546E-02	0.8873 0.4051E-03	0.9075 0.5019E-03	0.9163 0.3405E-03	0.9218 0.3023E-03	0.9240 0.9311E-03	0.9266 0.3429E-03	0.9299 0.4870E-03
STD	0.1911 0.3632E-02	0.6070E-01 0.6464E-03	0.4042E-01 0.3250E-03	0.3038E-01 0.7672E-03	0.2503E-01 0.3038E-04	0.2179E-01 0.6200E-03	0.1804E-01 0.3237E-03	0.1477E-01 0.4119E-03
M.S.E	0.2398 0.3818E-02	0.6151E-01 0.6699E-03	0.4173E-01 0.2689E-03	0.3591E-01 0.5178E-03	0.3510E-01 0.2329E-03	0.3461E-01 0.3784E-03	0.3451E-01 0.1345E-03	0.3584E-01 0.2908E-03
SKEWNESS	-1.952 0.1059	-1.616 0.1466	-1.180 0.5408E-01	-0.8993 0.8578E-01	-0.8168 0.6419E-01	-0.7408 0.4566E-01	-0.5865 0.1061E-01	-0.7526 0.1322
KURTOSIS	6.212 0.8925	5.401 1.646	2.357 0.2105	1.207 0.3502	0.9403 0.3114	0.7776 0.6897E-01	0.3729 0.9648E-01	1.387 0.8287
SER.COR.	-0.2655E-02 0.9546E-02	0.1576E-01 0.1496E-01	0.2193E-02 0.2121E-01	-0.2496E-01 0.9614E-02	-0.6999E-02 0.1930E-01	-0.3708E-01 0.4370E-01	0.2198E-01 0.6767E-02	0.1213E-01 0.2537E-01
QUANTILES								
0.010	0.7687E-01 0.1814E-01	0.6806 0.3056E-02	0.7779 0.4364E-02	0.8272 0.6567E-02	0.8423 0.5977E-02	0.8558 0.2763E-02	0.8733 0.3516E-02	0.8833 0.2723E-02
0.025	0.2431 0.1059E-01	0.7306 0.5015E-02	0.8118 0.2056E-02	0.8449 0.2862E-02	0.8646 0.1056E-02	0.8732 0.2729E-02	0.8871 0.1648E-02	0.8947 0.1658E-02
0.050	0.3701 0.7259E-02	0.7765 0.1132E-02	0.8304 0.2111E-03	0.8610 0.1791E-02	0.8760 0.1255E-02	0.8836 0.2462E-02	0.8940 0.1220E-02	0.9026 0.1877E-02
0.100	0.5037 0.6074E-02	0.8119 0.1325E-02	0.8538 0.7747E-03	0.8754 0.9083E-03	0.8878 0.1741E-02	0.8943 0.1785E-02	0.9016 0.9868E-03	0.9117 0.1912E-02
0.250	0.6773 0.1981E-03	0.8585 0.6534E-03	0.8868 0.1135E-02	0.8993 0.8607E-03	0.9072 0.1251E-02	0.9118 0.1655E-02	0.9155 0.7947E-03	0.9221 0.3499E-03
0.500	0.8046 0.2448E-03	0.8999 0.7477E-03	0.9148 0.8310E-04	0.9212 0.3722E-03	0.9246 0.1954E-03	0.9269 0.2354E-03	0.9289 0.1967E-03	0.9310 0.7974E-03
0.750	0.8855 0.7422E-03	0.9299 0.1082E-03	0.9362 0.5390E-03	0.9388 0.1171E-03	0.9399 0.3172E-03	0.9394 0.5282E-03	0.9400 0.2182E-03	0.9403 0.2732E-03
0.900	0.9299 0.6629E-03	0.9502 0.7287E-03	0.9517 0.6016E-03	0.9507 0.4503E-03	0.9508 0.6459E-03	0.9499 0.3020E-03	0.9480 0.2753E-03	0.9470 0.1510E-03
0.950	0.9482 0.3642E-03	0.9597 0.6708E-03	0.9597 0.8155E-03	0.9571 0.3398E-03	0.9561 0.2394E-03	0.9550 0.9684E-03	0.9529 0.3480E-03	0.9521 0.3831E-03
0.975	0.9604 0.2178E-03	0.9663 0.4831E-03	0.9652 0.1121E-02	0.9632 0.2711E-03	0.9603 0.2852E-03	0.9590 0.1244E-02	0.9561 0.6296E-03	0.9557 0.5524E-03
0.990	0.9707 0.2251E-03	0.9733 0.9364E-03	0.9717 0.1268E-02	0.9669 0.8901E-03	0.9656 0.6467E-03	0.9639 0.1816E-02	0.9614 0.3041E-03	0.9592 0.2664E-03

MEAN OF REGRESSION ON AVERAGES	0.9354 0.1516E-02	-3.002 0.9717	47.83 155.0	-3081. 7809.	-1280. 3489.
STD DEV OF REGRESSION	0.2964E-02 0.2850E-03	1.716 0.2114	278.1 40.05	0.1442E+05 2147.	6547. 978.8
REGRESSION ON VARIANCE	0.1420 0.1400	-1.998 6.245	36.17 94.89	-197.8 581.7	575.5 1197.

ESTIMATOR: ***Rho(.8972) of SqrtBeta L laplace AR r.vs., A=.82, L= 3 Using Cressie Estimator

Figure 20. Summary Statistics, L-Laplace samples. Cressie Estimator.



SUBSAMPLE SIZE	20	60	100	150	200	250	350	500
MEAN	0.8639	0.8971	0.9040	0.9068	0.9077	0.9086	0.9082	0.9116
STD	0.1340	0.5989E-01	0.4403E-01	0.3297E-01	0.2959E-01	0.2773E-01	0.2411E-01	0.1837E-01
STD MEAN	0.1730E-02	0.1339E-02	0.1271E-02	0.1166E-02	0.1208E-02	0.1266E-02	0.1307E-02	0.1186E-02
M.S.E	0.1381	0.5989E-01	0.4456E-01	0.3434E-01	0.3140E-01	0.2998E-01	0.2652E-01	0.2332E-01
SKEWNESS	-1.8089	-1.1840	-0.7870	-0.7609	-0.6988	-0.6105	-0.5412	-0.5822
KURTOSIS	6.7212	3.7834	1.0934	2.3463	1.3290	0.8887	0.9548	0.4148

MEAN OF REGRESSION ON AVERAGES	0.9143	-2.146	212.2	-0.1120E+05	-5090.
VARIANCE OF REGRESSION	0.8933E-05	2.465	0.6185E+05	0.1676E+09	0.3478E+08
STD DEV OF REGRESSION	0.2989E-02	1.570	248.7	0.1295E+05	5897.
REGRESSION ON VARIANCE	0.2016	0.2733	-22.29	237.7	-578.7

ESTIMATOR: ***Rho(.8972) of SqrtBeta L laplace AR r.v.s, A=.82, L= 3 Using Robust Least Squares estimator (C=4.2)
VERTICAL SCALE: YMIN = 0.2176
YMAX = 1.1921

Figure 21. Individual Plot, L-Laplace samples. Robust Least Squares

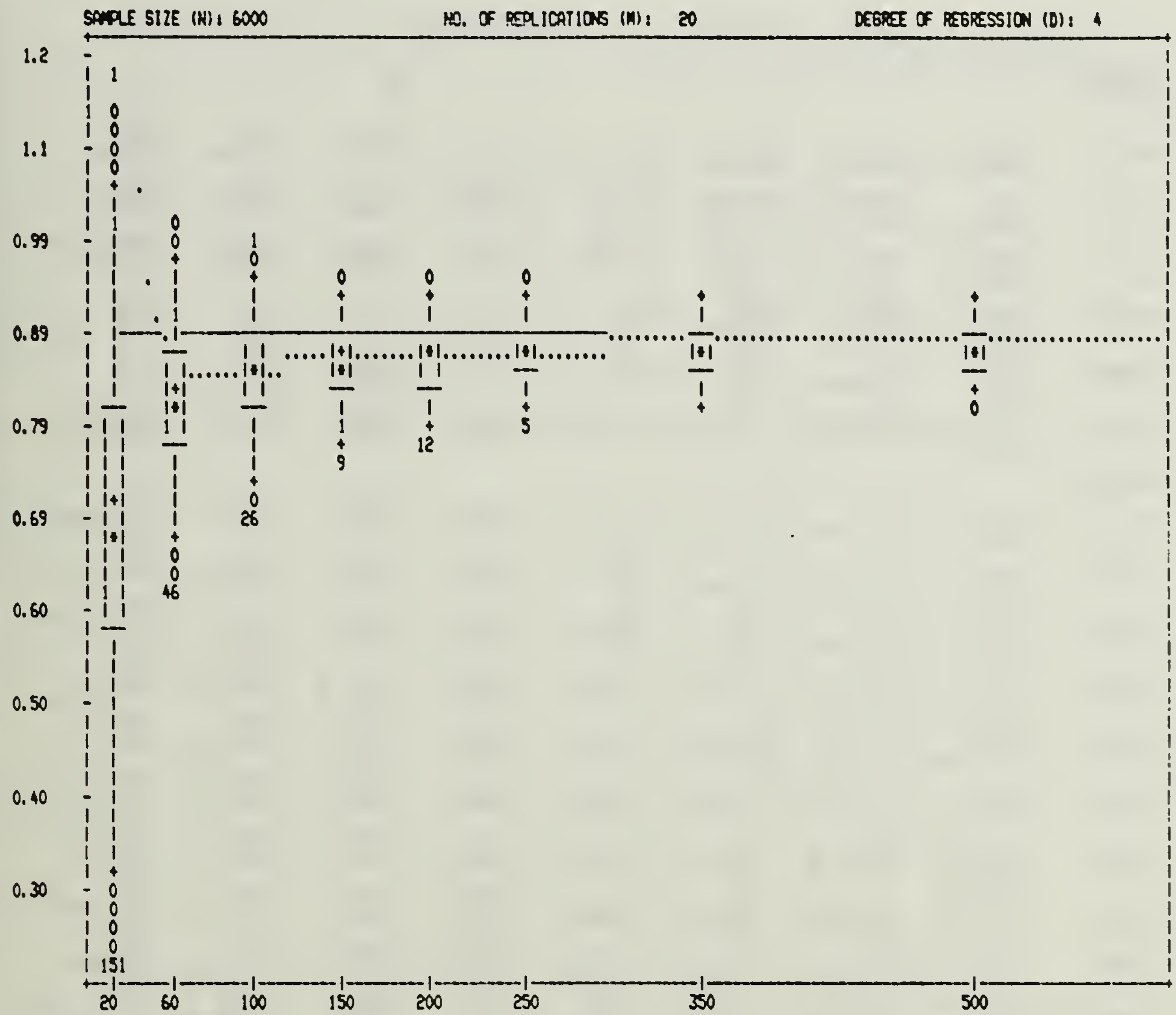
SUMMARY STATISTICS (MEAN/STD) 3 SUPER-REPLICATIONS

SUBSAMPLE SIZE	20	60	100	150	200	250	350	500
MEAN	0.8662 0.1292E-02	0.8978 0.5263E-03	0.9033 0.4428E-03	0.9077 0.7325E-03	0.9087 0.1299E-02	0.9093 0.9952E-03	0.9088 0.5672E-03	0.9100 0.8075E-03
STD	0.1305 0.1908E-02	0.6008E-01 0.1090E-02	0.4418E-01 0.1840E-03	0.3387E-01 0.5096E-03	0.2926E-01 0.3070E-03	0.2633E-01 0.7289E-03	0.2280E-01 0.6685E-03	0.1843E-01 0.9430E-03
M.S.E	0.1342 0.2054E-02	0.6009E-01 0.1084E-02	0.4461E-01 0.1355E-03	0.3546E-01 0.6932E-03	0.3150E-01 0.2215E-03	0.2901E-01 0.5359E-03	0.2561E-01 0.4665E-03	0.2244E-01 0.9031E-03
SKEWNESS	-1.718 0.5259E-01	-1.050 0.1236	-0.8212 0.2521E-01	-0.5970 0.8362E-01	-0.6678 0.1812E-01	-0.5663 0.2234E-01	-0.4949 0.2328E-01	-0.6070 0.1141
KURTOSIS	6.041 0.3734	2.497 0.8233	1.424 0.1929	1.106 0.6200	0.9466 0.2129	0.6101 0.1893	0.5894 0.1874	0.8156 0.3188
SER.COR.	-0.6327E-02 0.4302E-02	0.1028E-02 0.1456E-01	-0.1936E-01 0.1129E-01	0.2655E-01 0.2540E-01	-0.4921E-02 0.2317E-01	-0.1304E-01 0.2877E-01	-0.4848E-01 0.7375E-02	-0.8095E-01 0.4174E-01
QUANTILES								
0.010	0.4119 0.1300E-01	0.7087 0.9724E-02	0.7703 0.6874E-02	0.8073 0.2449E-02	0.8260 0.3988E-02	0.8310 0.2569E-02	0.8419 0.6051E-02	0.8542 0.6061E-02
0.025	0.5322 0.4240E-02	0.7521 0.4486E-02	0.8027 0.1326E-02	0.8308 0.6770E-03	0.8420 0.2107E-02	0.8501 0.1706E-02	0.8587 0.1186E-02	0.8686 0.2946E-02
0.050	0.6204 0.5216E-02	0.7875 0.1551E-02	0.8224 0.2062E-02	0.8493 0.2583E-02	0.8545 0.1639E-02	0.8638 0.2088E-02	0.8688 0.6406E-03	0.8761 0.3053E-02
0.100	0.7087 0.4647E-02	0.8220 0.1252E-02	0.8463 0.4762E-03	0.8642 0.1448E-02	0.8702 0.1004E-02	0.8754 0.2981E-02	0.8796 0.1762E-02	0.8863 0.1620E-02
0.250	0.8133 0.2222E-02	0.8653 0.4921E-03	0.8783 0.5644E-03	0.8865 0.5780E-03	0.8912 0.2077E-02	0.8933 0.1129E-02	0.8947 0.1478E-02	0.8991 0.8768E-03
0.500	0.8914 0.1199E-02	0.9071 0.9443E-03	0.9094 0.6588E-03	0.9106 0.1317E-02	0.9114 0.1180E-02	0.9114 0.9327E-03	0.9105 0.9443E-03	0.9110 0.9037E-03
0.750	0.9490 0.8401E-03	0.9386 0.6156E-03	0.9335 0.8418E-03	0.9311 0.1352E-02	0.9289 0.8576E-03	0.9280 0.4546E-03	0.9244 0.4008E-03	0.9230 0.1434E-02
0.900	0.9928 0.4341E-03	0.9641 0.1984E-03	0.9534 0.3329E-03	0.9475 0.1420E-02	0.9434 0.9452E-03	0.9411 0.1002E-02	0.9366 0.5967E-03	0.9321 0.1123E-02
0.950	1.019 0.4790E-04	0.9785 0.1943E-03	0.9647 0.2976E-03	0.9586 0.1317E-02	0.9521 0.1057E-02	0.9489 0.1775E-02	0.9442 0.1021E-02	0.9380 0.1199E-02
0.975	1.047 0.5747E-03	0.9914 0.2168E-03	0.9746 0.1079E-03	0.9659 0.1762E-02	0.9582 0.1351E-02	0.9540 0.1541E-02	0.9493 0.2039E-02	0.9412 0.1816E-02
0.990	1.081 0.2228E-02	1.005 0.4448E-03	0.9852 0.1892E-02	0.9746 0.2358E-02	0.9659 0.2136E-03	0.9594 0.1476E-02	0.9570 0.1839E-02	0.9465 0.1331E-02

MEAN OF REGRESSION ON AVERAGES	0.9080 0.3606E-02	1.238 2.097	-281.7 301.6	0.1293E+05 0.1440E+05	5584. 6301.
STD DEV OF REGRESSION	0.3379E-02 0.2371E-03	1.775 0.1592	271.0 25.08	0.1374E+05 1207.	6189. 527.7
REGRESSION ON VARIANCE	0.3316 0.1480	-6.580 5.923	90.36 82.03	-476.7 464.5	916.9 908.5

ESTIMATOR: ***Rho(.8972) of SqrtBeta L laplace AR r.v.s, A=.82, L= 3 Using Robust Least Squares estimator (C=4.2)

Figure 22. Summary Statistics, L-Laplace samples. Robust Least Squares



SUBSAMPLE SIZE	20	60	100	150	200	250	350	500
MEAN	0.6857	0.8285	0.8541	0.8697	0.8769	0.8844	0.8829	0.8880
STD	0.1944	0.8405E-01	0.6040E-01	0.4341E-01	0.3936E-01	0.3080E-01	0.2679E-01	0.2290E-01
STD MEAN	0.2510E-02	0.1879E-02	0.1743E-02	0.1535E-02	0.1607E-02	0.1406E-02	0.1453E-02	0.1478E-02
M.S.E	0.2873	0.1086	0.7422E-01	0.5136E-01	0.4427E-01	0.3337E-01	0.3035E-01	0.2468E-01
SKEWNESS	-0.8870	-0.9672	-0.9224	-0.5846	-0.6873	-0.3753	-0.4695	-0.2174
KURTOSIS	0.9891	1.4398	1.6753	0.3557	0.7104	0.1048	0.5516	-0.2563

MEAN OF REGRESSION ON AVERAGES	0.8889	0.4267	-693.6	0.3450E+05	0.1541E+05
VARIANCE OF REGRESSION	0.2059E-04	4.805	0.1060E+06	0.2668E+09	0.5391E+08
STD DEV OF REGRESSION	0.4538E-02	2.192	325.6	0.1634E+05	7342.
REGRESSION ON VARIANCE	0.7956	-25.45	396.4	-2330.	4731.

ESTIMATOR: ***Rho(.8972) of SqrtBeta L laplace AR r.v.s, A=.82, L= 3 Using moment estimator
VERTICAL SCALE: YMIN = 0.2176
YMAX = 1.1921

Figure 23. Individual Plot, L-Laplace samples. Moment Estimator.

SUMMARY STATISTICS (MEAN/STD)   3 SUPER-REPLICATIONS

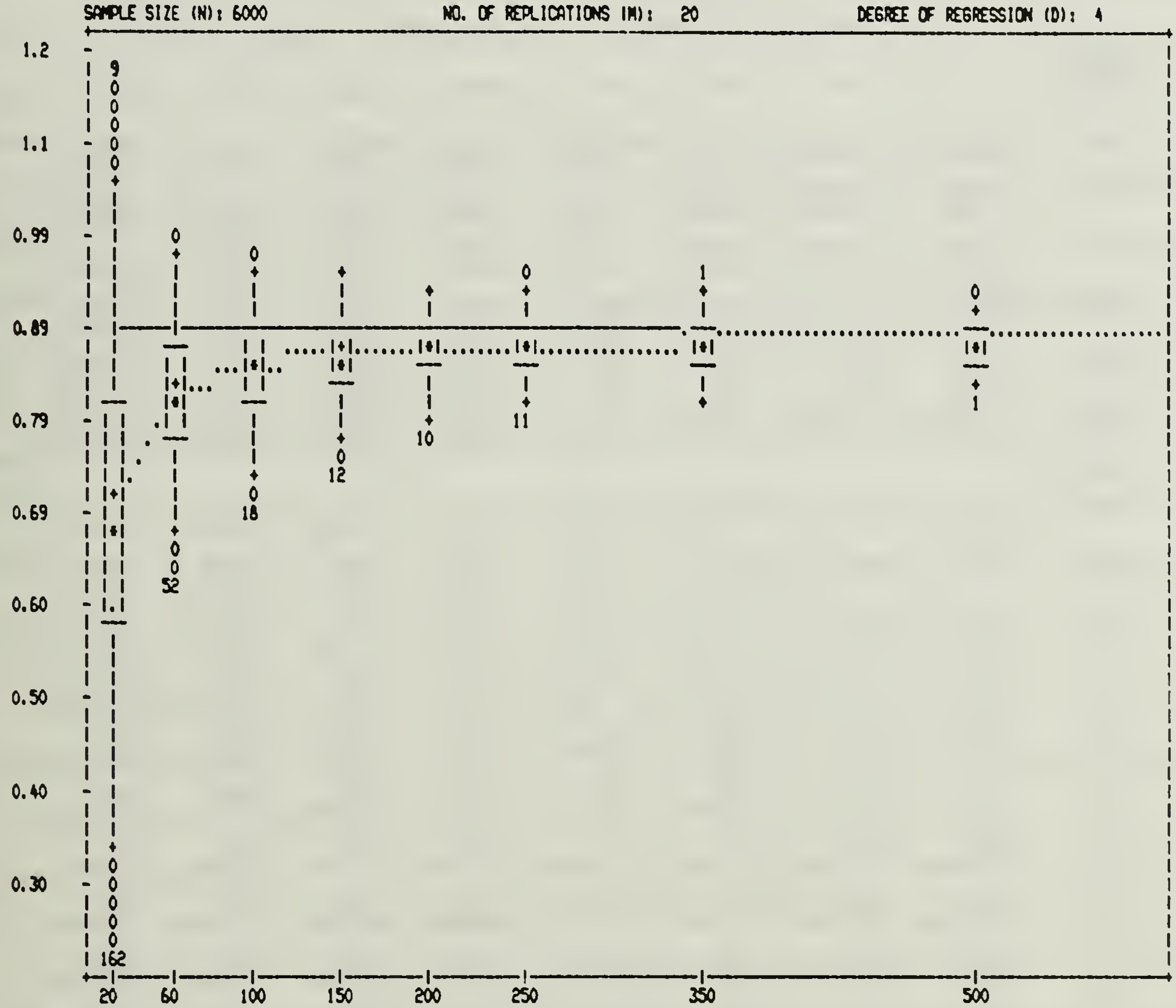
SUBSAMPLE SIZE	20	60	100	150	200	250	350	500
MEAN	0.6874	0.8299	0.8561	0.8690	0.8765	0.8827	0.8844	0.8891
	0.1849E-02	0.7716E-03	0.1025E-02	0.3937E-03	0.4811E-03	0.9492E-03	0.7484E-03	0.1160E-02
STD	0.1940	0.8322E-01	0.5983E-01	0.4518E-01	0.3875E-01	0.3148E-01	0.2704E-01	0.2259E-01
	0.1179E-02	0.5424E-03	0.1273E-02	0.1039E-02	0.3381E-03	0.7016E-03	0.5156E-03	0.5928E-03
M.S.E	0.2857	0.1070	0.7260E-01	0.5328E-01	0.4394E-01	0.3469E-01	0.2992E-01	0.2404E-01
	0.2135E-02	0.7895E-03	0.1286E-02	0.1066E-02	0.3695E-03	0.7562E-03	0.5400E-03	0.9264E-03
SKEWNESS	-0.9477	-0.9544	-0.9904	-0.7313	-0.8229	-0.4499	-0.4607	-0.5149
	0.3128E-01	0.2295E-01	0.6983E-01	0.9484E-01	0.8293E-01	0.3941E-01	0.1003	0.1740
KURTOSIS	1.232	1.540	2.065	1.181	1.360	0.2761	0.3169	1.052
	0.1278	0.5849E-01	0.3005	0.4990	0.5141	0.9601E-01	0.2207	0.7837
SER.COR.	0.3842E-03	0.2805E-02	0.1378E-01	-0.1813E-01	-0.3241E-02	-0.1749E-01	0.3371E-02	-0.5050E-01
	0.1160E-02	0.3988E-02	0.1311E-01	0.2933E-01	0.2326E-01	0.2286E-01	0.7194E-01	0.4585E-01
QUANTILES								
0.010	0.1065	0.5763	0.6799	0.7426	0.7585	0.7970	0.8096	0.8261
	0.3629E-02	0.2412E-02	0.8594E-02	0.2236E-02	0.3032E-02	0.5129E-02	0.7285E-02	0.1931E-02
0.025	0.2144	0.6343	0.7165	0.7681	0.7862	0.8136	0.8274	0.8434
	0.2442E-02	0.2174E-02	0.3386E-02	0.5216E-02	0.3658E-02	0.3298E-02	0.1880E-02	0.2985E-02
0.050	0.3181	0.6765	0.7466	0.7880	0.8061	0.8266	0.8371	0.8507
	0.5277E-02	0.2757E-02	0.2981E-02	0.2270E-02	0.2375E-02	0.2771E-02	0.9376E-03	0.2608E-02
0.100	0.4234	0.7183	0.7781	0.8082	0.8264	0.8411	0.8475	0.8600
	0.4652E-02	0.3028E-02	0.2193E-02	0.2071E-02	0.1909E-02	0.1364E-02	0.1144E-02	0.2905E-02
0.250	0.5821	0.7841	0.8232	0.8414	0.8539	0.8639	0.8683	0.8738
	0.2262E-02	0.9969E-03	0.9590E-03	0.1653E-02	0.5614E-03	0.1355E-02	0.6845E-03	0.1711E-02
0.500	0.7213	0.8425	0.8636	0.8744	0.8811	0.8845	0.8863	0.8900
	0.1430E-02	0.9129E-03	0.1127E-02	0.1168E-02	0.4967E-03	0.1451E-02	0.1021E-02	0.1244E-02
0.750	0.8248	0.8900	0.8990	0.9020	0.9036	0.9051	0.9039	0.9055
	0.1548E-02	0.1043E-02	0.1829E-02	0.7345E-03	0.5512E-03	0.2939E-03	0.1049E-02	0.5232E-03
0.900	0.9013	0.9236	0.9240	0.9216	0.9207	0.9216	0.9173	0.9169
	0.4793E-03	0.1075E-02	0.1375E-02	0.7097E-03	0.1056E-02	0.1085E-02	0.1167E-02	0.7682E-03
0.950	0.9413	0.9401	0.9362	0.9330	0.9310	0.9302	0.9234	0.9239
	0.1731E-02	0.8416E-03	0.7853E-03	0.1567E-02	0.1773E-02	0.1723E-02	0.8162E-03	0.1458E-02
0.975	0.9766	0.9550	0.9472	0.9406	0.9397	0.9364	0.9305	0.9287
	0.2496E-02	0.1713E-02	0.8573E-03	0.1154E-02	0.1459E-02	0.9242E-03	0.1200E-02	0.4263E-03
0.990	1.015	0.9685	0.9568	0.9530	0.9508	0.9468	0.9398	0.9355
	0.2242E-02	0.2126E-02	0.1795E-02	0.1786E-02	0.1856E-02	0.4480E-02	0.3142E-02	0.7145E-03

MEAN OF REGRESSION ON AVERAGES	0.8963	-3.500	-109.3	6293.	3009.
	0.4464E-02	2.242	324.8	0.1558E+05	6842.
STD DEV OF REGRESSION .	0.4508E-02	2.362	366.2	0.1876E+05	8496.
	0.2209E-03	0.1113	21.49	1235.	581.5
REGRESSION ON VARIANCE	0.5691	-16.17	269.5	-1630.	3421.
	0.1381	6.730	110.7	719.2	1548.

ESTIMATOR: ***Rho(.8972) of SqrtBeta L laplace AR r.v.s, A=.82, L= 3 Using moment estimator

Figure 24. Summary Statistics, L-Laplace samples. Moment Estimator.

SAMPLE SIZE (N): 6000　　NO. OF REPLICATIONS (M): 20　　DEGREE OF REGRESSION (D): 4



SUBSAMPLE SIZE	20	60	100	150	200	250	350	500
MEAN	0.6913	0.8264	0.8575	0.8716	0.8766	0.8804	0.8844	0.8869
STD	0.1950	0.8382E-01	0.5933E-01	0.4484E-01	0.3693E-01	0.3246E-01	0.2664E-01	0.2279E-01
STD MEAN	0.2518E-02	0.1874E-02	0.1713E-02	0.1585E-02	0.1508E-02	0.1482E-02	0.1445E-02	0.1471E-02
M.S.E	0.2836	0.1098	0.7140E-01	0.5161E-01	0.4229E-01	0.3657E-01	0.2954E-01	0.2500E-01
SKEWNESS	-0.9798	-1.1145	-0.9425	-0.7104	-0.6864	-0.5908	-0.3731	-0.3441
KURTOSIS	2.2236	2.1594	1.4534	0.6675	0.3702	0.4647	-0.0325	0.0045

MEAN OF REGRESSION ON AVERAGES	0.8927	-2.758	-71.13	-671.0	-1089.
VARIANCE OF REGRESSION	0.1416E-04	3.591	0.7942E+05	0.2024E+09	0.4126E+08
STD DEV OF REGRESSION	0.3763E-02	1.895	281.8	0.1423E+05	6423.
REGRESSION ON VARIANCE	0.6364	-18.20	283.9	-1634.	3307.

ESTIMATOR: ***Rho(.8972) of SqrtBeta L laplace AR r.vs., A=.82, L= 3 Using Priestly Estimator
VERTICAL SCALE: YMIN = 0.2176
YMAX = 1.1921

Figure 25. Individual Plot, L-Laplace samples. Priestley Estimator.

SUMMARY STATISTICS (MEAN/STD)   3 SUPER-REPLICATIONS

SUBSAMPLE SIZE	20	60	100	150	200	250	350	500
MEAN	0.6917 0.2270E-03	0.8270 0.3948E-03	0.8566 0.4744E-03	0.8719 0.5483E-03	0.8770 0.2863E-03	0.8816 0.8209E-03	0.8867 0.1175E-02	0.8879 0.5948E-03
STD	0.1950 0.6702E-04	0.8168E-01 0.1534E-02	0.5835E-01 0.8978E-03	0.4501E-01 0.1589E-03	0.3768E-01 0.3879E-03	0.3322E-01 0.4926E-03	0.2639E-01 0.3214E-03	0.2262E-01 0.9844E-04
M.S.E	0.2833 0.1870E-03	0.1077 0.1417E-02	0.7108E-01 0.5181E-03	0.5166E-01 0.3800E-03	0.4275E-01 0.2881E-03	0.3670E-01 0.5030E-03	0.2844E-01 0.6775E-03	0.2447E-01 0.3157E-03
SKEWNESS	-0.9581 0.1387E-01	-0.9541 0.8558E-01	-0.9443 0.2442E-01	-0.7927 0.6570E-01	-0.6319 0.5495E-01	-0.6095 0.4084E-01	-0.4438 0.6199E-01	-0.3528 0.9349E-02
KURTOSIS	1.823 0.2022	1.388 0.3995	1.608 0.2350	0.8451 0.2434	0.4770 0.8291E-01	0.4435 0.2125E-01	0.9895E-01 0.1717	0.1564E-01 0.8717E-01
SER.COR.	-0.6990E-02 0.3530E-02	0.6734E-02 0.1441E-01	0.2050E-02 0.1161E-01	-0.7611E-02 0.1191E-01	0.5194E-02 0.1459E-01	0.1208E-01 0.5170E-01	0.1226E-01 0.3999E-01	0.6532E-01 0.1630E-01
QUANTILES								
0.010	0.1042 0.7244E-02	0.5754 0.8325E-02	0.6838 0.6605E-02	0.7340 0.3983E-02	0.7671 0.5578E-02	0.7831 0.1219E-02	0.8174 0.2599E-02	0.8289 0.9947E-03
0.025	0.2200 0.7299E-02	0.6277 0.3223E-02	0.7208 0.2237E-02	0.7700 0.2273E-02	0.7939 0.2057E-02	0.8065 0.4058E-02	0.8277 0.2317E-02	0.8397 0.1557E-02
0.050	0.3225 0.1957E-02	0.6749 0.3507E-02	0.7492 0.1186E-02	0.7910 0.1672E-02	0.8099 0.1375E-02	0.8211 0.1597E-02	0.8390 0.3010E-02	0.8462 0.1933E-02
0.100	0.4308 0.1486E-02	0.7189 0.2779E-02	0.7768 0.8848E-03	0.8106 0.1745E-02	0.8251 0.1084E-02	0.8369 0.1081E-02	0.8487 0.1735E-02	0.8586 0.9670E-03
0.250	0.5886 0.2043E-02	0.7824 0.3129E-03	0.8235 0.3865E-03	0.8462 0.1929E-02	0.8542 0.1346E-02	0.8613 0.3063E-03	0.8704 0.1149E-02	0.8733 0.7283E-03
0.500	0.7239 0.2230E-02	0.8400 0.8701E-03	0.8662 0.1235E-02	0.8779 0.6014E-03	0.8811 0.3518E-03	0.8854 0.7980E-03	0.8881 0.1525E-02	0.8889 0.2623E-03
0.750	0.8280 0.2780E-03	0.8858 0.8113E-03	0.8989 0.1135E-02	0.9051 0.5592E-03	0.9040 0.6763E-03	0.9055 0.1474E-02	0.9060 0.9489E-03	0.9041 0.8847E-03
0.900	0.9031 0.4087E-03	0.9186 0.1109E-02	0.9228 0.1678E-02	0.9240 0.2218E-03	0.9216 0.2337E-02	0.9202 0.5077E-03	0.9185 0.1148E-02	0.9152 0.8643E-03
0.950	0.9464 0.1333E-02	0.9359 0.6841E-03	0.9356 0.2097E-02	0.9335 0.8015E-03	0.9328 0.1769E-02	0.9301 0.5609E-03	0.9248 0.1190E-02	0.9233 0.3677E-03
0.975	0.9852 0.1892E-02	0.9490 0.3994E-03	0.9442 0.1547E-02	0.9414 0.1164E-02	0.9401 0.7899E-03	0.9369 0.1525E-02	0.9318 0.1145E-02	0.9269 0.1114E-02
0.990	1.030 0.2861E-02	0.9636 0.1556E-02	0.9548 0.1812E-02	0.9504 0.1054E-02	0.9470 0.2529E-02	0.9439 0.1290E-02	0.9402 0.1289E-02	0.9355 0.2436E-02

MEAN OF REGRESSION ON AVERAGES	0.8936 0.1545E-02	-2.205 0.6609	-226.6 106.2	8801. 5636.	3438. 2592.
STD DEV OF REGRESSION	0.4253E-02 0.3847E-03	2.225 0.1972	329.9 25.74	0.1633E+05 1065.	7285. 431.5
REGRESSION ON VARIANCE	0.3184 0.2538	-4.712 11.89	92.89 188.6	-558.6 1197.	1238. 2546.

ESTIMATOR: ***Rho(.8972) of SqrtBeta L laplace AR r.vs., A=.82, L= 3 Using Priestly Estimator

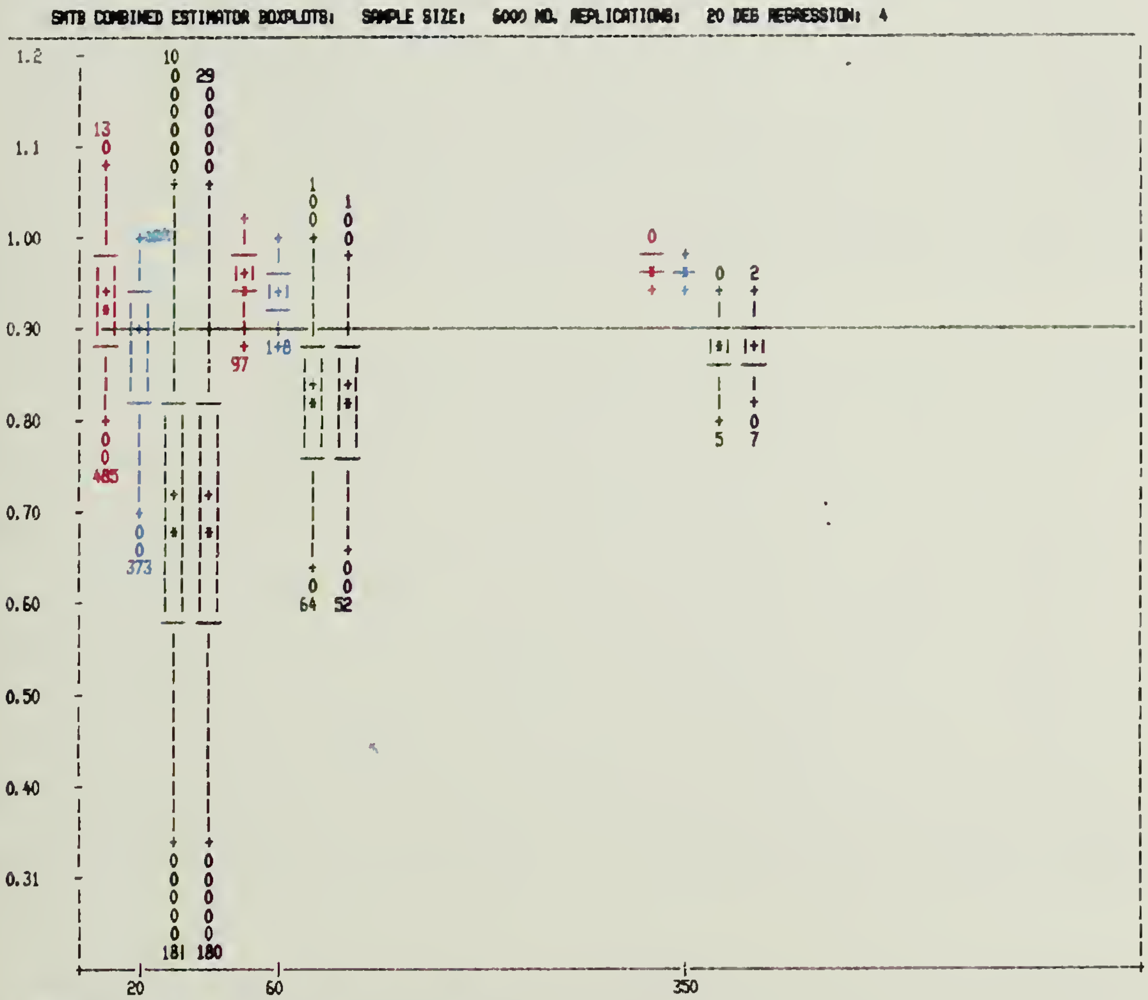
Figure 26. Summary Statistics, L-Laplace samples. Priestley Estimator.

2. Parameter l = 1.

This set of parameters is the first encounter with the fact that the random generators, for large samples, will generate one or more numbers which are smaller than (closer to zero than) the smallest number representable in the Ryan-McFarland Fortran compiler for micros. The number then becomes a NAN (Not A Number) and propagates through the computations as ????????. Figure 27 is a color combined SIMTBED plot, and it shows the undefined results which ripple through all calculations when an invalid number is returned by the random number generator. This is believed to result from an underflow error because the parameters of the underlying probability distribution that we are asking the FORTRAN subroutines to generate are very small. This results in random numbers generated which are very close to, but not equal to zero. The random number generators must perform transformations involving logarithms and square roots to generate these Gamma and Beta distributions, and when the parameters become very small, and the generators are operated very frequently, as they are with SIMTBED and these simulations, it happens that an underflow occurs and an invalid result is returned. This ripples through every mathematical operation and appears in the results, an example of which is Figure 27. In this particular example, the parameter l is set to one, or very nearly one (0.95 was used to try and circumvent the problem in this particular example). Since there are two Beta random streams required to generated the l-BELAR(1) process, with parameters $(\alpha l,(1-\alpha)l)$, it is true that when l is one, and we want either a high or low correlation, then either α or $(1-\alpha)$ will be small. This causes the Beta generator (which generates Beta numbers as the ratio of Gammas) to underflow. This is a rare event, but the number of operations of a random number generator by SIMTBED is extremely large (they are at the heart of the statistical simulation process), and only one occurrence is needed to invalidate the entire simulation run. For these reasons, we were not able to obtain acceptable results for this parameter of the BELAR(1) process. Every available random number generator was tried, and all of them have problems generating Gamma random numbers with very small parameters. Therefore, we continue with the next case.

3. Parameter l = 0.4.

This case did not exhibit the type of random number generator problem described above. Figure 28 is the color combined estimator plot for the case of $l = 0.4$, with strongly correlated BELAR(1) samples. From this plot it is immediately obvious that there is a difference in the asymptotic behavior of the estimators. It can be seen that the robust regression approaches 1.0, when the true value is 0.8954, and further,




```
SUBSAMPLE SIZE:                  20           60           350
ESTIMATOR 1:   MEAN:         0.9180       0.9548       0.9708
              STD:          0.1160       0.4408E-01   0.1173E-01
              STD MEAN:     0.1497E-02   0.9857E-03   0.6360E-03

SUBSAMPLE SIZE:                  20           60           350
ESTIMATOR 2:   MEAN:      ???????????????????????????  0.9676
              STD:       ???????????????????????????  0.9166E-02
              STD MEAN:  ???????????????????????????  0.4971E-03

SUBSAMPLE SIZE:                  20           60           350
ESTIMATOR 3:   MEAN:         0.6955       0.8228       0.8810
              STD:          0.2022       0.9387E-01   0.3732E-01
              STD MEAN:     0.2610E-02   0.2099E-02   0.2024E-02

SUBSAMPLE SIZE:                  20           60           350
ESTIMATOR 4:   MEAN:         0.6976       0.8263    ??????????????
              STD:          0.2110       0.8991E-01??????????????
              STD MEAN:     0.2724E-02   0.2011E-02??????????????

ESTIMATOR 1:  ***Rho(.8963) of SqrtBeta L laplace AR r.v.s, A=.84, L= .95 Using Robust Least Squares estimator (C=4.2)
ESTIMATOR 2:  ***Rho(.8963) of SqrtBeta L laplace AR r.vs., A=.84, L= .95 Using Cressie Estimator
ESTIMATOR 3:  ***Rho(.8963) of SqrtBeta L laplace AR r.v.s, A=.84, L= .95 Using moment estimator
ESTIMATOR 4:  ***Rho(.8963) of SqrtBeta L laplace AR r.vs., A=.84, L= .95 Using Priestly Estimator

*** WIDEST Y VALUES FOUND:  YMIN=0.7611     -  YMAX= 2.160       *** SMTB ***
*** THIS WAS A RESTART RUN. NSR START/END=         1           3
```

Figure 27.   Combined Plot, L-laplace samples. Rho = 0.8963. L = 0.95.

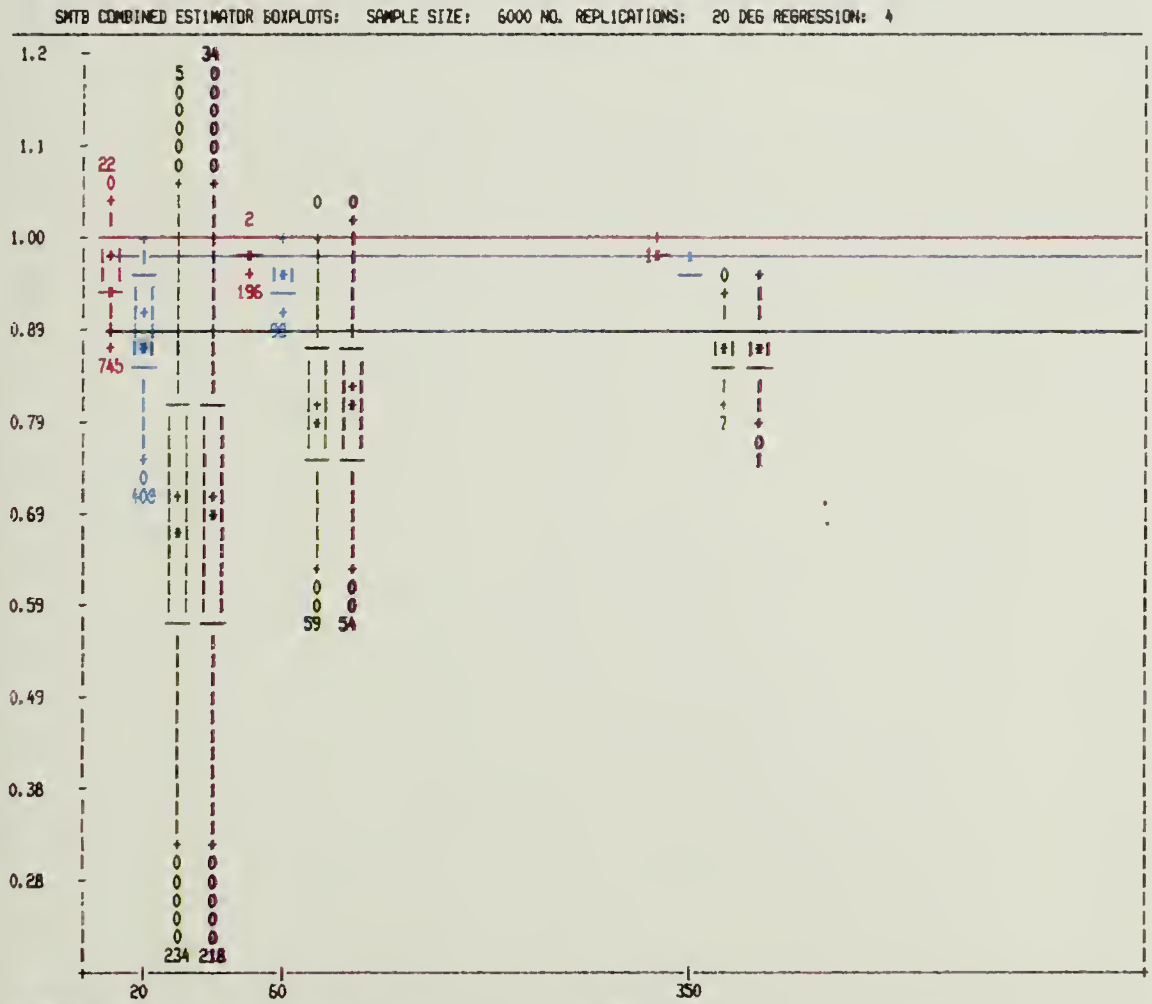
that the small-sample bias of the robust regression is also unacceptable. The Cressie estimator also has serious asymptotic bias with respect to sample size, but is better behaved with small-sample bias. Still, it would appear that both of these estimator are very poorly behaved with extremely 'heavy-tailed', non-Normal samples such as these. The two estimators that *are* asymptotically unbiased are the moment and MLE estimators, but they are beginning to show larger and larger sub-sample sizes required to reach the true value. Also, as we depart from Normal samples, the small-sample bias of these estimators becomes larger.

Figures 29 and 30 clearly show the biased behavior of the robust regression estimator. This bias is referred to by Denby and Martin, when discussing the "M-estimate," and the "GM-estimate." This robust regression model is the "M-estimate," and hence demonstrates the asymptotic biased behavior referred to by these authors. [Ref. 9, p. 141] The further from Normality the sample distribution becomes (in terms of the l parameter of the l-Laplace distribution and "heavy-tailedness" of the underlying probability distribution), the worse the behavior of these estimators becomes. Figures 31 and 32 are the individual plots for the Cressie estimator. With only three super-replications, we can see good stability in the simulation results (in terms of the standard deviations reported with every statistic in the super-replications summary (Figures 30 and 32).

Figures 33 through 36 are the individual plots for the two "convergent" estimators, the moment and the MLE estimators. Here, we can begin to compare these and see if perhaps one estimator is becoming unbiased faster than the other (again, with respect to sample size). It appears from Figures 33 and 35 that the MSEs of both estimators are decreasing at about the same rate. The asymptotic regression estimate for the moment estimator (0.9048) is larger than that for the Priestley estimator (0.8941), but the Priestley estimator is underestimating the actual correlation, while the moment estimator is now over-estimating the true value. Both estimators appear to be exhibiting similar behavior. (The reader will undoubtedly note the volume of output required to be analyzed when computers are used to conduct simulations, and hopefully will begin to appreciate the purpose of such a package as SIMTBED.)

4. Parameter l = 0.1.

As the final case of the strongly correlated, non-Normal samples on which we estimate serial correlation, the extreme non-Normal case where parameter $l = 0.1$ was run with SIMTBED. The results begin with Figure 37, which is a color combined estimator plot of this simulation. It is clear from Figure 37 that the estimators are split into



SUBSAMPLE SIZE:		20	60	350
ESTIMATOR 1:	MEAN:	0.9383	0.9799	0.9927
	STD:	0.1366	0.3824E-01	0.4681E-02
	STD MEAN:	0.1763E-02	0.8551E-03	0.2539E-03

SUBSAMPLE SIZE:		20	60	350
ESTIMATOR 2:	MEAN:	0.8946	0.9617	0.9778
	STD:	0.1165	0.2879E-01	0.7491E-02
	STD MEAN:	0.1504E-02	0.6437E-03	0.4063E-03

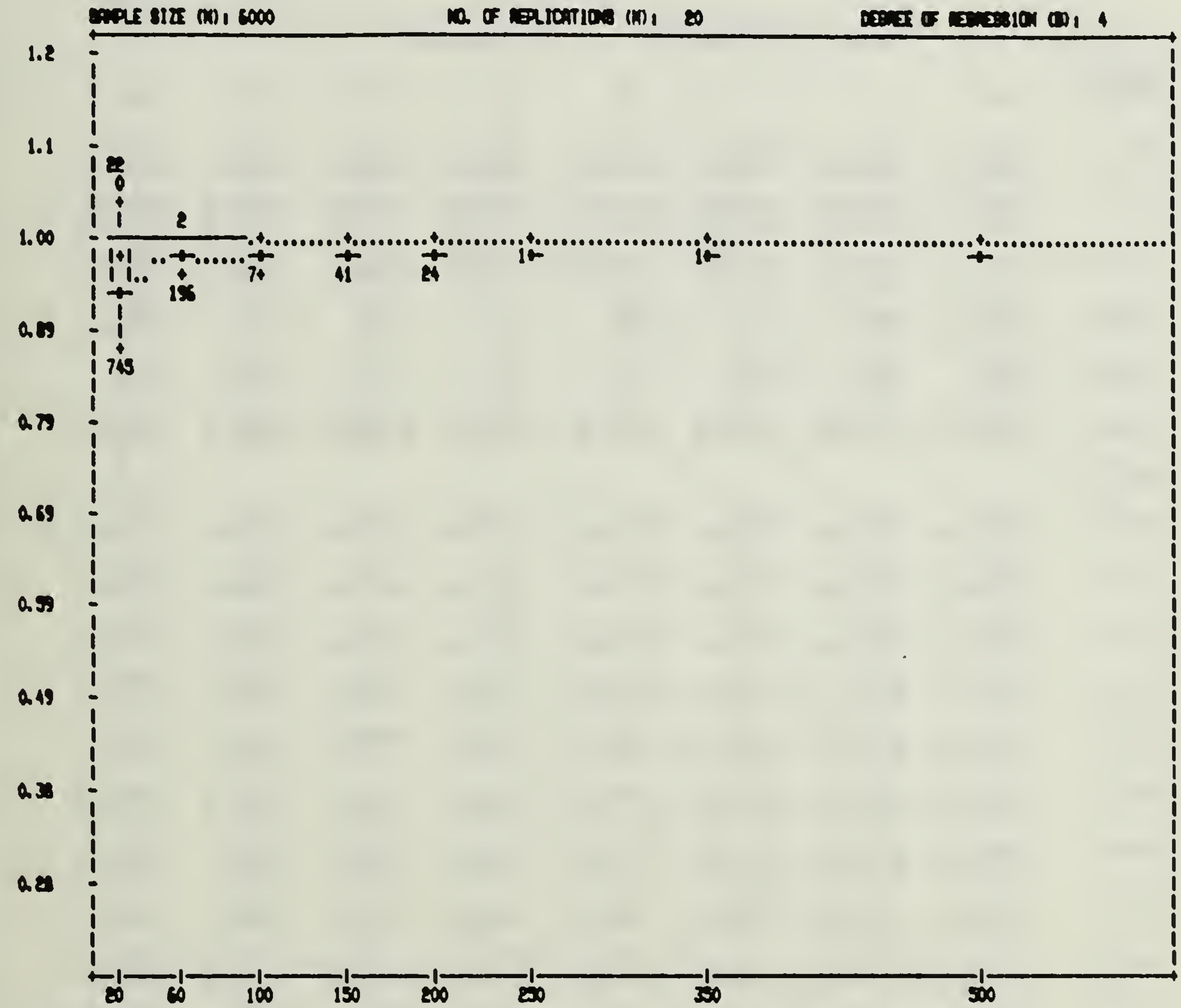
SUBSAMPLE SIZE:		20	60	350
ESTIMATOR 3:	MEAN:	0.6777	0.8120	0.8814
	STD:	0.2132	0.1003	0.3627E-01
	STD MEAN:	0.2753E-02	0.2244E-02	0.1367E-02

SUBSAMPLE SIZE:		20	60	350
ESTIMATOR 4:	MEAN:	0.6916	0.8159	0.8795
	STD:	0.2207	0.1060	0.3849E-01
	STD MEAN:	0.2850E-02	0.2370E-02	0.2087E-02

ESTIMATOR 1: ***Rho(.8954) of SqrtBeta L laplace AR r.v.s, A=.86, L= .4 Using Robust Least Squares estimator
ESTIMATOR 2: ***Rho(.8954) of SqrtBeta L laplace AR r.vs., A=.86, L= .4 Using Cressie Estimator
ESTIMATOR 3: ***Rho(.8954) of SqrtBeta L laplace AR r.v.s, A=.86, L= .4 Using moment estimator
ESTIMATOR 4: ***Rho(.8954) of SqrtBeta L laplace AR r.vs., A=.86, L= .4 Using Priestly Estimator

*** WIDEST Y VALUES FOUND: YMIN=-1.569 - YMAX= 4.262 *** SMTB ***
*** THIS WAS A RESTART RUN. NOR START/END= 1 3

Figure 28. Combined Plot, L-laplace samples. Rho = 0.8954. L = 0.4.



SUBSAMPLE SIZE	20	60	100	150	200	250	350	500
MEAN	0.9383	0.9799	0.9873	0.9899	0.9909	0.9917	0.9927	0.9923
STD	0.1366	0.3824E-01	0.1651E-01	0.9702E-02	0.7716E-02	0.5789E-02	0.4681E-02	0.4028E-02
STD MEAN	0.1763E-02	0.8551E-03	0.4766E-03	0.3430E-03	0.3150E-03	0.2642E-03	0.2539E-03	0.2600E-03
M.S.E	0.1431	0.9279E-01	0.9335E-01	0.9501E-01	0.9577E-01	0.9645E-01	0.9737E-01	0.9701E-01
SKEWNESS	-3.6540	-6.5591	-3.4430	-2.1945	-2.4747	-1.1305	-1.3901	-0.9688
KURTOSIS	16.1645	83.5072	17.4649	7.2355	13.0522	1.6626	2.7905	1.3881

MEAN OF REGRESSION ON AVERAGES	0.9932	-0.2511	-38.36	288.5	-55.78
VARIANCE OF REGRESSION	0.8659E-06	0.2539	6265.	0.1661E+08	0.3415E+07
STD DEV OF REGRESSION	0.9305E-03	0.5039	79.15	4075.	1848.
REGRESSION ON VARIANCE	-0.1140	6.248	-112.6	821.8	-1787.

ESTIMATOR: ***Rho(.8954) of SqrtBeta L laplace AR r.v.s, A=.86, L= .4 Using Robust Least Squares estimator
VERTICAL SCALE: YMIN = 0.2001
YMAX = 1.2010

Figure 29. Individual Plot, L-Laplace samples. Robust Least Squares.

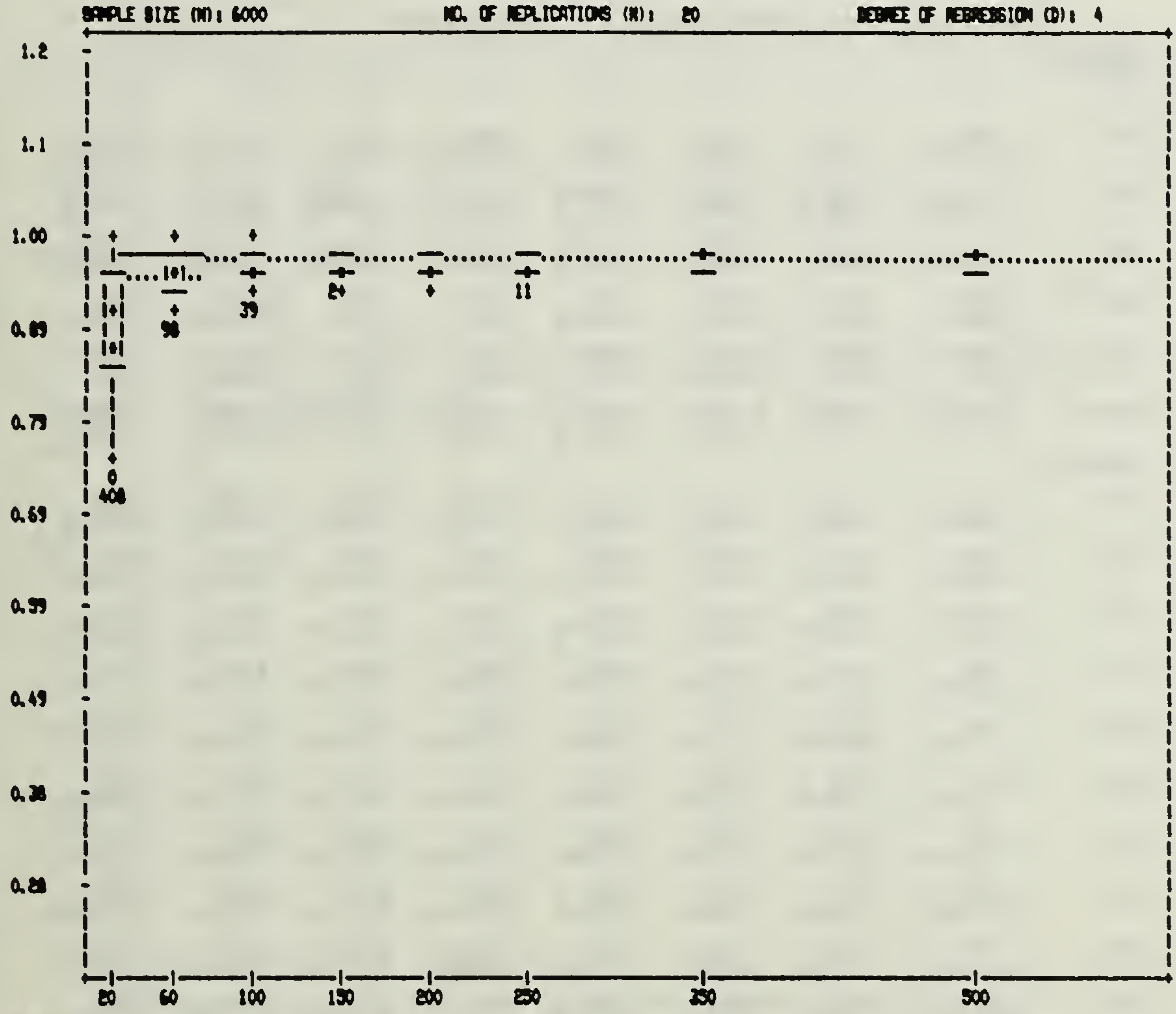
SUMMARY STATISTICS (MEAN/STD)   3 SUPER-REPLICATIONS

SUBSAMPLE SIZE	20	60	100	150	200	250	350	500
MEAN	0.9371 0.6304E-03	0.9808 0.8906E-03	0.9871 0.8325E-04	0.9897 0.1621E-03	0.9907 0.1403E-03	0.9915 0.1243E-03	0.9925 0.1191E-03	0.9926 0.1687E-03
STD	0.1371 0.2736E-03	0.3503E-01 0.3075E-02	0.1740E-01 0.4902E-03	0.1088E-01 0.5931E-03	0.8394E-02 0.3708E-03	0.5941E-02 0.1246E-03	0.4977E-02 0.1630E-03	0.3898E-02 0.1568E-03
M.S.E	0.1433 0.9580E-04	0.9245E-01 0.2890E-03	0.9337E-01 0.7448E-04	0.9497E-01 0.1278E-03	0.9565E-01 0.8260E-04	0.9627E-01 0.1165E-03	0.9723E-01 0.1123E-03	0.9728E-01 0.1648E-03
SKEWNESS	-3.573 0.4442E-01	-5.204 0.7657	-4.042 0.3019	-3.221 0.5171	-2.637 0.1130	-1.339 0.1059	-1.764 0.4060	-0.9322 0.7794E-01
KURTOSIS	15.33 0.4233	48.23 18.28	27.07 5.209	20.04 6.601	13.07 0.7897	2.997 0.7108	7.238 4.255	1.351 0.4587
SER. COR.	0.9515E-02 0.7875E-02	0.2988E-01 0.1309E-01	-0.2111E-01 0.3606E-02	0.3568E-02 0.2443E-01	-0.4817E-02 0.1687E-01	-0.9878E-02 0.4464E-01	-0.3034E-02 0.2259E-01	0.2385E-01 0.5096E-01
QUANTILES								
0.010	0.2706 0.4904E-02	0.8248 0.1809E-01	0.9123 0.2424E-02	0.9482 0.2018E-02	0.9582 0.4593E-02	0.9702 0.9989E-03	0.9758 0.1092E-02	0.9797 0.1214E-02
0.025	0.4935 0.5939E-02	0.8899 0.9849E-02	0.9418 0.1349E-03	0.9601 0.1472E-02	0.9702 0.7354E-03	0.9762 0.8863E-03	0.9804 0.1093E-02	0.9833 0.4855E-03
0.050	0.6702 0.4078E-02	0.9254 0.2771E-02	0.9581 0.8197E-03	0.9706 0.1166E-02	0.9760 0.9353E-03	0.9804 0.2772E-03	0.9831 0.9500E-03	0.9852 0.5372E-03
0.100	0.8145 0.4148E-02	0.9538 0.1893E-02	0.9711 0.9459E-03	0.9785 0.3947E-03	0.9814 0.1919E-03	0.9838 0.1545E-03	0.9866 0.2523E-03	0.9875 0.2296E-03
0.250	0.9402 0.1412E-02	0.9787 0.3249E-03	0.9837 0.3010E-03	0.9866 0.1753E-03	0.9878 0.4612E-04	0.9886 0.1511E-03	0.9903 0.1706E-03	0.9905 0.2432E-03
0.500	0.9850 0.3947E-03	0.9906 0.6940E-04	0.9916 0.1845E-03	0.9923 0.1784E-03	0.9926 0.5490E-04	0.9926 0.1938E-04	0.9934 0.9815E-04	0.9932 0.1185E-03
0.750	0.9966 0.2067E-03	0.9968 0.2125E-04	0.9964 0.4990E-04	0.9960 0.4869E-04	0.9958 0.1192E-03	0.9957 0.1229E-03	0.9958 0.2586E-04	0.9954 0.1395E-03
0.900	1.008 0.2537E-03	1.001 0.1674E-03	0.9996 0.8008E-05	0.9987 0.7019E-04	0.9982 0.2249E-03	0.9977 0.1900E-03	0.9976 0.5544E-04	0.9970 0.1584E-03
0.950	1.017 0.7537E-03	1.004 0.2724E-03	1.001 0.9344E-05	1.000 0.8422E-04	0.9994 0.2772E-03	0.9987 0.4049E-04	0.9985 0.1622E-03	0.9981 0.1672E-03
0.975	1.029 0.1095E-02	1.007 0.4649E-03	1.003 0.2601E-03	1.002 0.1741E-03	1.000 0.3225E-03	0.9995 0.1201E-03	0.9994 0.1564E-03	0.9987 0.2725E-03
0.990	1.049 0.2144E-02	1.013 0.1656E-03	1.006 0.4328E-03	1.004 0.4489E-03	1.002 0.5962E-03	1.001 0.3119E-03	1.000 0.1863E-03	0.9996 0.1033E-03

MEAN OF REGRESSION ON AVERAGES	0.9940 0.4247E-03	-0.6013 0.2135	-2.920 28.90	-676.6 1579.	-357.7 764.5
STD DEV OF REGRESSION	0.9613E-03 0.5986E-04	0.5352 0.2587E-01	84.34 2.875	4342. 140.1	1970. 64.06
REGRESSION ON VARIANCE	-0.4011E-01 0.4940E-01	2.249 2.614	-42.13 47.55	320.9 342.9	-636.1 796.9

ESTIMATOR: ***Rho(.8954) of SqrtBeta L laplace AR r.v.s, A=.86, L= .4 Using Robust Least Squares estimator

Figure 30. Summary Statistics, L-Laplace samples. Robust Least Squares.



SUBSAMPLE SIZE	20	60	100	150	200	250	350	500
MEAN	0.8946	0.9617	0.9694	0.9731	0.9746	0.9754	0.9778	0.9775
STD	0.1165	0.2879E-01	0.2046E-01	0.1437E-01	0.1134E-01	0.9521E-02	0.7491E-02	0.6612E-02
STD MEAN	0.1504E-02	0.6437E-03	0.5908E-03	0.5080E-03	0.4629E-03	0.4346E-03	0.4063E-03	0.4268E-03
M.S.E	0.1165	0.7230E-01	0.7675E-01	0.7904E-01	0.8001E-01	0.8057E-01	0.8274E-01	0.8232E-01
SKEWNESS	-3.0335	-1.8097	-1.9141	-1.4087	-0.8415	-0.8807	-0.7470	-0.6919
KURTOSIS	14.3689	4.5232	6.0581	2.0748	0.5361	1.7996	0.6636	1.1419

MEAN OF REGRESSION ON AVERAGES	0.9798	-0.9499	-12.84	457.8	345.5
VARIANCE OF REGRESSION	0.2058E-05	0.6451	0.1471E+05	0.3649E+08	0.7286E+07
STD DEV OF REGRESSION	0.1435E-02	0.8032	121.3	6041.	2699.
REGRESSION ON VARIANCE	0.1507	-6.408	106.8	-695.6	1597.

ESTIMATOR: ***Rho(.8954) of SqrtBeta L laplace AR r.vs., A=.86, L= .4 Using Cressie Estimator
VERTICAL SCALE: YMIN = 0.2001
YMAX = 1.2010

Figure 31. Individual Plot, L-Laplace samples. Cressie Estimator.

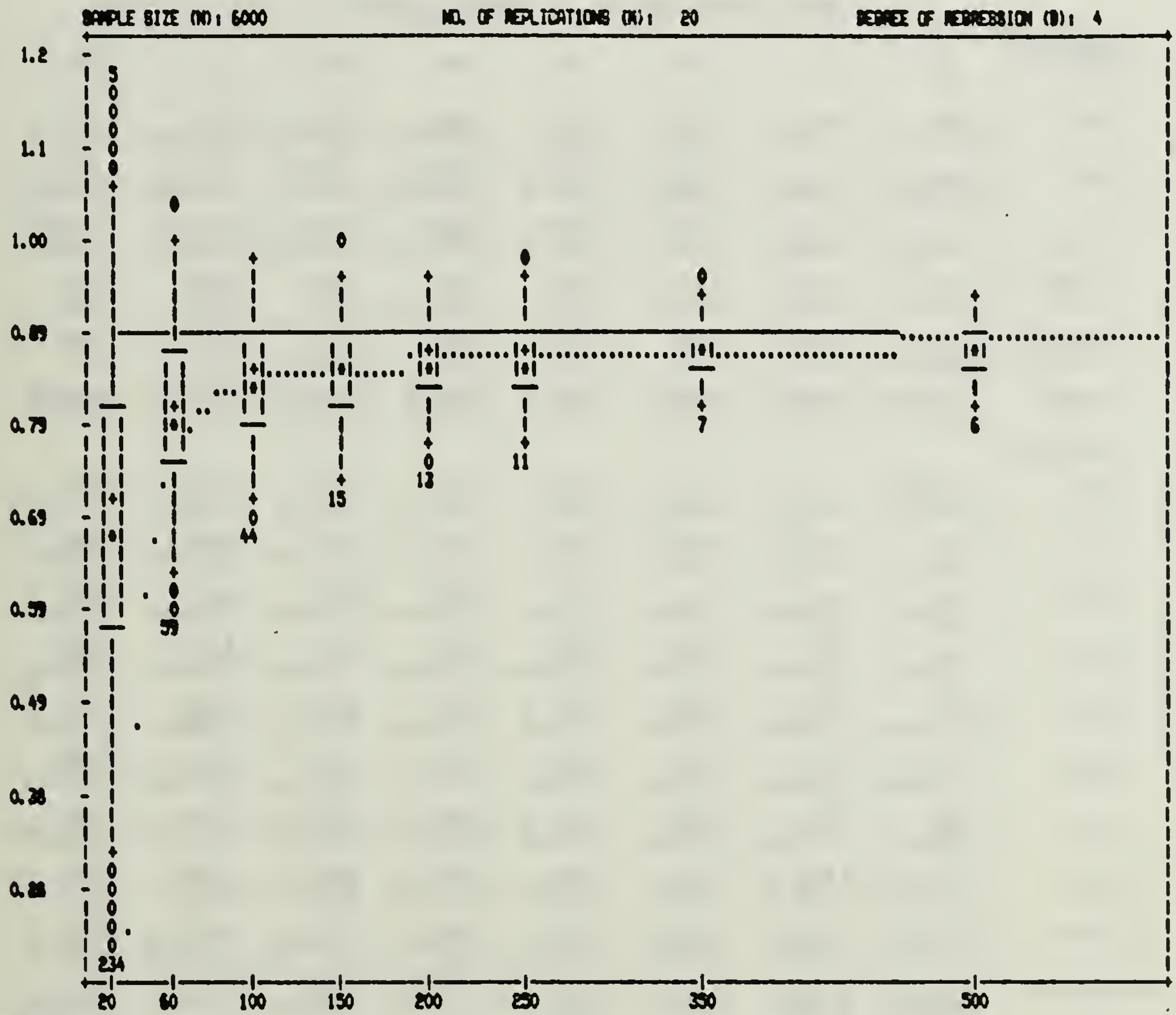
SUMMARY STATISTICS (MEAN/STD)   3 SUPER-REPLICATIONS

SUBSAMPLE SIZE	20	60	100	150	200	250	350	500
MEAN	0.8935 0.5395E-03	0.9613 0.2346E-03	0.9702 0.4149E-03	0.9730 0.1625E-03	0.9750 0.2346E-03	0.9754 0.2575E-03	0.9769 0.4698E-03	0.9775 0.2972E-04
STD	0.1218 0.3095E-02	0.2968E-01 0.1030E-02	0.1930E-01 0.5884E-03	0.1412E-01 0.2613E-03	0.1166E-01 0.1816E-03	0.1004E-01 0.3674E-03	0.8091E-02 0.2261E-03	0.6364E-02 0.5198E-04
M.S.E	0.1218 0.3103E-02	0.7225E-01 0.4029E-03	0.7722E-01 0.2555E-03	0.7890E-01 0.1321E-03	0.8048E-01 0.2488E-03	0.8066E-01 0.2271E-03	0.8198E-01 0.4444E-03	0.8237E-01 0.2892E-04
SKEWNESS	-3.519 0.2556	-1.961 0.2283	-1.688 0.1157	-1.346 0.4288E-01	-1.116 0.1443	-1.029 0.1362	-0.7205 0.8449E-01	-0.6380 0.2697E-01
KURTOSIS	22.28 3.961	6.323 2.047	4.627 0.7311	2.984 0.2334	1.735 0.6552	1.897 0.4843	0.5982 0.2222	0.8072 0.2203
SER.COR.	-0.6320E-02 0.7054E-02	-0.1290E-01 0.1832E-01	-0.7381E-02 0.1078E-01	-0.2741E-01 0.1051E-01	0.3370E-02 0.1136E-01	-0.1403E-01 0.2238E-01	-0.2304E-01 0.3231E-01	-0.3180E-01 0.4325E-01
QUANTILES								
0.010	0.3980 0.9688E-02	0.8506 0.6015E-02	0.9002 0.3616E-02	0.9260 0.2299E-02	0.9400 0.2145E-02	0.9413 0.2598E-02	0.9528 0.1155E-02	0.9592 0.8355E-03
0.025	0.5636 0.8170E-02	0.8829 0.3519E-02	0.9213 0.2976E-02	0.9393 0.8122E-03	0.9454 0.1993E-02	0.9513 0.8275E-03	0.9579 0.1030E-02	0.9621 0.6621E-03
0.050	0.6742 0.3448E-02	0.9035 0.1540E-02	0.9350 0.7193E-03	0.9457 0.6156E-03	0.9533 0.6739E-03	0.9579 0.1260E-02	0.9617 0.1269E-02	0.9652 0.7866E-03
0.100	0.7598 0.6201E-03	0.9234 0.2717E-03	0.9451 0.3866E-03	0.9544 0.6635E-03	0.9590 0.6702E-03	0.9627 0.5895E-03	0.9658 0.9379E-03	0.9694 0.2248E-03
0.250	0.8667 0.2330E-03	0.9496 0.5503E-03	0.9611 0.9030E-03	0.9660 0.3822E-03	0.9687 0.4662E-03	0.9698 0.4695E-03	0.9721 0.5942E-03	0.9733 0.3250E-03
0.500	0.9324 0.1547E-03	0.9688 0.6635E-03	0.9750 0.1126E-03	0.9758 0.2625E-03	0.9772 0.3635E-03	0.9769 0.4073E-03	0.9782 0.5316E-03	0.9781 0.1768E-03
0.750	0.9667 0.2095E-03	0.9819 0.3268E-03	0.9838 0.1882E-03	0.9830 0.1607E-03	0.9834 0.4274E-03	0.9828 0.3041E-03	0.9827 0.3181E-03	0.9824 0.1063E-03
0.900	0.9828 0.4012E-03	0.9893 0.1916E-03	0.9894 0.7857E-04	0.9883 0.1860E-03	0.9879 0.1691E-03	0.9869 0.2919E-03	0.9863 0.9857E-04	0.9853 0.1153E-03
0.950	0.9885 0.2251E-03	0.9921 0.6558E-04	0.9919 0.3558E-03	0.9907 0.1786E-03	0.9898 0.4511E-03	0.9891 0.1911E-03	0.9881 0.1982E-03	0.9872 0.3447E-03
0.975	0.9918 0.1965E-03	0.9939 0.1643E-03	0.9933 0.4250E-03	0.9921 0.3713E-04	0.9911 0.7445E-04	0.9903 0.3288E-03	0.9899 0.6768E-03	0.9887 0.3935E-03
0.990	0.9945 0.1644E-03	0.9955 0.1038E-03	0.9947 0.4426E-03	0.9935 0.1427E-03	0.9932 0.2829E-03	0.9923 0.8515E-03	0.9917 0.5539E-03	0.9904 0.2808E-03

MEAN OF REGRESSION ON AVERAGES	0.9799 0.1722E-03	-1.272 0.1905	68.92 45.40	-4593. 2758.	-2084. 1320.
STD DEV OF REGRESSION	0.1475E-02 0.3300E-04	0.7896 0.2183E-01	118.2 3.909	5889. 217.2	2635. 100.3
REGRESSION ON VARIANCE	0.3474E-01 0.6042E-01	-0.8783 2.865	18.33 45.87	-135.3 292.6	421.7 619.5

ESTIMATOR: ***Rho(.8954) of SqrtBeta L laplace AR r.vs., A=.86, L= .4 Using Cressie Estimator

Figure 32. Summary Statistics, L-Laplace samples. Cressie Estimator.



SUBSAMPLE SIZE	20	60	100	150	200	250	350	500
MEAN	0.6777	0.8120	0.8458	0.8563	0.8665	0.8723	0.8814	0.8842
STD	0.2132	0.1003	0.7440E-01	0.6005E-01	0.5264E-01	0.4691E-01	0.3627E-01	0.3354E-01
STD MEAN	0.2753E-02	0.2244E-02	0.2148E-02	0.2123E-02	0.2149E-02	0.2141E-02	0.1967E-02	0.2294E-02
M.S.E	0.3047	0.1305	0.8944E-01	0.7166E-01	0.6005E-01	0.5228E-01	0.3889E-01	0.3725E-01
SKEWNESS	-1.1896	-1.3018	-1.1956	-1.0101	-1.1866	-0.9298	-0.6557	-0.7391
KURTOSIS	2.2851	2.9369	2.1627	1.8837	3.0106	1.8344	0.4754	1.0993

MEAN OF REGRESSION ON AVERAGES	0.9048	-11.04	853.0	-0.3998E+05	-0.1754E+05
VARIANCE OF REGRESSION	0.5354E-04	14.15	0.3112E+06	0.7590E+09	0.1507E+09
STD DEV OF REGRESSION	0.7317E-02	3.761	557.9	0.2755E+05	0.1228E+05
REGRESSION ON VARIANCE	1.076	-20.91	278.7	-1307.	2971.

ESTIMATOR: ***Rho(.8954) of SortData L laplace AR r.v.s, A=.86, L= .4 Using moment estimator
VERTICAL SCALE: YMIN = 0.2001
YMAX = 1.2010

Figure 33. Individual Plot, L-Laplace samples. Moment Estimator.

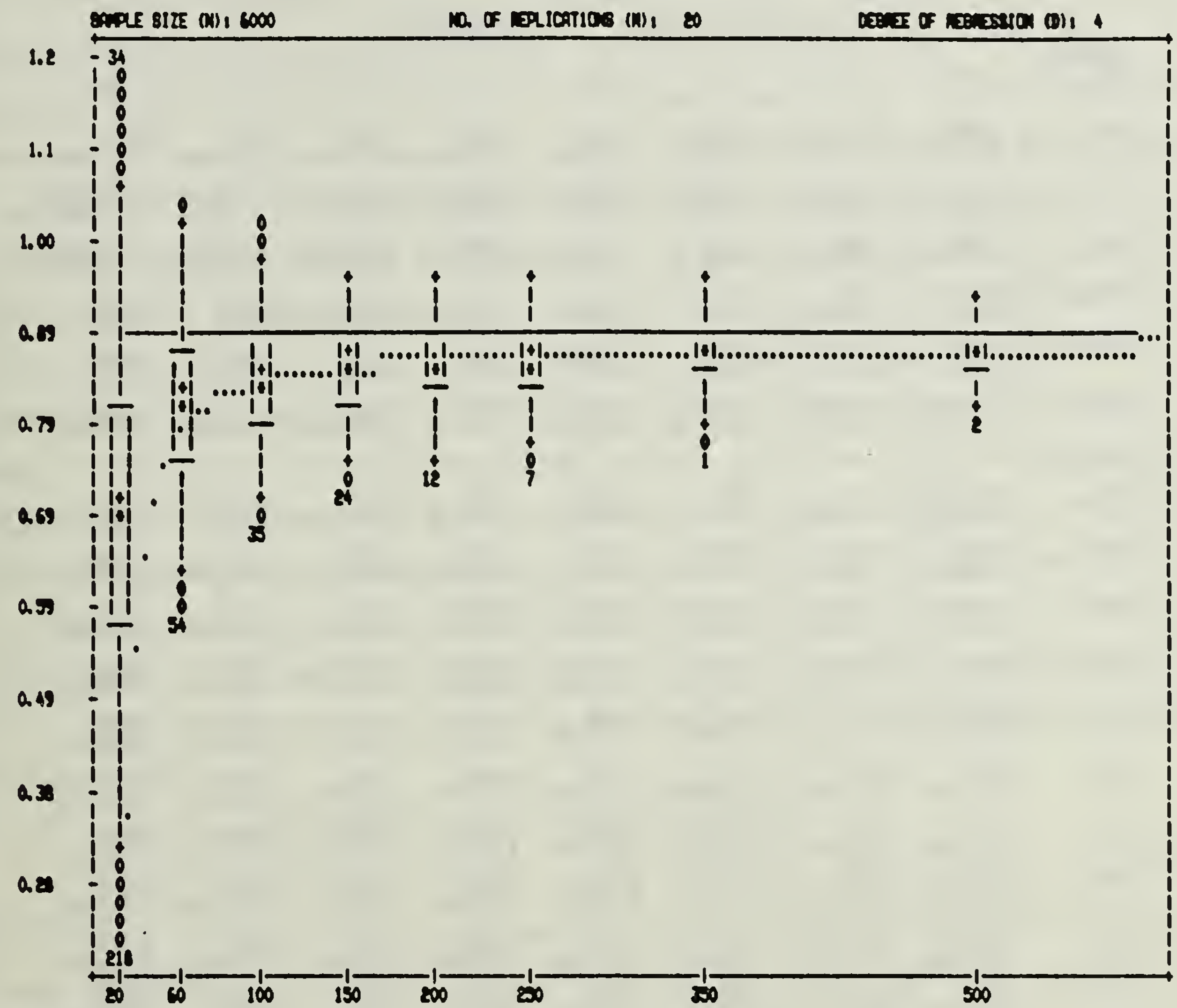
SUMMARY STATISTICS (MEAN/STD) 3 SUPER-REPLICATIONS

SUBSAMPLE SIZE	20	60	100	150	200	250	300	500
MEAN	0.6780 0.1911E-02	0.8097 0.1431E-02	0.8424 0.1761E-02	0.8564 0.4364E-03	0.8675 0.8314E-03	0.8722 0.3537E-03	0.8792 0.1195E-02	0.8843 0.1315E-03
STD	0.2092 0.2847E-02	0.1041 0.1918E-02	0.7424E-01 0.3836E-03	0.6070E-01 0.5734E-03	0.5187E-01 0.6330E-03	0.4574E-01 0.5868E-03	0.3678E-01 0.1219E-02	0.3291E-01 0.1362E-02
M.S.E	0.3017 0.3148E-02	0.1349 0.2338E-02	0.9124E-01 0.1093E-02	0.7217E-01 0.3027E-03	0.5889E-01 0.9496E-03	0.5127E-01 0.5508E-03	0.4021E-01 0.1448E-02	0.3473E-01 0.1295E-02
SKEWNESS	-1.234 0.4417E-01	-1.462 0.8155E-01	-1.147 0.2810E-01	-0.9669 0.3273E-01	-1.011 0.8888E-01	-1.064 0.1202	-0.6392 0.1193	-0.5218 0.1423
KURTOSIS	3.415 0.6658	4.495 0.7795	2.480 0.2335	1.566 0.1604	2.228 0.7921	2.228 1.074	0.6608 0.3768	0.8281 0.1460
SER.COR.	0.1091E-02 0.9779E-02	-0.1286E-01 0.9311E-02	-0.3747E-02 0.2269E-01	-0.1115E-01 0.1050E-01	-0.3047E-01 0.3217E-01	-0.2917E-01 0.3039E-01	-0.2409E-01 0.2516E-01	-0.3269E-01 0.2135E-01
QUANTILES								
0.010	0.1745E-01 0.1193E-01	0.4673 0.5535E-02	0.6081 0.7203E-02	0.6710 0.2910E-02	0.7104 0.4527E-02	0.7366 0.1237E-02	0.7694 0.8422E-02	0.7942 0.8788E-02
0.025	0.1445 0.1493E-01	0.5522 0.1178E-01	0.6564 0.6193E-02	0.7117 0.3634E-02	0.7468 0.4933E-02	0.7667 0.3513E-02	0.7974 0.6365E-02	0.8102 0.8349E-02
0.050	0.2706 0.1259E-01	0.6126 0.3814E-02	0.7020 0.4603E-02	0.7439 0.3593E-02	0.7757 0.3582E-02	0.7911 0.3096E-02	0.8126 0.4229E-02	0.8273 0.5324E-02
0.100	0.4034 0.3955E-02	0.6765 0.2001E-02	0.7498 0.1674E-02	0.7748 0.7604E-03	0.8007 0.1168E-02	0.8149 0.2345E-02	0.8277 0.2122E-02	0.8430 0.2504E-02
0.250	0.5750 0.1214E-02	0.7597 0.1241E-02	0.8038 0.3524E-02	0.8234 0.1144E-02	0.8372 0.2020E-02	0.8469 0.5481E-03	0.8583 0.1522E-02	0.8637 0.3822E-03
0.500	0.7184 0.9326E-03	0.8299 0.1055E-02	0.8530 0.3266E-02	0.8650 0.9500E-03	0.8758 0.1305E-02	0.8763 0.9293E-03	0.8834 0.1758E-02	0.8859 0.8213E-03
0.750	0.8204 0.2220E-02	0.8826 0.1069E-03	0.8953 0.1696E-02	0.8998 0.5473E-03	0.9039 0.3689E-03	0.9058 0.4503E-03	0.9040 0.9161E-03	0.9080 0.8165E-03
0.900	0.8967 0.1318E-02	0.9177 0.2028E-03	0.9241 0.7723E-03	0.9260 0.1363E-02	0.9252 0.2907E-03	0.9244 0.6383E-03	0.9225 0.1420E-02	0.9244 0.1414E-02
0.950	0.9388 0.2298E-02	0.9360 0.1354E-03	0.9385 0.5640E-03	0.9373 0.1605E-02	0.9379 0.2345E-02	0.9364 0.2287E-02	0.9337 0.1546E-02	0.9346 0.1490E-02
0.975	0.9737 0.3595E-02	0.9505 0.7772E-03	0.9514 0.4390E-03	0.9477 0.1183E-02	0.9494 0.2847E-02	0.9437 0.2963E-02	0.9435 0.2258E-02	0.9418 0.1900E-02
0.990	1.019 0.3949E-02	0.9671 0.1739E-02	0.9642 0.1779E-02	0.9576 0.1129E-02	0.9588 0.2148E-02	0.9519 0.3983E-02	0.9496 0.3233E-02	0.9503 0.2112E-02

MEAN OF REGRESSION ON AVERAGES	0.8994 0.2727E-02	-8.065 1.583	370.7 262.3	-0.1590E+05 0.1315E+05	-6828. 5858.
STD DEV OF REGRESSION	0.6854E-02 0.2440E-03	3.506 0.1347	519.0 19.46	0.2565E+05 969.9	0.1143E+05 443.9
REGRESSION ON VARIANCE	0.2302 0.5168	13.27 21.70	-220.0 319.3	1548. 1938.	-3453. 4025.

ESTIMATOR: ***Rho(.8954) of SqrtBeta L laplace AR r.v.s, A=.86, L= .4 Using moment estimator

Figure 34. Summary Statistics, L-Laplace samples. Moment Estimator.



SUBSAMPLE SIZE	20	60	100	150	200	250	350	500
MEAN	0.6916	0.8159	0.8475	0.8622	0.8668	0.8736	0.8795	0.8821
STD	0.2207	0.1060	0.7792E-01	0.6166E-01	0.5145E-01	0.5020E-01	0.3849E-01	0.3113E-01
STD MEAN	0.2850E-02	0.2370E-02	0.2249E-02	0.2180E-02	0.2100E-02	0.2291E-02	0.2087E-02	0.2010E-02
M.S.E	0.3004	0.1325	0.9146E-01	0.7004E-01	0.5886E-01	0.5475E-01	0.4165E-01	0.3384E-01
SKEWNESS	-1.0495	-1.2991	-1.3054	-1.1431	-0.7851	-1.2163	-0.4819	-0.3851
KURTOSIS	4.2038	2.7093	3.5868	1.8081	0.6682	3.4218	-0.1259	0.1872

MEAN OF REGRESSION ON AVERAGES	0.8941	-6.197	283.8	-0.1500E+05	-6993.
VARIANCE OF REGRESSION	0.2596E-04	6.107	0.1254E+06	0.3067E+09	0.6150E+08
STD DEV OF REGRESSION	0.5095E-02	2.471	354.2	0.1751E+05	7842.
REGRESSION ON VARIANCE	-0.2214	30.63	-433.5	2626.	-5333.

ESTIMATOR: ***Rho(.8954) of SqrtData L laplace AR r.vs., A=.86, L= .4 Using Priestly Estimator
VERTICAL SCALE: YMIN = 0.2001
YMAX = 1.2010

Figure 35. Individual Plot, L-Laplace samples. Priestley Estimator.

SUMMARY STATISTICS (MEAN/STD)   3 SUPER-REPLICATIONS

SUBSAMPLE SIZE	20	60	100	150	200	250	350	500
MEAN	0.6922 0.1996E-02	0.8173 0.1005E-02	0.8465 0.6582E-03	0.8613 0.8907E-03	0.8672 0.2472E-03	0.8747 0.5956E-03	0.8793 0.8040E-03	0.8843 0.2046E-02
STD	0.2179 0.1521E-02	0.1045 0.7617E-03	0.7754E-01 0.9896E-03	0.6123E-01 0.4925E-03	0.5115E-01 0.5878E-03	0.4745E-01 0.1383E-02	0.3993E-01 0.1592E-02	0.3178E-01 0.1918E-02
M.S.E	0.2980 0.2173E-02	0.1305 0.1081E-02	0.9165E-01 0.1044E-02	0.7011E-01 0.7615E-03	0.5839E-01 0.6019E-03	0.5176E-01 0.1498E-02	0.4308E-01 0.1764E-02	0.3373E-01 0.2319E-02
SKEWNESS	-0.6748 0.3461	-1.309 0.1208E-01	-1.312 0.1067	-1.077 0.4553E-01	-0.7717 0.4269E-01	-1.060 0.9149E-01	-0.7598 0.1563	-0.5140 0.1586
KURTOSIS	7.173 3.784	3.002 0.1793	3.377 0.6567	1.632 0.2451	0.6659 0.1025	2.446 0.7007	1.071 0.6361	0.8035 0.6352
SER. COR.	-0.7771E-02 0.8092E-02	-0.2922E-02 0.2298E-01	0.3977E-02 0.7994E-02	-0.5766E-02 0.1218E-01	0.2362E-01 0.1875E-01	-0.8591E-01 0.1755E-01	-0.1704E-01 0.2345E-01	-0.4054E-03 0.4468E-01
QUANTILES								
0.010	0.2906E-01 0.5127E-02	0.4773 0.1222E-01	0.5975 0.1115E-01	0.6678 0.9300E-02	0.7097 0.2291E-02	0.7271 0.4341E-02	0.7544 0.1293E-01	0.7918 0.1073E-01
0.025	0.1636 0.6942E-02	0.5619 0.2401E-02	0.6579 0.7570E-02	0.7061 0.3901E-02	0.7465 0.3780E-02	0.7652 0.8022E-02	0.7825 0.3971E-02	0.8189 0.4818E-02
0.050	0.2814 0.1212E-01	0.6210 0.5963E-02	0.7015 0.2258E-02	0.7437 0.2443E-02	0.7743 0.4263E-02	0.7888 0.5360E-02	0.8065 0.7377E-02	0.8293 0.4588E-02
0.100	0.4141 0.5720E-02	0.6796 0.1130E-02	0.7447 0.2840E-02	0.7817 0.1727E-02	0.7982 0.6298E-03	0.8120 0.2318E-02	0.8274 0.2835E-02	0.8418 0.3207E-02
0.250	0.5865 0.2647E-02	0.7661 0.1312E-02	0.8075 0.9928E-03	0.8285 0.2186E-02	0.8366 0.1343E-03	0.8479 0.2117E-02	0.8564 0.1633E-02	0.8649 0.2932E-02
0.500	0.7294 0.6251E-03	0.8367 0.5752E-03	0.8605 0.5234E-03	0.8730 0.1419E-02	0.8743 0.1043E-02	0.8823 0.1018E-02	0.8843 0.2041E-02	0.8861 0.2000E-02
0.750	0.8312 0.1333E-03	0.8922 0.5014E-03	0.9006 0.2348E-03	0.9053 0.6194E-03	0.9036 0.5596E-03	0.9088 0.8939E-03	0.9080 0.9828E-03	0.9063 0.1212E-02
0.900	0.9157 0.1410E-02	0.9271 0.2286E-02	0.9294 0.1657E-02	0.9288 0.7199E-03	0.9273 0.1114E-02	0.9281 0.5149E-03	0.9251 0.1889E-02	0.9250 0.1955E-02
0.950	0.9636 0.1015E-02	0.9451 0.2129E-02	0.9443 0.1206E-02	0.9398 0.1301E-02	0.9380 0.5003E-03	0.9391 0.1147E-02	0.9340 0.2261E-02	0.9334 0.6556E-03
0.975	1.006 0.9600E-03	0.9609 0.2363E-02	0.9551 0.9767E-03	0.9490 0.1330E-02	0.9460 0.4723E-03	0.9468 0.1002E-02	0.9444 0.2663E-02	0.9408 0.9818E-03
0.990	1.080 0.5931E-02	0.9832 0.2849E-02	0.9700 0.2060E-02	0.9579 0.1155E-02	0.9564 0.2176E-02	0.9548 0.1444E-02	0.9560 0.3260E-02	0.9480 0.2170E-02

MEAN OF REGRESSION ON AVERAGES	0.8968 0.3346E-02	-6.769 1.514	280.0 227.1	-0.1216E+05 0.1177E+05	-5242. 5380.
STD DEV OF REGRESSION	0.6000E-02 0.6609E-03	3.036 0.3539	444.5 55.04	0.2205E+05 2766.	9859. 1234.
REGRESSION ON VARIANCE	0.5931 0.7454	-3.307 31.71	53.19 468.7	-245.6 2826.	472.9 5802.

ESTIMATOR: ***Rho(.8954) of SqrtBeta L laplace AR r.vs., A=.86, L= .4 Using Priestly Estimator

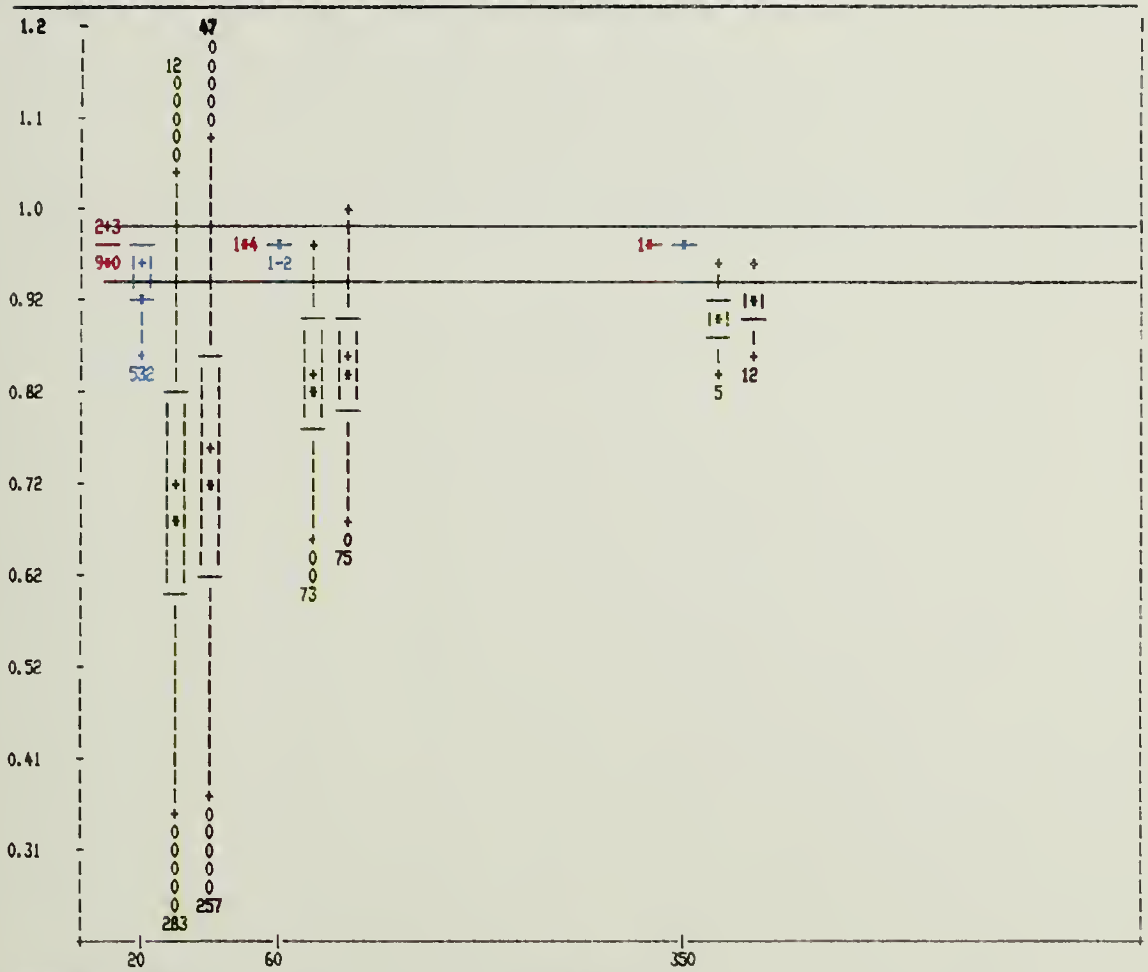
Figure 36. Summary Statistics, L-Laplace samples. Priestley Estimator.

two distinct classes: the robust regression and the Cressie estimator comprise one group, and the moment and MLE estimators comprise the other. The first group is strongly biased, while the second group converges, but with slower and slower progress, as this particular sample distribution (BELAR(1)) becomes more extreme. Recall that when the parameter l is small, and the samples are correlated, the behavior of the sample stream is such that there are long runs of numbers very close to zero, and then peaks of a few positive (or negative) numbers are interspersed. Figure 38 shows the unacceptably biased behavior of the robust regression, and Figure 39 has the super-replications summary for this estimator's run. Figures 40 and 41 show the similar behavior for the Cressie estimator. Recall that the Cressie estimator showed very sensitive behavior with respect to Normality, in that it immediately began to show bias when l was set to three. Figures 42 and 43 represent the moment estimator. One might say, without the previous simulation results, that this estimator also is exhibiting bias, but I would comment that the MSE is still decreasing at sub-sample size 500, and is still roughly twice the corresponding standard deviation, so it is possible that this estimator is converging, but very slowly. The same comments apply to the MLE estimator, results for which are in Figures 44 and 45.

5. Large-Sample Simulation with Robust Regression.

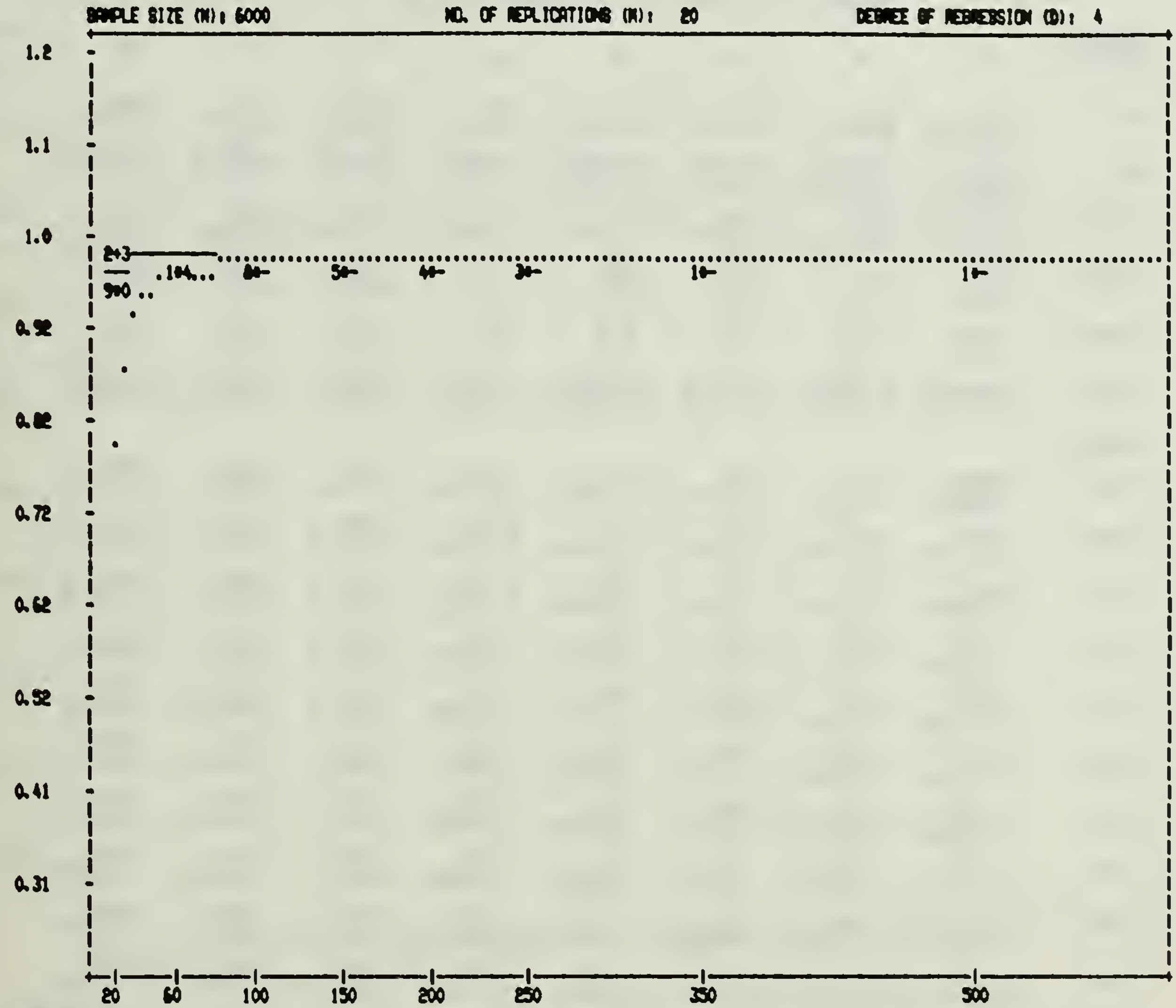
Before presenting the results on the simulation involving the uncorrelated BELAR(1) processes, we present the results for a large-sample simulation involving the robust regression. It was desired to determine if the robust regression estimator was actually converging, but in the process it might have been actually deviating more from the true value for some range of sample sizes. In other words, we were investigating the possibility of a non-monotonic convergance to the true correlation value. To do this, we ran a SIMTBED simulation with only the robust regression estimator, and used very large sample sizes, as can be seen in Figure 46. Here, sub-sample sizes up to and including 5000 were used, with the overall sample size for the simulation increased to 10000, and the number of replications increase to 50. Thus, at sub-sample size 5000, there are 200 evaluations of the estimator represented by that boxplot. This seemed an acceptable minimum number of estimator evaluations. Figure 46 shows that there is apparently no convergance, even at this extreme, and Figure 47 contains the super-replications summary statistics, for three super-replications. The parameter l used was 0.4, a midranged value that did not cause any problems, and the simulation was conducted on two separate versions of the robust regression estimator: one, with no additional scaling, and the other, with the 0.6745 scaling, referred to in Chapter IV, and used

SMTB COMBINED ESTIMATOR BOXPLOTS: SAMPLE SIZE: 6000 NO. REPLICATIONS: 20 DEG REGRESSION: 4




```
SUBSAMPLE SIZE:                    20            60            350
ESTIMATOR 1:   MEAN:           0.9715        0.9957        0.9984
               STD:            0.1257        0.2775E-01    0.1394E-02
               STD MEAN:       0.1623E-02    0.6204E-03    0.7559E-04

SUBSAMPLE SIZE:                    20            60            350
ESTIMATOR 2:   MEAN:           0.9424        0.9881        0.9953
               STD:            0.1013        0.1296E-01    0.2487E-02
               STD MEAN:       0.1308E-02    0.2898E-03    0.1349E-03

SUBSAMPLE SIZE:                    20            60            350
ESTIMATOR 3:   MEAN:           0.6922        0.8380        0.9162
               STD:            0.2172        0.1032        0.3393E-01
               STD MEAN:       0.2804E-02    0.2307E-02    0.1840E-02

SUBSAMPLE SIZE:                  . 20            60            350
ESTIMATOR 4:   MEAN:           0.7208        0.8524        0.9273
               STD:            0.2671        0.1023        0.3063E-01
               STD MEAN:       0.3448E-02    0.2287E-02    0.1661E-02

ESTIMATOR 1: +++Rho(.8920) of SqrtBeta L laplace AR r.v.s, A=.88, L= .1 Using Robust Least Squares estimator
ESTIMATOR 2: +++Rho(.8920) of SqrtBeta L laplace AR r.vs., A=.88, L= .1 Using Cressie Estimator
ESTIMATOR 3: +++Rho(.8920) of SqrtBeta L laplace AR r.v.s, A=.88, L= .1 Using moment estimator
ESTIMATOR 4: +++Rho(.8920) of SqrtBeta L laplace AR r.vs., A=.88, L= .1 Using Priestly Estimator

+++ WIDEST Y VALUES FOUND: YMIN=-8.614    - YMAX= 13.41     +++ SMTB +++
+++ THIS WAS A RESTART RUN. NSR START/END=      1         3
```

Figure 37. Combined Plot, L-laplace samples. Rho = 0.8920. L = 0.1.



SUBSAMPLE SIZE	20	60	100	150	200	250	350	500
MEAN	0.9715	0.9957	0.9979	0.9978	0.9982	0.9984	0.9984	0.9985
STD	0.1257	0.2775E-01	0.4695E-02	0.5166E-02	0.3133E-02	0.1621E-02	0.1794E-02	0.1246E-02
STD MEAN	0.1623E-02	0.6204E-03	0.1355E-03	0.1827E-03	0.1279E-03	0.7399E-04	0.7559E-04	0.8043E-04
M.S.E	0.1488	0.1073	0.1060	0.1060	0.1063	0.1065	0.1064	0.1066
SKEWNESS	-4.7388	-22.8426	-5.2782	-9.4658	-10.0054	-4.2355	-3.1936	-2.9853
KURTOSIS	44.8429	689.0498	57.2943	108.4055	146.8451	35.4088	16.7977	14.0313

MEAN OF REGRESSION ON AVERAGES	0.9993	-0.4586	65.11	-3880.	-1745.
VARIANCE OF REGRESSION	0.9372E-07	0.3012E-01	829.7	0.2725E+07	0.6210E+06
STD DEV OF REGRESSION	0.3061E-03	0.1736	28.80	1651.	788.1
REGRESSION ON VARIANCE	-0.2001	9.641	-159.2	1047.	-2154.

ESTIMATOR: ***Rho(.8920) of SqrtBeta L laplace AR r.v.s, A=.88, L= .1 Using Robust Least Squares estimator
VERTICAL SCALE: YMIN = 0.2286
YMAX = 1.2314

Figure 38. Individual Plot, L-Laplace samples. Robust Least Squares.

SUMMARY STATISTICS (MEAN/STD)   3 SUPER-REPLICATIONS

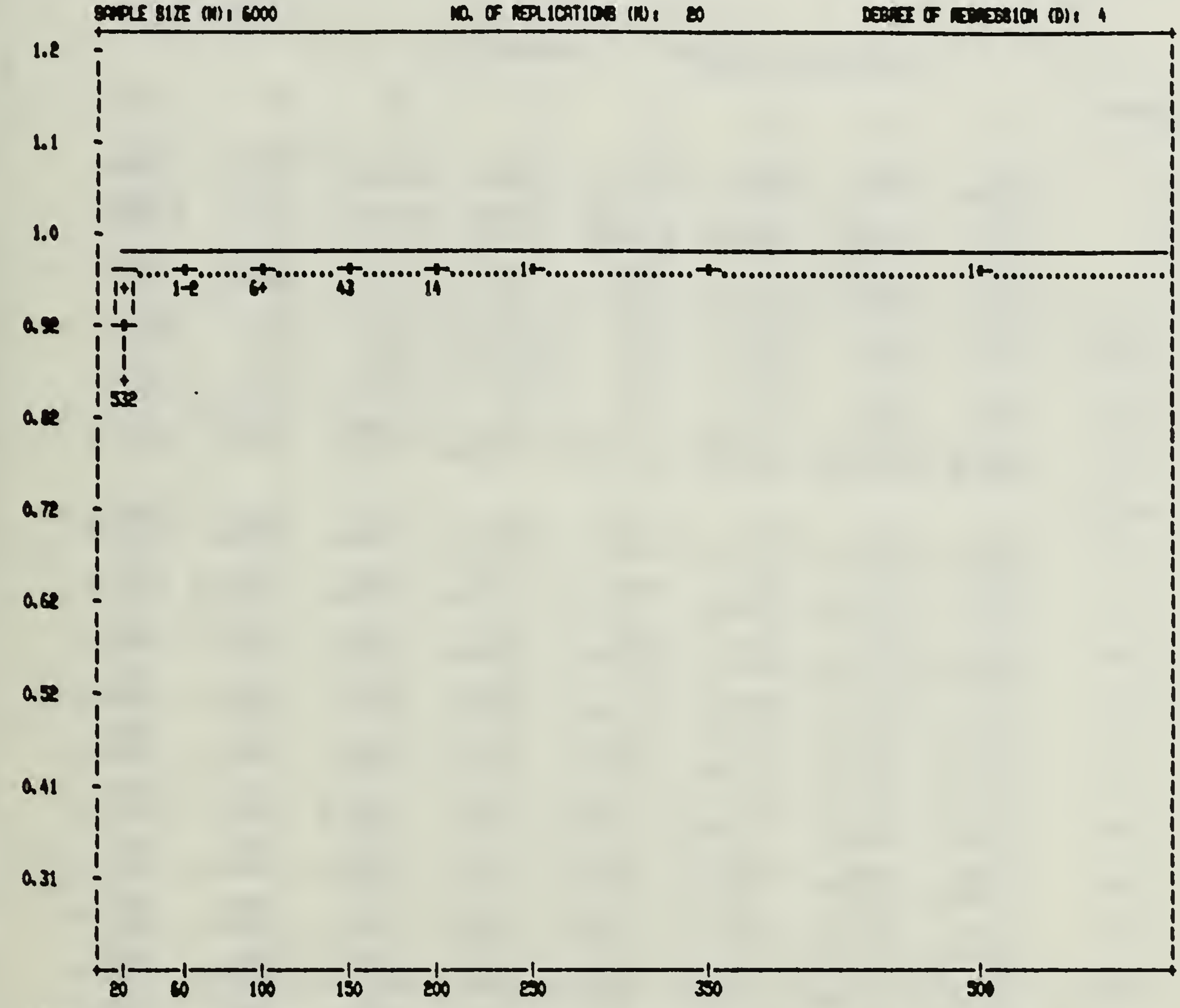
SUBSAMPLE SIZE	20	60	100	150	200	250	350	500
MEAN	0.9713 0.5492E-03	0.9958 0.8935E-04	0.9978 0.1253E-03	0.9979 0.6012E-04	0.9984 0.7812E-04	0.9984 0.4512E-04	0.9984 0.2877E-04	0.9985 0.1307E-04
STD	0.1252 0.6745E-03	0.2141E-01 0.3267E-02	0.5740E-02 0.7118E-03	0.4755E-02 0.2400E-03	0.2502E-02 0.3507E-03	0.1774E-02 0.1305E-03	0.1397E-02 0.2194E-04	0.1161E-02 0.4401E-04
M.S.E	0.1482 0.3783E-03	0.1061 0.6198E-03	0.1059 0.8543E-04	0.1060 0.5618E-04	0.1064 0.6981E-04	0.1064 0.4304E-04	0.1064 0.2853E-04	0.1066 0.1309E-04
SKEWNESS	-4.767 0.6725	-15.01 2.932	-8.406 1.567	-8.249 0.7570	-7.091 1.527	-4.760 0.4189	-3.834 0.2719	-2.391 0.4581
KURTOSIS	46.25 2.054	335.4 177.3	112.2 28.17	84.13 14.40	81.41 33.42	40.47 6.555	16.75 2.433	10.13 2.625
SER.COR.	-0.8004E-02 0.7671E-02	-0.1397E-01 0.7133E-02	-0.7664E-02 0.1340E-02	-0.2928E-01 0.2067E-01	-0.4550E-01 0.1494E-01	-0.8640E-02 0.1191E-01	0.6603E-01 0.1198E-01	-0.3674E-01 0.3410E-01
QUANTILES								
0.010	0.2227 0.1897E-01	0.9296 0.1463E-02	0.9720 0.4566E-02	0.9765 0.2491E-02	0.9895 0.8932E-03	0.9922 0.8967E-03	0.9924 0.4880E-03	0.9935 0.6432E-03
0.025	0.6248 0.9648E-02	0.9715 0.1189E-02	0.9874 0.2554E-03	0.9912 0.4285E-03	0.9937 0.4723E-03	0.9945 0.5468E-03	0.9951 0.1222E-03	0.9956 0.1352E-03
0.050	0.8395 0.5568E-02	0.9872 0.6076E-03	0.9928 0.1789E-03	0.9941 0.3058E-03	0.9953 0.2900E-03	0.9956 0.2952E-03	0.9963 0.7460E-04	0.9963 0.1458E-03
0.100	0.9557 0.2446E-02	0.9930 0.3384E-03	0.9952 0.1261E-03	0.9960 0.1960E-03	0.9967 0.7536E-04	0.9968 0.1637E-03	0.9970 0.5122E-04	0.9972 0.9856E-04
0.250	0.9930 0.2871E-03	0.9971 0.3133E-04	0.9976 0.5879E-04	0.9978 0.1040E-03	0.9979 0.5461E-04	0.9980 0.4616E-04	0.9979 0.8444E-04	0.9981 0.5306E-04
0.500	0.9991 0.5909E-04	0.9990 0.3071E-04	0.9990 0.4003E-04	0.9989 0.3532E-04	0.9989 0.3976E-04	0.9988 0.4092E-04	0.9987 0.1948E-04	0.9988 0.2988E-04
0.750	1.001 0.5545E-04	0.9999 0.2085E-04	0.9997 0.2186E-04	0.9995 0.2506E-04	0.9995 0.1893E-04	0.9994 0.2671E-04	0.9993 0.2194E-04	0.9993 0.1019E-04
0.900	1.005 0.1304E-03	1.001 0.1465E-04	1.000 0.1714E-04	1.0000 0.1260E-04	0.9999 0.1288E-04	0.9998 0.3062E-04	0.9997 0.1876E-04	0.9996 0.1064E-04
0.950	1.010 0.4874E-04	1.002 0.6724E-04	1.001 0.6691E-04	1.000 0.3458E-04	1.000 0.1402E-04	0.9999 0.1439E-04	0.9998 0.3906E-04	0.9997 0.1415E-04
0.975	1.019 0.5671E-03	1.003 0.3986E-04	1.001 0.7207E-04	1.000 0.4158E-04	1.000 0.6147E-04	1.000 0.5559E-05	1.0000 0.8099E-04	0.9999 0.5081E-05
0.990	1.039 0.1120E-02	1.004 0.1600E-03	1.002 0.2215E-03	1.001 0.5776E-04	1.001 0.7210E-04	1.000 0.6023E-04	1.000 0.8152E-04	1.0000 0.2028E-04

MEAN OF REGRESSION ON AVERAGES	0.9989 0.2287E-03	-0.2135 0.1446	24.89 23.82	-1771. 1237.	-781.7 562.9
STD DEV OF REGRESSION	0.2987E-03 0.2739E-04	0.1737 0.1612E-01	28.86 2.343	1537. 129.2	715.7 63.86
REGRESSION ON VARIANCE	-0.8225E-01 0.5893E-01	3.906 2.868	-62.73 48.21	392.3 227.2	-691.1 731.6

ESTIMATOR: ***Rho(.8920) of SqrtBeta L laplace AR r.v.s, A=.88, L= .1 Using Robust Least Squares estimator

Figure 39. Summary Statistics, L-Laplace samples. Robust Least Squares.

SAMPLE SIZE (N): 6000 NO. OF REPLICATIONS (M): 20 DEGREE OF REGRESSION (D): 4



SUBSAMPLE SIZE	20	60	100	150	200	250	350	500
MEAN	0.9424	0.9881	0.9920	0.9933	0.9944	0.9948	0.9953	0.9955
STD	0.1013	0.1296E-01	0.7529E-02	0.5375E-02	0.3793E-02	0.3295E-02	0.2487E-02	0.2279E-02
STD MEAN	0.1308E-02	0.2898E-03	0.2174E-03	0.1901E-03	0.1549E-03	0.1504E-03	0.1349E-03	0.1471E-03
M.S.E	0.1132	0.9697E-01	0.1003	0.1015	0.1025	0.1029	0.1033	0.1035
SKEWNESS	-7.7622	-2.8478	-2.9736	-2.3316	-2.2688	-1.7591	-1.4646	-1.2795
KURTOSIS	105.0954	12.0436	16.4570	8.2732	10.6281	5.6092	2.9702	1.5526

MEAN OF REGRESSION ON AVERAGES	0.9963	-0.3724	-6.121	-27.50	90.89
VARIANCE OF REGRESSION	0.2394E-06	0.7193E-01	1762.	0.4631E+07	0.9473E+06
STD DEV OF REGRESSION	0.4893E-03	0.2682	41.97	2152.	973.3
REGRESSION ON VARIANCE	0.2762E-01	-1.328	24.43	-187.3	539.0

ESTIMATOR: ***Rho(.8920) of SqrtBeta L laplace AR r.vs., A=.88, L= .1 Using Cressie Estimator
VERTICAL SCALE: YMIN = 0.2286
YMAX = 1.2314

Figure 40. Individual Plot, L-Laplace samples. Cressie Estimator.

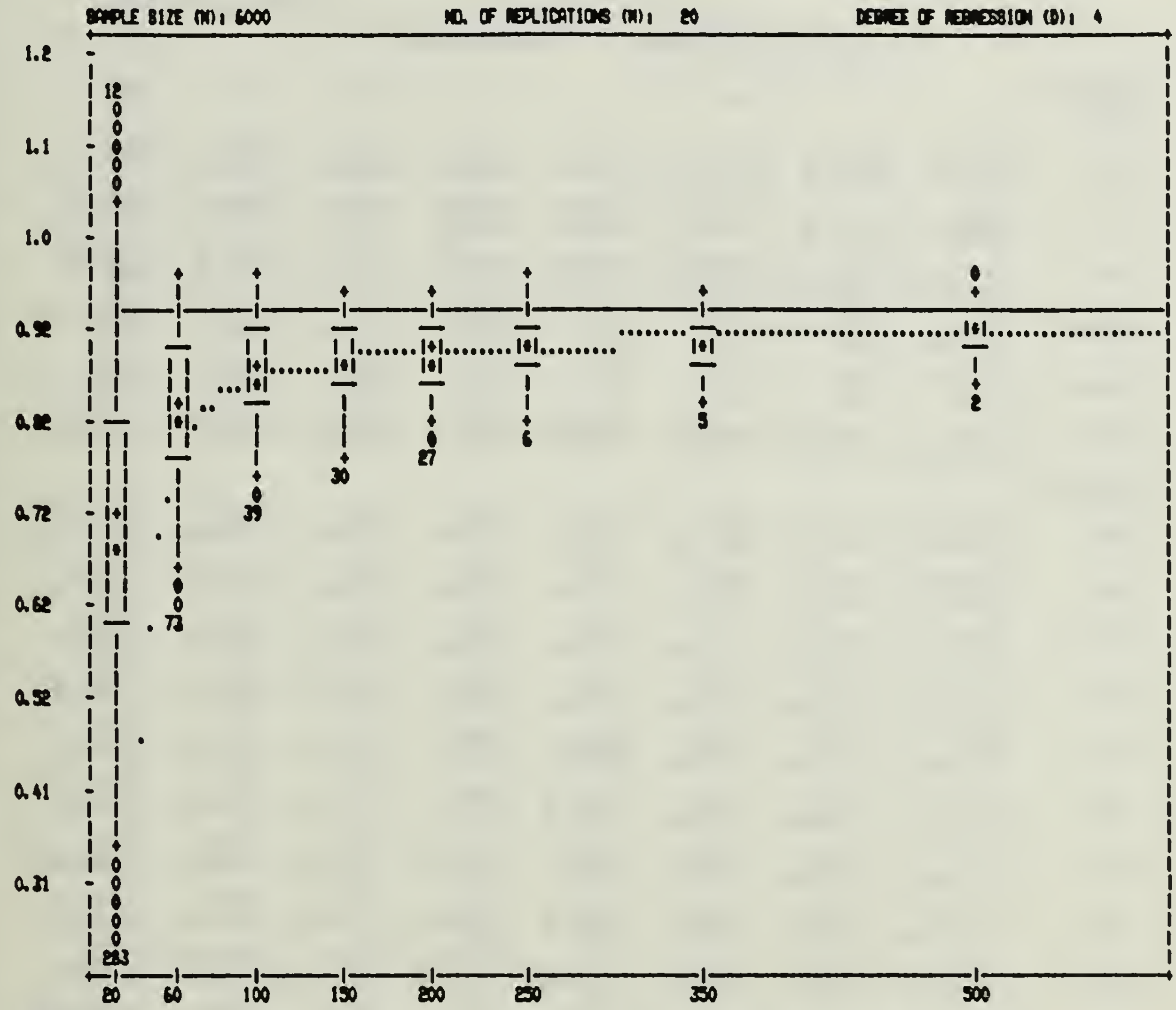
SUMMARY STATISTICS (MEAN/STD) 3 SUPER-REPLICATIONS

SUBSAMPLE SIZE	20	60	100	150	200	250	300	500
MEAN	0.9406 0.9420E-03	0.9880 0.2439E-03	0.9919 0.8044E-04	0.9936 0.1795E-03	0.9943 0.6282E-04	0.9948 0.5820E-04	0.9953 0.2280E-04	0.9956 0.1109E-03
STD	0.1020 0.1380E-02	0.1322E-01 0.2693E-03	0.7408E-02 0.1906E-03	0.4991E-02 0.1988E-03	0.4022E-02 0.1709E-03	0.3319E-02 0.1311E-03	0.2519E-02 0.4513E-04	0.2074E-02 0.1319E-03
M.S.E	0.1130 0.1311E-02	0.9694E-01 0.2096E-03	0.1002 0.7414E-04	0.1018 0.1708E-03	0.1024 0.7908E-04	0.1028 0.5458E-04	0.1033 0.2390E-04	0.1036 0.1087E-03
SKEWNESS	-7.909 1.179	-3.133 0.1845	-2.437 0.3107	-2.069 0.1431	-2.098 0.8548E-01	-1.787 0.1184	-1.484 0.1714	-1.272 0.4614E-01
KURTOSIS	114.2 42.54	15.31 2.130	10.20 2.473	6.394 1.028	7.556 1.537	5.338 0.8520	4.296 0.6463	2.578 0.6541
SER.COR.	0.1180E-01 0.4621E-02	-0.5708E-02 0.4222E-02	0.2217E-02 0.6245E-02	0.2188E-01 0.1741E-01	-0.3723E-01 0.1264E-01	-0.1600E-01 0.1608E-01	-0.2424E-01 0.5491E-01	-0.1003E-01 0.6622E-01
QUANTILES								
0.010	0.5479 0.1462E-01	0.9346 0.1082E-02	0.9642 0.7932E-03	0.9745 0.1786E-02	0.9793 0.1244E-02	0.9827 0.7265E-03	0.9859 0.8598E-03	0.9883 0.6522E-03
0.025	0.6959 0.6402E-02	0.9516 0.1728E-02	0.9718 0.1698E-03	0.9801 0.7242E-03	0.9839 0.1067E-02	0.9859 0.5057E-03	0.9888 0.3841E-03	0.9901 0.6315E-03
0.050	0.7858 0.4643E-02	0.9624 0.1083E-02	0.9772 0.3203E-03	0.9842 0.6546E-03	0.9868 0.7659E-03	0.9885 0.5688E-03	0.9910 0.5489E-04	0.9915 0.6983E-03
0.100	0.8594 0.3922E-02	0.9739 0.8169E-03	0.9826 0.2729E-03	0.9875 0.4370E-04	0.9896 0.2569E-03	0.9907 0.2497E-03	0.9922 0.4819E-04	0.9930 0.3052E-03
0.250	0.9336 0.7610E-03	0.9851 0.3619E-03	0.9894 0.1814E-03	0.9918 0.1954E-03	0.9927 0.2299E-03	0.9933 0.8934E-04	0.9940 0.1490E-03	0.9945 0.1129E-03
0.500	0.9717 0.4111E-03	0.9921 0.9027E-04	0.9942 0.1598E-04	0.9950 0.1194E-03	0.9954 0.1429E-04	0.9955 0.6802E-04	0.9957 0.6390E-04	0.9960 0.1288E-03
0.750	0.9883 0.1925E-03	0.9961 0.5869E-04	0.9968 0.3537E-04	0.9970 0.1190E-03	0.9971 0.3440E-04	0.9971 0.1077E-03	0.9971 0.1248E-04	0.9971 0.6525E-04
0.900	0.9948 0.9667E-04	0.9980 0.5396E-04	0.9984 0.4495E-04	0.9982 0.1184E-03	0.9981 0.4054E-04	0.9981 0.1542E-04	0.9980 0.6691E-04	0.9978 0.3074E-04
0.950	0.9968 0.1687E-04	0.9987 0.4160E-04	0.9989 0.3551E-04	0.9986 0.8959E-04	0.9986 0.4184E-04	0.9985 0.3437E-04	0.9984 0.8994E-04	0.9982 0.2256E-04
0.975	0.9979 0.1701E-04	0.9990 0.4454E-04	0.9992 0.3104E-04	0.9990 0.6248E-04	0.9989 0.3441E-04	0.9989 0.2800E-04	0.9987 0.4590E-04	0.9985 0.5719E-04
0.990	0.9987 0.3739E-04	0.9993 0.1856E-04	0.9995 0.3444E-04	0.9993 0.5445E-04	0.9992 0.2231E-04	0.9992 0.1781E-04	0.9990 0.9981E-04	0.9987 0.7232E-04

MEAN OF REGRESSION ON AVERAGES	0.9963 0.5399E-03	-0.3382 0.2904	-11.37 43.18	208.6 2196.	200.3 993.8
STD DEV OF REGRESSION ·	0.5007E-03 0.2895E-04	0.2781 0.8730E-02	42.43 0.8919	2148. 29.63	969.5 10.14
REGRESSION ON VARIANCE	0.1783E-01 0.1195E-01	-0.8824 0.5372	17.43 8.222	-143.1 50.37	446.2 101.9

ESTIMATOR: ***Rho(.8920) of SqrtBeta L laplace AR r.vs., A=.88, L= .1 Using Cressie Estimator

Figure 41. Summary Statistics, L-Laplace samples. Cressie Estimator.



SUBSAMPLE SIZE	20	60	100	150	200	250	350	500
MEAN	0.6922	0.8380	0.8776	0.8914	0.9034	0.9114	0.9162	0.9258
STD	0.2172	0.1032	0.6838E-01	0.5799E-01	0.4955E-01	0.4150E-01	0.3393E-01	0.2774E-01
STD MEAN	0.2804E-02	0.2307E-02	0.1974E-02	0.2050E-02	0.2023E-02	0.1894E-02	0.1840E-02	0.1791E-02
M.S.E	0.2951	0.1165	0.6988E-01	0.5799E-01	0.5085E-01	0.4582E-01	0.4167E-01	0.4376E-01
SKEWNESS	-1.4629	-2.1191	-1.4026	-1.7898	-1.6836	-1.0257	-0.7269	-0.6056
KURTOSIS	4.1398	9.0264	2.9646	7.2113	4.5876	1.4091	0.5493	0.4609

MEAN OF REGRESSION ON AVERAGES	0.9471	-12.50	929.5	-0.4369E+05	-0.1928E+05
VARIANCE OF REGRESSION	0.4200E-04	11.77	0.2688E+06	0.6678E+09	0.1234E+09
STD DEV OF REGRESSION	0.6480E-02	3.430	518.4	0.2584E+05	0.1155E+05
REGRESSION ON VARIANCE	-1.363	76.75	-1148.	7150.	-0.1495E+05

ESTIMATOR: ***Rho(.8920) of SqrtBeta L laplace AR r.v.s, A=.88, L= .1 Using moment estimator
VERTICAL SCALE: YMIN = 0.2286
YMAX = 1.2314

Figure 42. Individual Plot, L-Laplace samples. Moment Estimator.

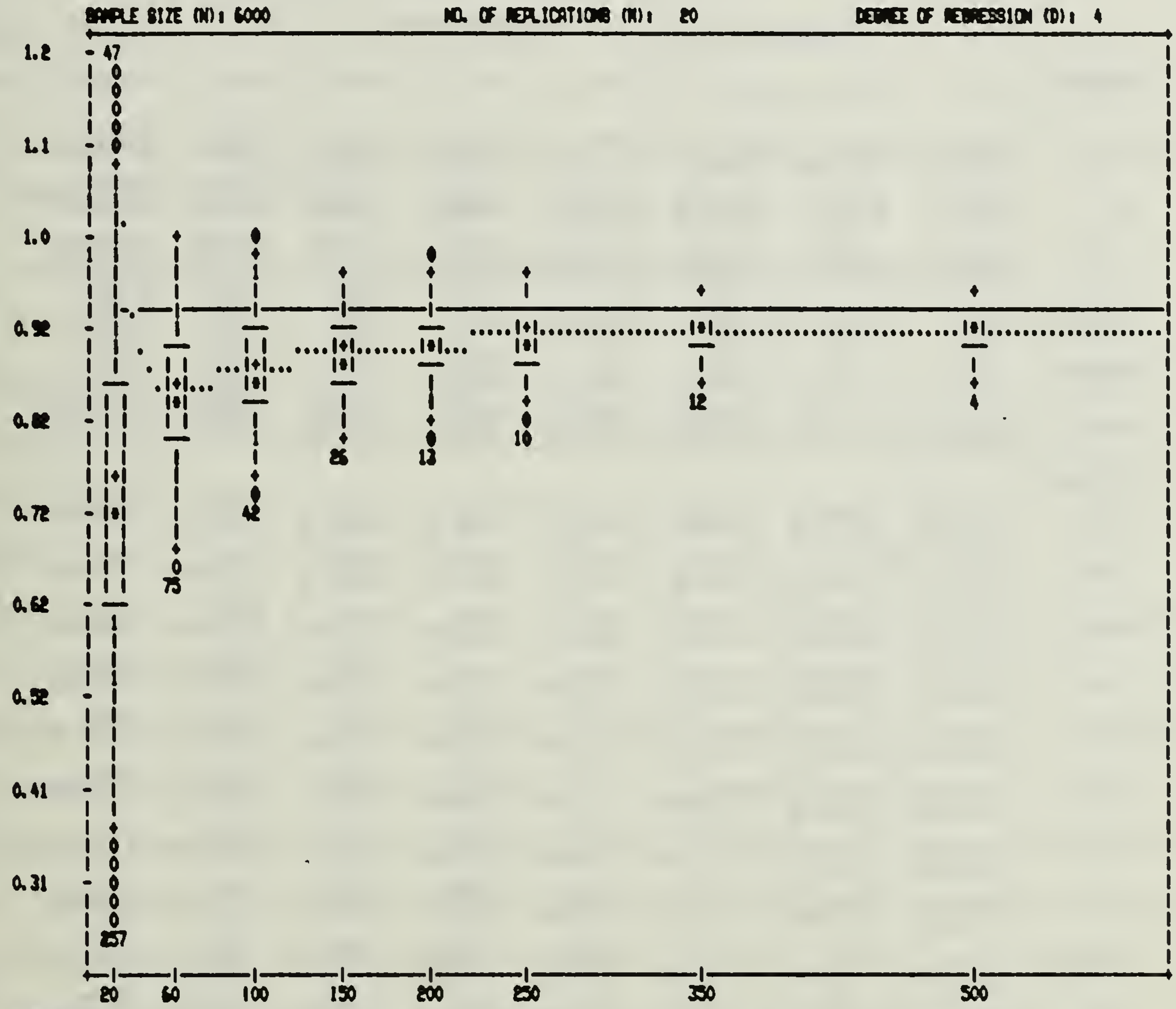
SUMMARY STATISTICS (MEAN/STD)   3 SUPER-REPLICATIONS

SUBSAMPLE SIZE	20	60	100	150	200	250	350	500
MEAN	0.6911 0.7457E-03	0.8390 0.7985E-03	0.8751 0.1288E-02	0.8934 0.1738E-02	0.9033 0.1020E-03	0.9107 0.6700E-03	0.9177 0.7620E-03	0.9246 0.7355E-03
STD	0.2163 0.8915E-03	0.1019 0.2742E-02	0.7215E-01 0.1913E-02	0.5729E-01 0.2291E-02	0.4761E-01 0.1250E-02	0.4045E-01 0.7989E-03	0.3538E-01 0.7374E-03	0.2741E-01 0.3552E-03
M.S.E	0.2952 0.9025E-03	0.1149 0.2768E-02	0.7413E-01 0.2148E-02	0.5736E-01 0.2222E-02	0.4895E-01 0.1205E-02	0.4459E-01 0.6867E-03	0.4374E-01 0.1034E-02	0.4260E-01 0.5872E-03
SKEWNESS	-1.424 0.4621E-01	-1.902 0.1155	-1.638 0.1313	-1.672 0.1770	-1.460 0.1423	-1.037 0.1212E-01	-1.169 0.2609	-0.7060 0.6878E-01
KURTOSIS	3.469 0.2648	6.877 1.081	4.589 0.9852	5.171 1.277	3.767 0.6313	1.608 0.1055	2.517 1.488	0.7175 0.2042
SER.COR.	-0.7540E-02 0.6988E-02	-0.3623E-01 0.1601E-01	-0.1903E-01 0.1521E-02	-0.3625E-02 0.1619E-01	0.1657E-02 0.2580E-01	0.1306E-01 0.1631E-01	-0.1655E-01 0.4430E-01	-0.4494E-01 0.1992E-01
QUANTILES								
0.010	-0.1294E-01 0.8736E-02	0.4857 0.1970E-01	0.6249 0.5939E-02	0.6903 0.2633E-01	0.7352 0.8955E-02	0.7837 0.3619E-02	0.8067 0.7611E-02	0.8413 0.7241E-02
0.025	0.9385E-01 0.5890E-02	0.5818 0.1425E-01	0.6919 0.5960E-02	0.7439 0.5882E-02	0.7842 0.7555E-02	0.8142 0.2814E-02	0.8304 0.6351E-03	0.8629 0.5765E-03
0.050	0.2459 0.7769E-02	0.6494 0.7445E-02	0.7391 0.6524E-02	0.7851 0.6514E-02	0.8188 0.2629E-02	0.8351 0.3494E-02	0.8535 0.4891E-02	0.8739 0.5872E-03
0.100	0.4118 0.3076E-02	0.7133 0.5155E-02	0.7831 0.3507E-02	0.8241 0.2000E-02	0.8433 0.2580E-02	0.8568 0.1727E-02	0.8708 0.1650E-02	0.8888 0.1501E-02
0.250	0.6007 0.1193E-02	0.7940 0.1685E-02	0.8430 0.1214E-02	0.8693 0.2570E-02	0.8802 0.4723E-03	0.8876 0.3211E-03	0.8996 0.2041E-02	0.9078 0.1743E-03
0.500	0.7406 0.2917E-03	0.8621 0.7235E-03	0.8902 0.1352E-02	0.9040 0.1253E-02	0.9129 0.1519E-02	0.9177 0.1145E-02	0.9242 0.1861E-02	0.9274 0.3309E-03
0.750	0.8350 0.5781E-03	0.9095 0.4625E-03	0.9253 0.3157E-03	0.9326 0.1176E-02	0.9362 0.4219E-03	0.9404 0.9506E-03	0.9420 0.9329E-03	0.9446 0.1528E-02
0.900	0.9065 0.8733E-03	0.9383 0.1165E-02	0.9479 0.4996E-03	0.9523 0.9458E-03	0.9532 0.8550E-03	0.9561 0.4437E-03	0.9573 0.8105E-03	0.9579 0.5156E-03
0.950	0.9413 0.1086E-02	0.9516 0.9114E-03	0.9586 0.4628E-03	0.9615 0.3612E-03	0.9619 0.4180E-03	0.9639 0.2731E-03	0.9643 0.7399E-03	0.9646 0.9110E-04
0.975	0.9704 0.1887E-02	0.9623 0.5899E-03	0.9677 0.1051E-02	0.9675 0.2072E-03	0.9689 0.8973E-03	0.9696 0.1234E-02	0.9696 0.1538E-02	0.9689 0.1760E-02
0.990	1.016 0.4894E-02	0.9747 0.1122E-02	0.9750 0.4941E-03	0.9726 0.1592E-02	0.9761 0.1250E-02	0.9738 0.1857E-02	0.9761 0.1291E-02	0.9730 0.2750E-02

MEAN OF REGRESSION ON AVERAGES	0.9411 0.3344E-02	-8.996 1.853	367.8 286.1	-0.1474E+05 0.1455E+05	-6170. 6570.
STD DEV OF REGRESSION .	0.5119E-02 0.8870E-03	2.923 0.3187	460.7 34.12	0.2361E+05 1240.	0.1068E+05 466.6
REGRESSION ON VARIANCE	0.8146E-01 0.7560	11.59 33.36	-146.5 505.1	953.2 3102.	-2028. 6462.

ESTIMATOR: ***Rho(.8920) of SqrtBeta L laplace AR r.v.s, A=.88, L= .1 Using moment estimator

Figure 43.   Summary Statistics, L-Laplace samples. Moment Estimator.



SUBSAMPLE SIZE	20	60	100	150	200	250	350	500
MEAN	0.7208	0.8524	0.8829	0.9032	0.9089	0.9173	0.9273	0.9276
STD	0.2671	0.1023	0.7093E-01	0.5448E-01	0.4538E-01	0.3903E-01	0.3063E-01	0.2778E-01
STD MEAN	0.3446E-02	0.2287E-02	0.2048E-02	0.1924E-02	0.1853E-02	0.1781E-02	0.1661E-02	0.1793E-02
M.S.E	0.3172	0.1097	0.7152E-01	0.5555E-01	0.4842E-01	0.4654E-01	0.4671E-01	0.4512E-01
SKEWNESS	-3.7461	-1.8249	-1.3187	-1.2756	-0.9145	-1.0376	-1.2950	-0.7172
KURTOSIS	76.8379	6.0725	2.7079	2.4129	1.0336	1.5950	2.7741	0.5353

MEAN OF REGRESSION ON AVERAGES	0.9403	-5.378	-121.5	0.1008E+05	4985.
VARIANCE OF REGRESSION	0.2673E-04	6.878	0.1626E+06	0.4261E+09	0.8735E+08
STD DEV OF REGRESSION	0.5170E-02	2.623	403.2	0.2064E+05	9346.
REGRESSION ON VARIANCE	0.6448	-14.75	244.4	-1410.	3049.

ESTIMATOR: ***Rho(.8920) of SqrtBeta L laplace AR r.vs., A=.88, L= .1 Using Priestly Estimator
VERTICAL SCALE: YMIN = 0.2286
YMAX = 1.2314

Figure 44. Individual Plot, L-Laplace samples. Priestley Estimator.

SUMMARY STATISTICS (MEAN/STD)   3 SUPER-REPLICATIONS

SUBSAMPLE SIZE	20	60	100	150	200	250	350	500
MEAN	0.7217 0.6687E-03	0.8539 0.1371E-02	0.8842 0.7063E-03	0.9018 0.1146E-02	0.9085 0.3401E-03	0.9159 0.6997E-03	0.9231 0.2098E-02	0.9254 0.1149E-02
STD	0.2914 0.2613E-01	0.1085 0.1143E-02	0.7090E-01 0.1239E-02	0.5450E-01 0.6362E-03	0.4720E-01 0.9126E-03	0.4088E-01 0.8063E-03	0.3289E-01 0.1159E-02	0.2838E-01 0.1079E-02
M.S.E	0.3380 0.2290E-01	0.1095 0.1459E-02	0.7134E-01 0.1268E-02	0.5540E-01 0.4825E-03	0.5001E-01 0.7980E-03	0.4689E-01 0.4464E-03	0.4536E-01 0.7007E-03	0.4381E-01 0.7871E-03
SKEWNESS	-0.8138 1.663	-1.863 0.1583	-1.559 0.2552	-1.332 0.3063E-01	-1.248 0.2249	-1.241 0.1922	-1.086 0.1060	-0.9355 0.2260
KURTOSIS	196.2 128.2	7.085 1.471	4.647 1.987	3.027 0.3195	3.247 1.900	2.563 1.426	2.165 0.3156	1.502 0.7991
SER.COR.	0.3258E-02 0.4035E-02	0.1096E-01 0.1196E-01	-0.1875E-02 0.1582E-01	-0.2973E-02 0.1742E-01	0.8853E-02 0.5755E-01	-0.3609E-01 0.3058E-01	-0.4765E-01 0.2040E-01	0.4485E-01 0.3466E-01
QUANTILES								
0.010	-0.4129E-01 0.2139E-02	0.4885 0.7678E-02	0.6463 0.7884E-02	0.7043 0.8184E-02	0.7616 0.7372E-02	0.7916 0.7889E-02	0.8170 0.2224E-02	0.8310 0.9778E-02
0.025	0.1196 0.1938E-02	0.5977 0.9323E-03	0.7074 0.4854E-02	0.7674 0.6452E-02	0.7910 0.7674E-02	0.8169 0.3813E-02	0.8455 0.5127E-02	0.8533 0.7985E-02
0.050	0.2861 0.1339E-02	0.6650 0.4474E-02	0.7531 0.5242E-02	0.8000 0.2298E-02	0.8168 0.3272E-02	0.8429 0.2645E-03	0.8632 0.4047E-02	0.8709 0.5501E-02
0.100	0.4414 0.1873E-03	0.7322 0.1866E-02	0.7906 0.2663E-02	0.8302 0.5739E-03	0.8480 0.4289E-03	0.8635 0.1550E-02	0.8805 0.3711E-02	0.8890 0.2417E-02
0.250	0.6257 0.2635E-02	0.8106 0.1412E-02	0.8526 0.1678E-02	0.8754 0.1839E-02	0.8848 0.9761E-03	0.8956 0.9210E-03	0.9057 0.3607E-02	0.9091 0.1217E-02
0.500	0.7624 0.3942E-03	0.8742 0.1238E-02	0.8981 0.5332E-03	0.9117 0.1925E-02	0.9159 0.9221E-03	0.9228 0.1251E-02	0.9283 0.2425E-02	0.9291 0.1436E-02
0.750	0.8685 0.7000E-03	0.9211 0.1610E-02	0.9330 0.1517E-02	0.9401 0.5784E-03	0.9424 0.2602E-03	0.9448 0.5930E-03	0.9472 0.1049E-02	0.9448 0.1368E-02
0.900	0.9425 0.7508E-03	0.9532 0.1698E-02	0.9561 0.3453E-03	0.9602 0.7918E-03	0.9599 0.6151E-03	0.9593 0.5732E-03	0.9603 0.4137E-03	0.9577 0.4553E-03
0.950	0.9833 0.1785E-02	0.9743 0.1079E-02	0.9685 0.7223E-03	0.9699 0.9344E-03	0.9689 0.1300E-02	0.9682 0.1007E-02	0.9665 0.6197E-03	0.9659 0.2021E-02
0.975	1.036 0.9100E-03	0.9899 0.1059E-02	0.9793 0.1910E-02	0.9765 0.8380E-03	0.9762 0.5875E-03	0.9728 0.3748E-03	0.9717 0.8038E-03	0.9710 0.1484E-02
0.990	1.157 0.1182E-01	1.007 0.2556E-02	0.9951 0.3069E-02	0.9871 0.1602E-02	0.9857 0.6744E-03	0.9789 0.1322E-02	0.9756 0.1397E-02	0.9770 0.1974E-02

MEAN OF REGRESSION ON AVERAGES	0.9368 0.1897E-02	-5.219 0.2228	-46.85 37.48	4882. 2997.	2459. 1564.
STD DEV OF REGRESSION	0.5238E-02 0.4358E-03	2.672 0.3252	408.1 48.94	0.2082E+05 2338.	9419. 1021.
REGRESSION ON VARIANCE	0.3705 0.1506	-0.2305 7.271	14.69 117.4	-31.85 736.9	408.7 1433.

ESTIMATOR: ***Rho(.8920) of SqrtBeta L laplace AR r.vs., A=.88, L= .1 Using Priestly Estimator

Figure 45. Summary Statistics, L-Laplace samples. Priestley Estimator.

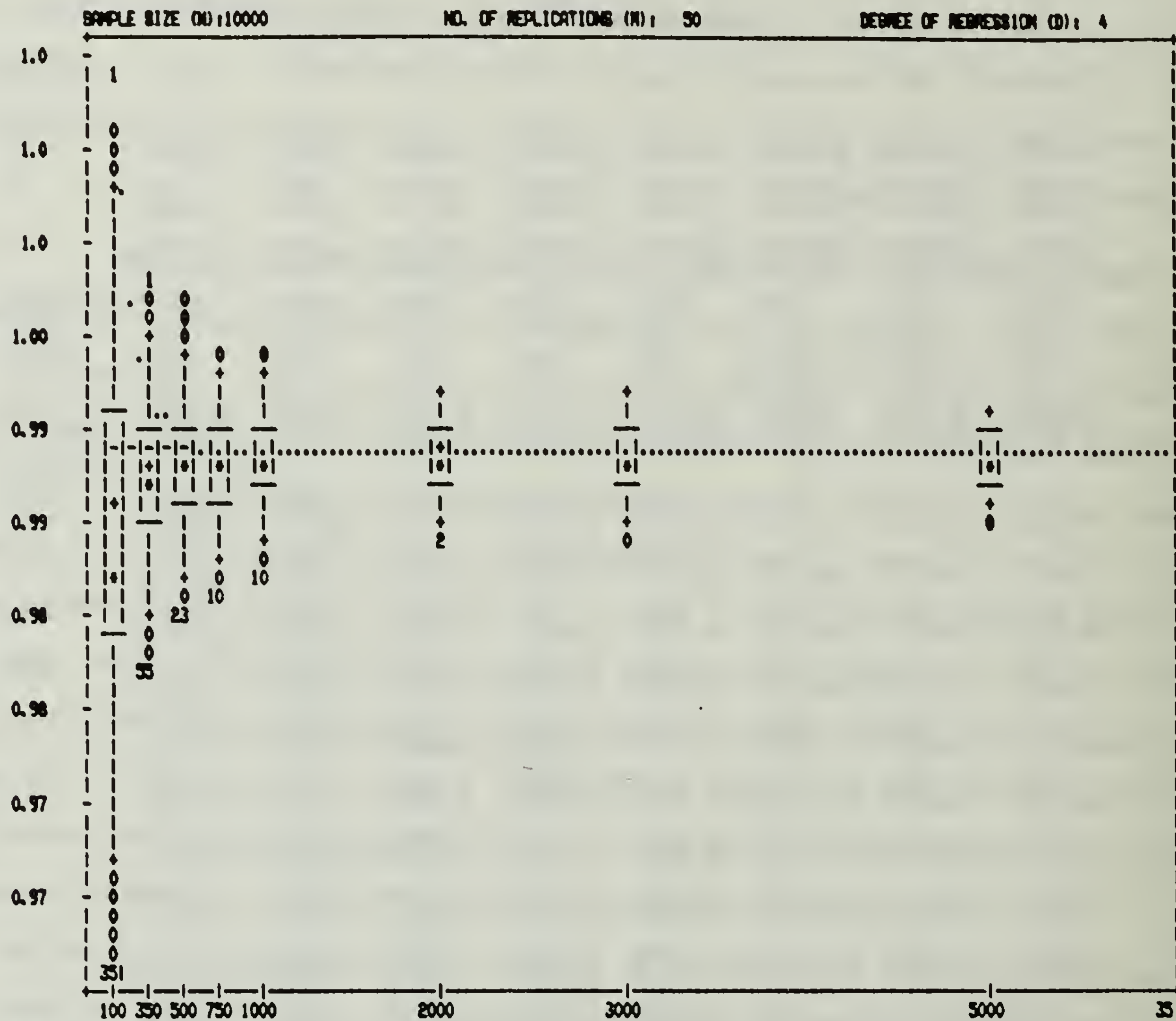
by Denby and Martin with their estimators [Ref. 9, p.141]. Figures 48 and 49 have the results with the scaling, and Figures 46 and 47 have no additional scaling. There is not much difference in the behavior of these two different approaches. The extreme bias of the robust regression is present in both cases, for the non-Normal BELAR(1) sample distribution being used here.

C. NON-NORMALLY DISTRIBUTED SAMPLES, UNCORRELATED

This section presents results for the case of the BELAR(1) process, with values of α chosen to produce a very small correlation, on the order of 0.05. In these cases, the problems associated with the random number generators previously discussed appear again, since the values of l we are interested in range from three down to 0.1. Thus, the cases for $l = 1$, and $l = 0.1$ produced underflow errors and invalid results. Results are available for the cases where $l = 3$ and $l = 0.4$. Figure 50 is a color combined estimator plot, with all four estimators, in the case of uncorrelated BELAR(1) samples. Here, the moment estimator seems to show the least degree of small-sample bias, but the Cressie estimator has the best asymptotic performance. Figures 51 through 54 are the super-replications summary statistics for each of the four estimator under conditions where $l = 3$. This picture is similar to that of the Normally distributed, uncorrelated samples. From the regression asymptotes, the Cressie estimator appears to show the best asymptotic behavior, although all estimators seem to be over-estimating the true value, indicating there is more correlation in the samples than is actually present. This occurs for all sub-sample sizes.

Figures 55 through 59 show the results for the uncorrelated BELAR(1) process, with $l = 0.4$. The results are similar to those above, and the Normal uncorrelated case. All of these estimators will tend to indicate a slight degree of correlation, even if the samples are completely independent, just due to the nature of performing calculations on the sample streams. It is very unlikely that they will indicate a zero correlation; something like 0.5 to 0.1 is more likely to be the case.

This concludes the presentation of simulation results on lag-1 serial correlation using SIMTBED. For the negatively correlated case, results similar to the positively correlated case are expected, including the random number generation difficulties.



SUBSAMPLE SIZE	100	350	500	750	1000	2000	3000	5000
MEAN	0.9867	0.9921	0.9929	0.9931	0.9931	0.9934	0.9934	0.9935
STD	0.1794E-01	0.5129E-02	0.3694E-02	0.2977E-02	0.2667E-02	0.1753E-02	0.1668E-02	0.1494E-02
STD MEAN	0.2538E-03	0.1371E-03	0.1168E-03	0.1168E-03	0.1193E-03	0.1109E-03	0.1362E-03	0.1494E-03
M.S.E	0.9305E-01	0.9684E-01	0.9759E-01	0.9771E-01	0.9778E-01	0.9805E-01	0.9804E-01	0.9815E-01
SKEWNESS	-4.8893	-1.5355	-1.2365	-1.0078	-0.6511	-0.5449	-0.1712	-0.2632
KURTOSIS	46.7935	4.3326	4.0920	2.8780	0.6981	0.0163	-0.5552	-0.4580

MEAN OF REGRESSION ON AVERAGES	0.9938	-1.252	920.4	5182.	-73.88
VARIANCE OF REGRESSION	0.4973E-07	0.5894	0.4577E+06	0.1106E+08	2111.
STD DEV OF REGRESSION	0.2230E-03	0.7677	676.5	3326.	45.94
REGRESSION ON VARIANCE	0.1882E-01	-0.8598	18.60	-138.3	517.0

ESTIMATOR: +++Rho(.8954) of SqrtBeta L laplace AR r.v.s, A=.86, L= .4 Using Rob. LS est.
VERTICAL SCALE: YMIN = 0.9641
YMAX = 1.0157

Figure 46. Individual Plot, No Additional Scaling. Robust Least Squares.

SUMMARY STATISTICS (MEAN/STD)   3 SUPER-REPLICATIONS

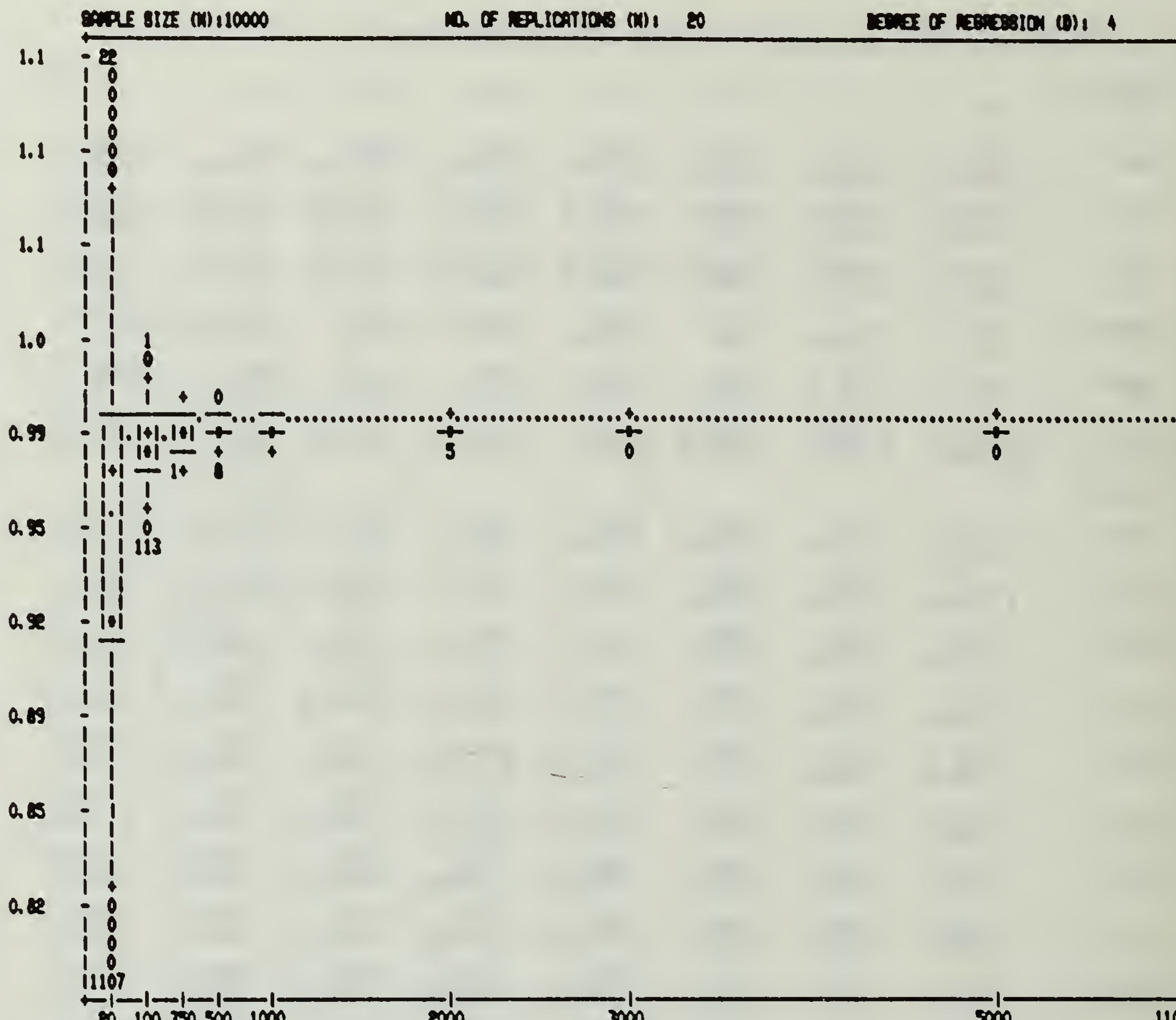
SUBSAMPLE SIZE	100	250	500	750	1000	2000	3000	5000
MEAN	0.9869 0.1628E-03	0.9922 0.7396E-04	0.9927 0.8792E-04	0.9929 0.9610E-04	0.9932 0.5845E-04	0.9934 0.3356E-04	0.9934 0.7818E-04	0.9934 0.6129E-04
STD	0.1782E-01 0.1858E-03	0.4872E-02 0.1325E-03	0.3836E-02 0.9442E-04	0.2899E-02 0.4507E-04	0.2582E-02 0.7203E-04	0.1845E-02 0.6245E-04	0.1558E-02 0.6314E-04	0.1533E-02 0.2229E-04
M.S.E	0.9323E-01 0.1270E-03	0.9693E-01 0.6854E-04	0.9742E-01 0.8516E-04	0.9758E-01 0.9634E-04	0.9782E-01 0.5944E-04	0.9799E-01 0.2294E-04	0.9800E-01 0.7880E-04	0.9803E-01 0.6103E-04
SKEWNESS	-4.627 0.1458	-1.512 0.3056E-01	-1.327 0.2762	-0.8622 0.7373E-01	-0.6105 0.2126E-01	-0.5025 0.1168	-0.2014 0.3505E-01	-0.6306E-01 0.1111
KURTOSIS	40.14 3.763	4.963 0.4185	5.094 2.310	1.887 0.4956	0.2697 0.2142	0.1113 0.1880	-0.2775 0.1593	-0.6824 0.1275
SER.COR.	0.9637E-02 0.9480E-02	-0.1519E-01 0.1279E-01	0.6148E-02 0.2956E-01	-0.4599E-02 0.4025E-01	-0.2771E-01 0.1332E-01	0.3748E-02 0.3051E-01	0.3378E-01 0.2758E-01	0.8543E-01 0.6788E-01
QUANTILES								
0.010	0.9120 0.1999E-02	0.9760 0.9181E-03	0.9803 0.1474E-02	0.9847 0.1733E-03	0.9859 0.4927E-03	0.9881 0.4441E-03	0.9895 0.5217E-04	0.9900 0.1938E-03
0.025	0.9430 0.9172E-03	0.9804 0.7309E-03	0.9840 0.3533E-03	0.9866 0.2317E-03	0.9871 0.2660E-03	0.9893 0.1623E-03	0.9899 0.9720E-04	0.9905 0.1939E-03
0.050	0.9575 0.6807E-03	0.9832 0.3739E-03	0.9860 0.2227E-03	0.9878 0.1588E-03	0.9884 0.1865E-03	0.9899 0.2522E-03	0.9906 0.2458E-04	0.9909 0.1228E-03
0.100	0.9708 0.2559E-03	0.9862 0.1398E-03	0.9881 0.1609E-03	0.9892 0.9493E-04	0.9896 0.5596E-04	0.9909 0.1314E-03	0.9913 0.1089E-03	0.9913 0.5192E-04
0.250	0.9835 0.1153E-03	0.9900 0.1545E-03	0.9907 0.1436E-03	0.9912 0.1245E-03	0.9916 0.4081E-04	0.9923 0.3146E-04	0.9924 0.1241E-03	0.9922 0.1340E-03
0.500	0.9914 0.1228E-03	0.9930 0.3632E-04	0.9932 0.5569E-04	0.9933 0.7728E-04	0.9935 0.2734E-04	0.9936 0.1032E-03	0.9934 0.7342E-04	0.9935 0.6346E-04
0.750	0.9963 0.1175E-03	0.9955 0.5857E-04	0.9954 0.5840E-04	0.9950 0.9903E-04	0.9950 0.1197E-03	0.9947 0.3589E-04	0.9946 0.1469E-03	0.9946 0.6576E-04
0.900	0.9995 0.5556E-04	0.9974 0.3543E-04	0.9970 0.4548E-04	0.9962 0.1159E-03	0.9962 0.1685E-03	0.9956 0.3866E-04	0.9953 0.1352E-03	0.9954 0.4617E-04
0.950	1.001 0.6237E-04	0.9984 0.6472E-04	0.9979 0.2290E-04	0.9970 0.1341E-03	0.9969 0.2137E-03	0.9961 0.4720E-04	0.9958 0.1384E-03	0.9959 0.1869E-04
0.975	1.003 0.2397E-03	0.9992 0.8354E-04	0.9985 0.5098E-04	0.9977 0.1198E-03	0.9974 0.2411E-03	0.9964 0.1331E-03	0.9963 0.2275E-03	0.9964 0.1495E-03
0.990	1.006 0.2190E-03	1.000 0.1711E-03	0.9996 0.8306E-04	0.9984 0.8324E-04	0.9980 0.2192E-03	0.9969 0.1787E-03	0.9970 0.6597E-04	0.9967 0.8119E-04

MEAN OF REGRESSION ON AVERAGES	0.9935 0.1345E-03	-0.3595 0.4865	16.27 486.2	386.7 2557.	-6.140 36.02
STD DEV OF REGRESSION	0.2439E-03 0.1487E-04	0.7796 0.5378E-01	682.7 39.31	3344. 166.9	46.12 2.156
REGRESSION ON VARIANCE	0.3101E-01 0.6305E-02	-2.304 0.7510	74.75 29.24	-998.6 447.4	4823. 2237.

ESTIMATOR: ***Rho(.8954) of SqrtBeta L laplace AR r.v.s, A=.86, L= .4 Using Rob. LS est.

*** WIDEST Y VALUES FOUND: YMIN=0.6740 - YMAX= 1.035 *** SMTB ***
*** THIS WAS A RESTART RUN. NSR START/END= 1 3

Figure 47. Summary Stats, No Additional Scaling. Robust Least Squares.



SUBSAMPLE SIZE	20	100	350	500	1000	2000	3000	5000
MEAN	0.9230	0.9839	0.9897	0.9904	0.9910	0.9910	0.9908	0.9909
STD	0.1494	0.2005E-01	0.6282E-02	0.5170E-02	0.3462E-02	0.2393E-02	0.1856E-02	0.1705E-02
STD MEAN	0.1494E-02	0.4482E-03	0.2654E-03	0.2585E-03	0.2448E-03	0.2393E-03	0.2396E-03	0.2696E-03
M.S.E	0.1519	0.9076E-01	0.9453E-01	0.9519E-01	0.9568E-01	0.9560E-01	0.9543E-01	0.9552E-01
SKEWNESS	-2.8962	-4.8128	-1.2732	-1.4813	-0.5475	-1.1488	-0.1853	0.0394
KURTOSIS	12.2506	57.2576	2.4859	4.2970	0.4877	2.6050	-0.2264	0.2975

MEAN OF REGRESSION ON AVERAGES	0.9907	0.6313	-452.6	-654.4	2.050
VARIANCE OF REGRESSION	0.8804E-07	0.5345	0.9652E+05	0.2417E+06	2.433
STD DEV OF REGRESSION	0.2967E-03	0.7311	310.7	491.6	1.560
REGRESSION ON VARIANCE	0.1118E-01	0.9534E-01	-4.122	64.94	-42.53

ESTIMATOR: ***Rho(.8954) of SqrtBeta L laplace AR r.v.s, A=.86, L= .4 Using Rob. LS est. (.6745 SCALING)
VERTICAL SCALE: YMIN = 0.7927
YMAX = 1.1204

Figure 48. Individual Plot, Additional Scaling. Robust Least Squares.

SUMMARY STATISTICS (MEAN/STD) 3 SUPER-REPLICATIONS

SUBSAMPLE SIZE	20	100	250	500	1000	2000	3000	5000
MEAN	0.9227 0.4792E-03	0.9838 0.8154E-04	0.9896 0.1263E-03	0.9903 0.7060E-04	0.9910 0.3768E-04	0.9908 0.1487E-03	0.9909 0.6559E-04	0.9909 0.1593E-03
STD	0.1502 0.4821E-03	0.2061E-01 0.4258E-03	0.6592E-02 0.3306E-03	0.5002E-02 0.1291E-03	0.3431E-02 0.1310E-03	0.2315E-02 0.4318E-04	0.1883E-02 0.3776E-04	0.1693E-02 0.4295E-04
M.S.E	0.1526 0.4985E-03	0.9077E-01 0.2270E-04	0.9446E-01 0.1428E-03	0.9505E-01 0.7701E-04	0.9566E-01 0.4227E-04	0.9538E-01 0.1491E-03	0.9552E-01 0.6686E-04	0.9556E-01 0.1585E-03
SKEWNESS	-3.141 0.1710	-4.302 0.4029	-1.639 0.4471	-1.197 0.1691	-1.003 0.2813	-0.7329 0.2674	-0.1910 0.9793E-01	0.7163E-02 0.1704
KURTOSIS	15.20 3.458	40.05 9.507	6.192 2.239	3.087 0.6250	1.965 1.195	1.217 0.8685	-0.1230 0.2222	-0.3010 0.3394
SER.COR.	-0.3766E-03 0.4380E-02	0.1981E-01 0.2553E-01	-0.6930E-01 0.2861E-01	0.2890E-01 0.2543E-01	0.2218E-01 0.1313E-01	-0.9814E-01 0.7548E-01	-0.6333E-02 0.3276E-01	-0.1121 0.8645E-01
QUANTILES								
0.010	0.2428 0.6944E-02	0.9085 0.1703E-02	0.9683 0.1632E-02	0.9723 0.2434E-02	0.9784 0.1925E-02	0.9827 0.1073E-02	0.9861 0.5585E-03	0.9872 0.4735E-03
0.025	0.4404 0.6580E-02	0.9315 0.1769E-02	0.9748 0.8212E-03	0.9790 0.6509E-03	0.9828 0.7388E-03	0.9850 0.3115E-03	0.9868 0.2768E-03	0.9872 0.4602E-03
0.050	0.6160 0.4873E-02	0.9483 0.1537E-02	0.9779 0.4526E-03	0.9817 0.2361E-03	0.9849 0.2002E-03	0.9867 0.3088E-03	0.9875 0.1119E-03	0.9884 0.5946E-04
0.100	0.7724 0.8250E-03	0.9630 0.4260E-03	0.9810 0.4044E-03	0.9840 0.1562E-03	0.9864 0.2034E-03	0.9878 0.2591E-03	0.9882 0.1384E-03	0.9886 0.8376E-04
0.250	0.9144 0.3536E-03	0.9785 0.2552E-03	0.9863 0.3387E-03	0.9878 0.5709E-04	0.9892 0.2570E-03	0.9894 0.3050E-03	0.9896 0.8063E-04	0.9896 0.2606E-03
0.500	0.9772 0.4866E-03	0.9891 0.9878E-04	0.9908 0.2415E-03	0.9909 0.1211E-03	0.9914 0.2169E-04	0.9910 0.1688E-03	0.9910 0.7781E-04	0.9911 0.1783E-03
0.750	0.9973 0.1831E-03	0.9954 0.8916E-04	0.9941 0.2200E-03	0.9937 0.1049E-03	0.9935 0.1038E-03	0.9924 0.1531E-03	0.9922 0.5155E-04	0.9921 0.2218E-03
0.900	1.009 0.2775E-03	0.9995 0.2652E-04	0.9964 0.1387E-03	0.9960 0.1747E-03	0.9950 0.2297E-03	0.9936 0.8313E-04	0.9933 0.2055E-03	0.9932 0.1249E-03
0.950	1.020 0.7052E-03	1.002 0.1162E-03	0.9979 0.1279E-03	0.9972 0.8150E-04	0.9958 0.3274E-03	0.9941 0.2525E-03	0.9939 0.2638E-03	0.9941 0.1154E-03
0.975	1.034 0.9061E-03	1.005 0.3372E-03	0.9988 0.2101E-03	0.9979 0.7842E-04	0.9963 0.2859E-03	0.9947 0.1950E-03	0.9947 0.1272E-03	0.9945 0.1783E-03
0.990	1.058 0.1577E-02	1.009 0.3477E-03	0.9998 0.2522E-03	0.9988 0.2936E-03	0.9969 0.2742E-03	0.9955 0.1261E-03	0.9953 0.1486E-03	0.9946 0.1838E-03

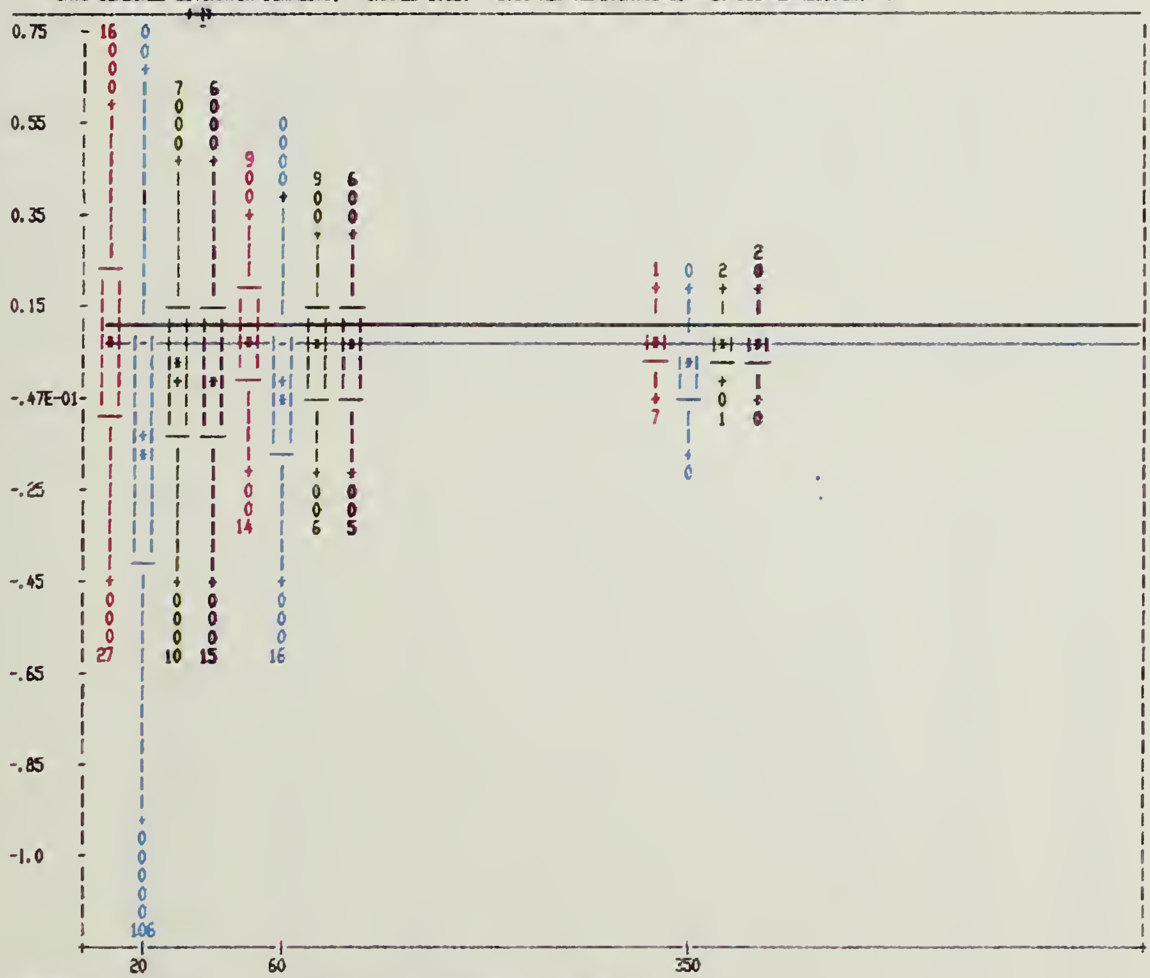
MEAN OF REGRESSION ON AVERAGES	0.9908 0.5404E-04	0.4012 0.1293	-370.4 41.18	-529.8 62.55	1.659 0.1972
STD DEV OF REGRESSION	0.2717E-03 0.1253E-04	0.6629 0.3528E-01	291.5 9.638	467.7 11.99	1.489 0.3574E-01
REGRESSION ON VARIANCE	0.1871E-01 0.8711E-02	-0.5937 0.7952	13.48 21.18	-85.52 192.1	338.9 502.9

ESTIMATOR: ***Rho(.8954) of SqrtBeta L laplace AR r.v.s, A=.86, L= .4 Using Rob. LS est. (.6745 SCALING)

*** WIDEST Y VALUES FOUND: YMIN=-1.863 - YMAX= 2.461 *** SMTB ***
*** THIS WAS A RESTART RUN. NSR START/END= 1 3

Figure 49. Summary Stats, Additional Scaling. Robust Least Squares.

SMTB COMBINED ESTIMATOR BOXPLOTS: SAMPLE SIZE: 6000 NO. REPLICATIONS: 20 DEG REGRESSION: 4



		20	60	350
SUBSAMPLE SIZE:		20	60	350
ESTIMATOR 1:	MEAN:	0.9261E-01	0.9595E-01	0.1000
	STD:	0.2439	0.1468	0.6287E-01
	STD MEAN:	0.3148E-02	0.3283E-02	0.3410E-02
SUBSAMPLE SIZE:		20	60	350
ESTIMATOR 2:	MEAN:	-0.1627	-0.9975E-02	0.3942E-01
	STD:	0.4204	0.2142	0.8305E-01
	STD MEAN:	0.5428E-02	0.4790E-02	0.4504E-02
SUBSAMPLE SIZE:		20	60	350
ESTIMATOR 3:	MEAN:	0.3348E-01	0.7832E-01	0.1087
	STD:	0.2256	0.1375	0.5941E-01
	STD MEAN:	0.2912E-02	0.3075E-02	0.3222E-02
SUBSAMPLE SIZE:		20	60	350
ESTIMATOR 4:	MEAN:	0.2949E-01	0.7902E-01	0.1100
	STD:	0.2268	0.1375	0.6331E-01
	STD MEAN:	0.2928E-02	0.3074E-02	0.3433E-02

ESTIMATOR 1: **Rho(.05924) of SqrtBeta L laplace AR r.v.s, A=.02, L= 3 Using Robust Least Squares estimator (C=4.2)
ESTIMATOR 2: **Rho(.05924) of SqrtBeta L laplace AR r.vs., A=.02, L= 3 Using Cressie Estimator
ESTIMATOR 3: **Rho(.05924) of SqrtBeta L laplace AR r.v.s, A=.02, L= 3 Using moment estimator
ESTIMATOR 4: **Rho(.05924) of SqrtBeta L laplace AR r.vs., A=.02, L= 3 Using Priestly Estimator

*** WIDEST Y VALUES FOUND: YMIN=-2.375 - YMAX= 1.056 *** SMTB ***
*** THIS WAS A RESTART RUN. NSR START/END= 1 3

Figure 50. Combined Plot, BELAR(1) Process. L = 3. Rho = 0.05924.

SUMMARY STATISTICS (MEAN/STD) 3 SUPER-REPLICATIONS

SUBSAMPLE SIZE	20	60	100	150	200	250	350	500
MEAN	0.9241E-01 0.2337E-02	0.9658E-01 0.1675E-02	0.9533E-01 0.1816E-02	0.9780E-01 0.2485E-02	0.9558E-01 0.2966E-02	0.1017 0.5672E-03	0.9847E-01 0.7941E-03	0.9954E-01 0.1718E-02
STD	0.2440 0.1803E-02	0.1446 0.2241E-02	0.1112 0.1042E-02	0.9269E-01 0.7145E-03	0.7888E-01 0.4717E-03	0.7003E-01 0.2124E-02	0.6040E-01 0.1262E-02	0.4879E-01 0.1128E-02
M.S.E	0.2462 0.1472E-02	0.1493 0.1764E-02	0.1169 0.1082E-02	0.1004 0.1036E-02	0.8694E-01 0.9580E-03	0.8190E-01 0.1915E-02	0.7202E-01 0.1491E-02	0.6335E-01 0.2808E-03
SKEWNESS	-0.2285E-01 0.1854E-01	-0.2678E-01 0.4710E-01	-0.3931E-01 0.3425E-01	0.2461E-01 0.1853E-01	0.1438 0.9216E-02	-0.6827E-01 0.9222E-01	-0.1638 0.1670	-0.1604E-01 0.4666E-01
KURTOSIS	0.3234E-01 0.1386E-01	0.1237 0.3547E-01	0.1485 0.1437E-01	-0.1165 0.1860E-01	-0.1521 0.1163	0.1026 0.5881E-01	0.1498 0.2096	0.7359E-01 0.2343
SER.COR.	0.6397E-02 0.1149E-01	-0.1503E-01 0.1033E-01	-0.2544E-01 0.1753E-01	0.1913E-01 0.2231E-01	-0.4665E-02 0.1683E-01	-0.1217E-02 0.3362E-01	-0.4981E-01 0.3908E-01	-0.7206E-02 0.3492E-01
QUANTILES								
0.010	-0.4853 0.7598E-02	-0.2501 0.1241E-01	-0.1759 0.3531E-02	-0.1239 0.2544E-02	-0.7237E-01 0.4774E-02	-0.7114E-01 0.1161E-01	-0.5441E-01 0.1044E-01	-0.2139E-01 0.4525E-02
0.025	-0.3879 0.4374E-02	-0.1878 0.7424E-02	-0.1262 0.2443E-02	-0.8350E-01 0.5818E-02	-0.4927E-01 0.3334E-02	-0.4016E-01 0.3491E-03	-0.3141E-01 0.1080E-01	0.3387E-02 0.4747E-02
0.050	-0.3095 0.4530E-02	-0.1486 0.7310E-02	-0.9098E-01 0.3084E-02	-0.5283E-01 0.6343E-02	-0.2991E-01 0.2886E-02	-0.1307E-01 0.2395E-02	-0.4206E-02 0.8082E-02	0.2002E-01 0.4948E-02
0.100	-0.2202 0.4536E-02	-0.8751E-01 0.7097E-02	-0.4361E-01 0.4412E-02	-0.1783E-01 0.2591E-02	-0.5552E-02 0.4890E-02	0.1298E-01 0.2735E-02	0.2373E-01 0.1346E-02	0.3867E-01 0.3929E-02
0.250	-0.7193E-01 0.4423E-02	0.1211E-02 0.2284E-02	0.2109E-01 0.1653E-02	0.3429E-01 0.1349E-02	0.4072E-01 0.3462E-02	0.5528E-01 0.1876E-02	0.5960E-01 0.2388E-02	0.6591E-01 0.2485E-02
0.500	0.9289E-01 0.3115E-02	0.9862E-01 0.1926E-02	0.9624E-01 0.1377E-02	0.9384E-01 0.1838E-02	0.9201E-01 0.2876E-02	0.1032 0.1492E-02	0.1001 0.2361E-02	0.1007 0.2331E-02
0.750	0.2581 0.1461E-02	0.1929 0.1404E-02	0.1701 0.1989E-02	0.1626 0.3490E-02	0.1488 0.1892E-02	0.1508 0.2708E-02	0.1391 0.1958E-02	0.1308 0.8258E-03
0.900	0.4057 0.6596E-03	0.2793 0.1263E-02	0.2365 0.2352E-02	0.2203 0.3767E-02	0.1991 0.2208E-02	0.1902 0.5020E-02	0.1750 0.2100E-02	0.1625 0.1261E-02
0.950	0.4899 0.2225E-02	0.3326 0.2355E-02	0.2747 0.3103E-02	0.2564 0.3211E-02	0.2291 0.5644E-02	0.2157 0.7788E-02	0.1959 0.5760E-02	0.1854 0.3583E-02
0.975	0.5649 0.3850E-02	0.3794 0.5437E-02	0.3112 0.3033E-02	0.2819 0.4901E-02	0.2511 0.4950E-02	0.2359 0.3520E-02	0.2168 0.5332E-02	0.1951 0.4064E-02
0.990	0.6591 0.6423E-02	0.4387 0.6965E-02	0.3517 0.6829E-02	0.3096 0.3268E-02	0.2911 0.1105E-03	0.2626 0.3312E-02	0.2369 0.5933E-02	0.2180 0.2086E-02

MEAN OF REGRESSION ON AVERAGES	0.9938E-01 0.8035E-02	0.3915 4.112	-175.7 581.7	0.1114E+05 0.2855E+05	5389. 0.1275E+05
STD DEV OF REGRESSION	0.8872E-02 0.9219E-03	4.533 0.3710	679.2 40.86	0.3417E+05 1751.	0.1536E+05 745.2
REGRESSION ON VARIANCE	0.4754 0.3610	31.42 16.21	-443.8 229.5	2590. 1260.	-5229. 2344.

ESTIMATOR: ==Rho(.05924) of SqrtBeta L laplace AR r.v.s, A=.02, L= 3 Using Robust Least Squares estimator (C=4.2)

Figure 51. Summary Statistics, L-Laplace Samples. Robust Least Squares.

SUMMARY STATISTICS (MEAN/STD)   3 SUPER-REPLICATIONS

SUBSAMPLE SIZE	20	60	100	150	200	250	350	500
MEAN	-0.1554 0.4335E-02	-0.1004E-01 0.1616E-02	0.1857E-01 0.2330E-02	0.2620E-01 0.1444E-02	0.3928E-01 0.1903E-02	0.3978E-01 0.1465E-02	0.4358E-01 0.2585E-02	0.5145E-01 0.3767E-02
STD	0.4109 0.4759E-02	0.2113 0.2328E-02	0.1592 0.7079E-03	0.1277 0.1060E-02	0.1136 0.2178E-02	0.9929E-01 0.6531E-03	0.8360E-01 0.3569E-03	0.6743E-01 0.1176E-02
M.S.E	0.4636 0.5980E-02	0.2224 0.2621E-02	0.1644 0.9427E-03	0.1319 0.7234E-03	0.1154 0.2275E-02	0.1012 0.7353E-03	0.8520E-01 0.6905E-03	0.6808E-01 0.1343E-02
SKEWNESS	-0.6819 0.2433E-01	-0.3342 0.2351E-01	-0.3486 0.4853E-01	-0.2418 0.6007E-02	-0.2343 0.6485E-01	-0.3957E-01 0.9143E-01	-0.3607E-01 0.1021	0.5383E-01 0.4372E-01
KURTOSIS	0.7092 0.4000E-01	0.3643E-01 0.2659E-01	0.1179 0.1288	0.1426 0.4575E-01	0.1098 0.6901E-01	-0.1005 0.2860E-01	-0.1669 0.9295E-01	-0.6008E-02 0.2054
SER.COR.	-0.1291E-01 0.8360E-02	0.1627E-02 0.8523E-02	-0.1238E-01 0.7973E-02	-0.1562E-01 0.3417E-02	0.5819E-02 0.2884E-01	-0.2932E-01 0.5194E-02	-0.4306E-02 0.2841E-01	-0.4712E-01 0.2615E-01
QUANTILES								
0.010	-1.305 0.3036E-01	-0.5504 0.9884E-02	-0.4004 0.1438E-01	-0.2951 0.1123E-01	-0.2457 0.1050E-01	-0.2019 0.1366E-01	-0.1507 0.3114E-02	-0.1021 0.7779E-02
0.025	-1.099 0.2408E-01	-0.4546 0.1161E-01	-0.3273 0.5808E-02	-0.2369 0.1954E-02	-0.2041 0.1005E-01	-0.1567 0.4588E-02	-0.1199 0.4971E-02	-0.7689E-01 0.6123E-02
0.050	-0.8978 0.1792E-01	-0.3787 0.7319E-02	-0.2619 0.6346E-02	-0.1937 0.1569E-02	-0.1600 0.2627E-02	-0.1229 0.5845E-02	-0.1024 0.7365E-02	-0.6056E-01 0.6938E-02
0.100	-0.6981 0.1278E-01	-0.2898 0.6051E-02	-0.1915 0.8050E-02	-0.1413 0.3767E-02	-0.1059 0.3211E-02	-0.8506E-01 0.4501E-02	-0.6529E-01 0.7049E-02	-0.3717E-01 0.4356E-02
0.250	-0.4021 0.3054E-02	-0.1476 0.4652E-02	-0.8052E-01 0.2786E-02	-0.5445E-01 0.1579E-02	-0.3405E-01 0.5255E-02	-0.2894E-01 0.1608E-02	-0.1551E-01 0.4974E-02	0.3472E-02 0.3625E-02
0.500	-0.1117 0.2229E-02	0.9363E-03 0.1732E-02	0.2688E-01 0.3274E-02	0.3029E-01 0.3610E-02	0.4208E-01 0.3879E-02	0.4055E-01 0.3115E-02	0.4530E-01 0.2225E-02	0.5517E-01 0.4854E-02
0.750	0.1452 0.3222E-02	0.1431 0.9173E-03	0.1314 0.3475E-02	0.1117 0.2672E-02	0.1218 0.4338E-02	0.1085 0.1318E-02	0.1022 0.2275E-02	0.9911E-01 0.5615E-02
0.900	0.3349 0.5984E-02	0.2538 0.2348E-02	0.2140 0.2899E-02	0.1881 0.8800E-03	0.1817 0.1841E-02	0.1706 0.2245E-02	0.1483 0.4519E-02	0.1310 0.4923E-02
0.950	0.4310 0.2526E-02	0.3153 0.6456E-02	0.2622 0.1877E-02	0.2281 0.4898E-02	0.2203 0.4257E-02	0.2045 0.4489E-02	0.1752 0.8193E-02	0.1593 0.5728E-02
0.975	0.5099 0.3473E-02	0.3704 0.5131E-02	0.3072 0.7924E-02	0.2619 0.7275E-03	0.2502 0.8628E-03	0.2313 0.9660E-02	0.2020 0.9043E-02	0.1892 0.2777E-02
0.990	0.5886 0.3922E-02	0.4202 0.5924E-02	0.3541 0.7066E-02	0.3042 0.8360E-02	0.2840 0.2379E-02	0.2624 0.9286E-02	0.2363 0.1234E-01	0.2191 0.7052E-02

MEAN OF REGRESSION ON AVERAGES	0.6476E-01 0.1048E-01	-8.164 3.981	599.4 440.2	-0.2884E+05 0.1873E+05	-0.1251E+05 7899.
STD DEV OF REGRESSION	0.1182E-01 0.8462E-03	6.490 0.3601	979.0 44.84	0.4929E+05 1965.	0.2217E+05 838.8
REGRESSION ON VARIANCE	-0.2149 2.539	109.4 108.0	-1574. 1581.	9436. 9475.	-0.1906E+05 0.1937E+05

ESTIMATOR: ##Rho(.05924) of SqrtBeta L laplace AR r.vs., A=.02, L= 3 Using Cressie Estimator

Figure 52. Summary Statistics, L-Laplace Samples. Cressie Estimator.

SUMMARY STATISTICS (MEAN/STD)   3 SUPER-REPLICATIONS

SUBSAMPLE SIZE	20	60	100	150	200	250	350	500
MEAN	0.3371E-01 0.7167E-03	0.7932E-01 0.2185E-02	0.9393E-01 0.1764E-02	0.9688E-01 0.8921E-03	0.9869E-01 0.1802E-02	0.1024 0.1461E-02	0.1050 0.1881E-02	0.1027 0.1607E-02
STD	0.2259 0.1217E-02	0.1351 0.1263E-02	0.1070 0.2055E-02	0.8780E-01 0.6422E-03	0.7442E-01 0.1084E-02	0.6885E-01 0.1198E-02	0.6031E-01 0.4678E-03	0.4981E-01 0.1298E-02
M.S.E	0.2273 0.1130E-02	0.1366 0.1103E-02	0.1125 0.2237E-02	0.9554E-01 0.5342E-03	0.8426E-01 0.1291E-02	0.8133E-01 0.3486E-03	0.7577E-01 0.7799E-03	0.6613E-01 0.1312E-02
SKEWNESS	0.4909E-02 0.1708E-01	-0.2869E-01 0.5613E-01	0.8582E-01 0.6183E-01	0.8012E-01 0.6554E-01	0.7773E-01 0.4219E-01	-0.5645E-01 0.5666E-01	0.5581E-01 0.7157E-01	0.6325E-01 0.1679E-01
KURTOSIS	-0.1919 0.2155E-01	-0.2390E-01 0.4804E-01	-0.1310E-01 0.7038E-01	0.1626 0.7343E-01	0.1140 0.7770E-01	0.1881 0.1822	0.8671E-01 0.2487E-01	0.1466 0.4152
SER. COR.	-0.1075E-01 0.6763E-02	-0.1153E-01 0.5158E-02	-0.8746E-02 0.9456E-02	0.2011E-01 0.2376E-01	-0.1141E-01 0.1879E-01	-0.4457E-02 0.1909E-01	0.1723E-02 0.1240E-01	0.1024E-01 0.3575E-01
QUANTILES								
0.010	-0.4770 0.7174E-02	-0.2354 0.6382E-02	-0.1379 0.2608E-02	-0.1109 0.4833E-02	-0.7421E-01 0.7111E-02	-0.6732E-01 0.8896E-02	-0.4428E-01 0.6724E-02	-0.1352E-01 0.3548E-02
0.025	-0.3988 0.5407E-02	-0.1917 0.2063E-02	-0.1072 0.4891E-02	-0.7096E-01 0.4849E-02	-0.4773E-01 0.5589E-02	-0.3385E-01 0.2579E-02	-0.1289E-01 0.2655E-02	0.4641E-02 0.2820E-02
0.050	-0.3389 0.6298E-02	-0.1481 0.1665E-02	-0.8140E-01 0.4143E-02	-0.4339E-01 0.3725E-02	-0.2389E-01 0.2613E-02	-0.1406E-01 0.3177E-02	0.8513E-02 0.2504E-02	0.1558E-01 0.3752E-02
0.100	-0.2595 0.1989E-02	-0.9660E-01 0.8821E-03	-0.4462E-01 0.5264E-02	-0.1168E-01 0.2302E-02	0.3030E-02 0.3622E-02	0.1370E-01 0.3880E-02	0.3145E-01 0.1734E-02	0.4027E-01 0.3643E-02
0.250	-0.1230 0.1322E-02	-0.1192E-01 0.4892E-02	0.1936E-01 0.2087E-02	0.3736E-01 0.1236E-02	0.4891E-01 0.2147E-02	0.5619E-01 0.1943E-02	0.6427E-01 0.2759E-02	0.6953E-01 0.3346E-02
0.500	0.3302E-01 0.1600E-02	0.8119E-01 0.3204E-02	0.9299E-01 0.1563E-02	0.9640E-01 0.2427E-02	0.9878E-01 0.1855E-02	0.1032 0.1990E-02	0.1046 0.2813E-02	0.1018 0.2481E-02
0.750	0.1917 0.1806E-02	0.1706 0.3319E-02	0.1670 0.5211E-02	0.1549 0.1663E-02	0.1474 0.2099E-02	0.1489 0.1914E-02	0.1442 0.2579E-02	0.1344 0.6741E-03
0.900	0.3278 0.1292E-02	0.2506 0.1123E-02	0.2293 0.1524E-02	0.2086 0.2040E-02	0.1981 0.3182E-02	0.1875 0.1182E-02	0.1830 0.2676E-02	0.1689 0.3105E-02
0.950	0.4052 0.1885E-02	0.2990 0.4607E-02	0.2677 0.4961E-02	0.2429 0.1363E-02	0.2236 0.2990E-02	0.2158 0.7825E-03	0.2057 0.9373E-03	0.1889 0.2998E-02
0.975	0.4732 0.3947E-02	0.3393 0.5003E-02	0.3089 0.9372E-02	0.2713 0.1417E-02	0.2429 0.2598E-02	0.2405 0.1187E-02	0.2297 0.3610E-02	0.2028 0.3268E-02
0.990	0.5484 0.1590E-02	0.3921 0.8493E-02	0.3591 0.1341E-01	0.3138 0.1085E-01	0.2745 0.4239E-02	0.2628 0.4282E-02	0.2533 0.4469E-02	0.2245 0.8163E-02

MEAN OF REGRESSION ON AVERAGES	0.1081 0.9141E-02	-2.159 4.711	154.7 682.6	-0.1014E+05 0.3328E+05	-5027. 0.1474E+05
STD DEV OF REGRESSION	0.8962E-02 0.1363E-02	4.804 0.6471	741.8 71.83	0.3771E+05 2636.	0.1701E+05 992.3
REGRESSION ON VARIANCE	2.458 0.9637	-49.18 39.31	669.6 563.7	-3865. 3324.	7718. 6714.

ESTIMATOR: ##Rho(.05924) of SqrtBeta L laplace AR r.v.s, A=.02, L= 3 Using moment estimator

Figure 53. Summary Statistics, L-Laplace Samples. Moment Estimator.

SUMMARY STATISTICS (MEAN/STD)   3 SUPER-REPLICATIONS

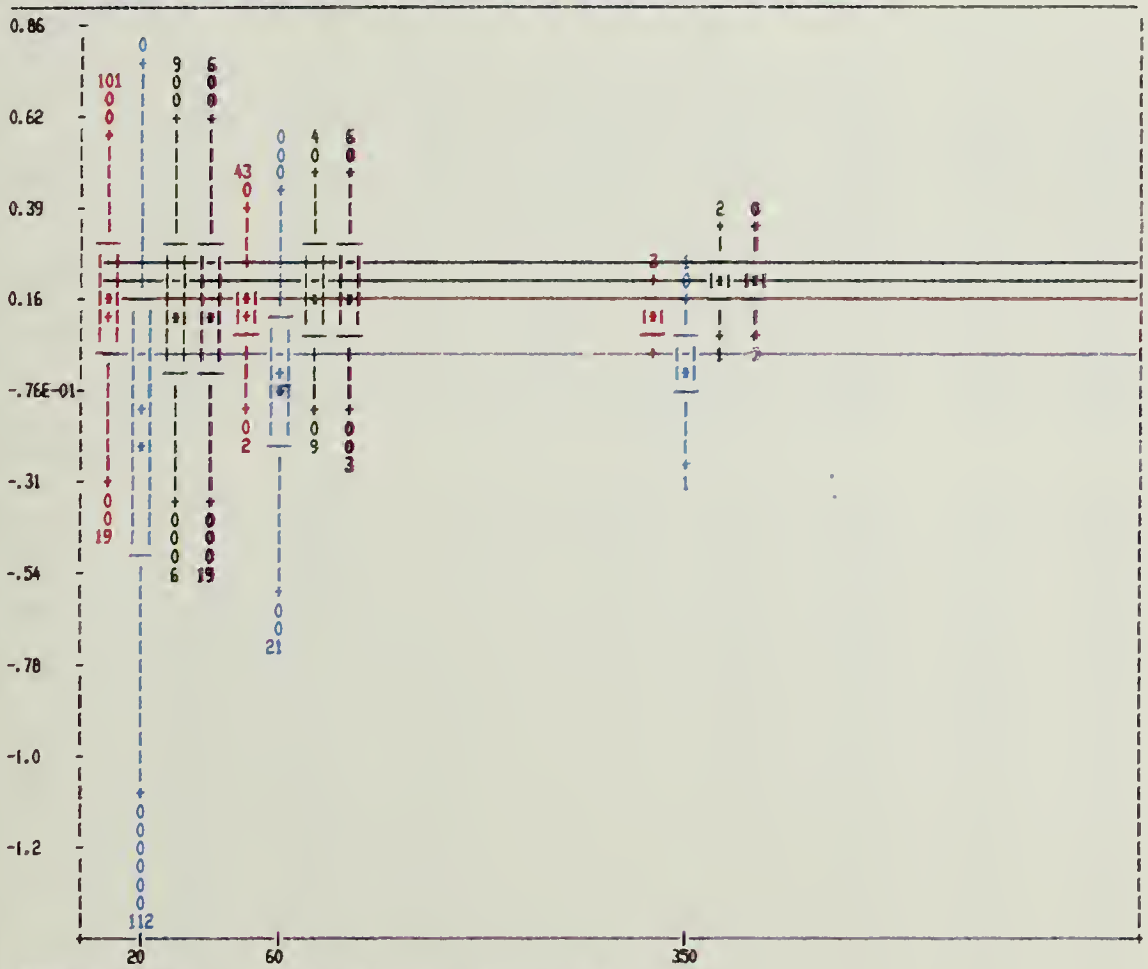
SUBSAMPLE SIZE	20	60	100	150	200	250	350	500
MEAN	0.3151E-01 0.1904E-02	0.8260E-01 0.1793E-02	0.9279E-01 0.1128E-02	0.9600E-01 0.1444E-02	0.9738E-01 0.2771E-02	0.1021 0.1988E-02	0.1050 0.2501E-02	0.1031 0.1646E-02
STD	0.2264 0.7034E-03	0.1365 0.4919E-03	0.1071 0.8740E-03	0.8729E-01 0.7676E-03	0.7680E-01 0.1095E-02	0.6765E-01 0.7668E-03	0.6068E-01 0.1366E-02	0.4770E-01 0.6300E-03
M.S.E	0.2281 0.9149E-03	0.1386 0.2278E-03	0.1123 0.8949E-03	0.9474E-01 0.9113E-03	0.8579E-01 0.2190E-02	0.8011E-01 0.1631E-02	0.7608E-01 0.2592E-02	0.6484E-01 0.1457E-02
SKEWNESS	-0.3311E-01 0.1963E-01	0.2568E-01 0.1605E-01	-0.3629E-01 0.3842E-01	-0.1199E-01 0.2473E-01	-0.6362E-01 0.7207E-01	-0.1120E-01 0.2926E-01	0.2414E-01 0.1107	-0.1810 0.3544E-01
KURTOSIS	-0.2294 0.3321E-01	-0.4161E-01 0.6496E-01	-0.1111 0.4240E-01	-0.9447E-01 0.4077E-01	0.2447 0.1888	0.2243 0.2295	0.2599E-01 0.1764	0.2447E-02 0.2386
SER.COR.	0.6017E-02 0.1095E-01	0.2018E-01 0.7065E-02	0.5898E-02 0.8605E-02	-0.3577E-01 0.7859E-02	0.1418E-01 0.2302E-01	-0.5940E-01 0.3413E-01	0.3304E-01 0.1895E-01	-0.4385E-02 0.8118E-02
QUANTILES								
0.010	-0.4878 0.4078E-02	-0.2252 0.2232E-02	-0.1546 0.1112E-01	-0.1134 0.5724E-02	-0.9803E-01 0.8828E-02	-0.6403E-01 0.7884E-02	-0.3123E-01 0.7122E-02	-0.1813E-01 0.4619E-02
0.025	-0.4076 0.3381E-02	-0.1827 0.1728E-02	-0.1197 0.1040E-01	-0.8006E-01 0.6557E-02	-0.5687E-01 0.9417E-02	-0.3704E-01 0.5546E-02	-0.1686E-01 0.5577E-02	0.3107E-02 0.4023E-02
0.050	-0.3405 0.3694E-02	-0.1447 0.1379E-02	-0.8621E-01 0.2918E-02	-0.4807E-01 0.4052E-02	-0.2981E-01 0.4378E-02	-0.7330E-02 0.4452E-03	0.3185E-03 0.4618E-02	0.2230E-01 0.4223E-02
0.100	-0.2631 0.4543E-02	-0.9494E-01 0.6547E-03	-0.4806E-01 0.1966E-02	-0.1408E-01 0.4989E-02	0.1033E-02 0.3637E-02	0.1939E-01 0.2672E-02	0.2279E-01 0.2246E-02	0.3910E-01 0.2236E-02
0.250	-0.1252 0.2584E-02	-0.8891E-02 0.3802E-02	0.1941E-01 0.1110E-02	0.3435E-01 0.3377E-03	0.4847E-01 0.1406E-02	0.5613E-01 0.2492E-02	0.6577E-01 0.2269E-02	0.7112E-01 0.3724E-02
0.500	0.3269E-01 0.2441E-02	0.8357E-01 0.2540E-02	0.9497E-01 0.2703E-02	0.9692E-01 0.1163E-02	0.9710E-01 0.1610E-02	0.1017 0.1579E-02	0.1053 0.2906E-02	0.1046 0.3451E-02
0.750	0.1896 0.5667E-03	0.1737 0.2161E-02	0.1637 0.1194E-02	0.1569 0.2791E-02	0.1474 0.4290E-02	0.1476 0.2925E-02	0.1449 0.2513E-02	0.1356 0.9652E-03
0.900	0.3256 0.1556E-02	0.2580 0.3879E-03	0.2293 0.1302E-02	0.2061 0.3629E-02	0.1964 0.6438E-02	0.1884 0.3388E-02	0.1816 0.4950E-02	0.1620 0.1165E-02
0.950	0.4021 0.4155E-02	0.3047 0.1352E-02	0.2668 0.2315E-02	0.2375 0.2206E-02	0.2234 0.4899E-02	0.2119 0.5928E-02	0.2010 0.3548E-02	0.1823 0.3112E-02
0.975	0.4707 0.4060E-02	0.3556 0.1082E-02	0.3006 0.6447E-02	0.2672 0.1954E-02	0.2518 0.5568E-02	0.2394 0.5289E-02	0.2261 0.5480E-02	0.1931 0.3087E-02
0.990	0.5312 0.5596E-02	0.4030 0.7716E-02	0.3347 0.5155E-02	0.2980 0.1341E-02	0.2754 0.9227E-02	0.2611 0.7766E-02	0.2545 0.1084E-01	0.2138 0.7799E-02

MEAN OF REGRESSION ON AVERAGES	0.1098 0.4056E-02	-2.830 2.520	177.0 376.2	-7902. 0.1828E+05	-3345. 8090.
STD DEV OF REGRESSION	0.9662E-02 0.1014E-02	4.953 0.5715	738.6 84.45	0.3695E+05 4068.	0.1657E+05 1784.
REGRESSION ON VARIANCE	1.076 0.2592	6.938 11.33	-127.2 162.7	814.3 962.7	-1738. 1964.

ESTIMATOR: **Rho(.05924) of SqrtBeta L laplace AR r.vs., A=.02, L= 3 Using Priestly Estimator

Figure 54. Summary Statistics, L-Laplace Samples. Priestley Estimator.

SMTB COMBINED ESTIMATOR BOXPLOTS: SAMPLE SIZE: 6000 NO. REPLICATIONS: 20 DEG REGRESSION: 4

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SUBSAMPLE SIZE:                    20          60          350
ESTIMATOR 1:   MEAN:           0.1804      0.1623      0.1393
               STD:            0.2336      0.1366      0.5340E-01
               STD MEAN:       0.3016E-02  0.3054E-02  0.2896E-02

SUBSAMPLE SIZE:                    20          60          350
ESTIMATOR 2:   MEAN:          -0.1786     -0.4485E-01 -0.3244E-02
               STD:            0.4901      0.2565      0.1034
               STD MEAN:       0.6327E-02  0.5736E-02  0.5609E-02

SUBSAMPLE SIZE:                    20          60          350
ESTIMATOR 3:   MEAN:           0.1434      0.2039      0.2276
               STD:            0.2294      0.1484      0.6985E-01
               STD MEAN:       0.2962E-02  0.3318E-02  0.3788E-02

SUBSAMPLE SIZE:                    20          60          350
ESTIMATOR 4:   MEAN:           0.1379      0.1975      0.2356
               STD:            0.2347      0.1484      0.7106E-01
               STD MEAN:       0.3030E-02  0.3319E-02  0.3854E-02

ESTIMATOR 1:  **Rho(.02911) of SqrtBeta L laplace AR r.v.s, A=.02, L= .4 Using Robust Least Squares estimator (C=4.2)
ESTIMATOR 2:  **Rho(.02911) of SqrtBeta L laplace AR r.vs., A=.02, L= .4 Using Cressie Estimator
ESTIMATOR 3:  **Rho(.02911) of SqrtBeta L laplace AR r.v.s, A=.02, L= .4 Using moment estimator
ESTIMATOR 4:  **Rho(.02911) of SqrtBeta L laplace AR r.vs., A=.02, L= .4 Using Priestly Estimator

*** WIDEST Y VALUES FOUND:  YMIN=-3.467     -  YMAX= 1.952      *** SMTB ***
*** THIS WAS A RESTART RUN. NSR START/END=        1          3
```

Figure 55. Combined Plot, BELAR(1) Process. L = 0.4. Rho = 0.02911.

SUMMARY STATISTICS (MEAN/STD)   3 SUPER-REPLICATIONS

SUBSAMPLE SIZE	20	60	100	150	200	250	350	500
MEAN	0.1799 0.1170E-02	0.1610 0.1576E-02	0.1550 0.8411E-03	0.1467 0.2663E-03	0.1405 0.1417E-02	0.1396 0.1357E-02	0.1369 0.2035E-02	0.1374 0.2405E-02
STD	0.2311 0.1294E-02	0.1356 0.2390E-02	0.1042 0.1734E-02	0.8304E-01 0.9612E-03	0.7004E-01 0.1288E-02	0.6374E-01 0.2217E-02	0.5378E-01 0.1226E-02	0.4588E-01 0.2097E-02
M.S.E	0.2799 0.1474E-02	0.1892 0.1034E-02	0.1634 0.1537E-02	0.1440 0.7685E-03	0.1316 0.1834E-02	0.1276 0.9754E-03	0.1205 0.2243E-02	0.1177 0.2877E-02
SKEWNESS	0.5206 0.2751E-01	0.8117 0.2847E-01	0.7446 0.4981E-01	0.6968 0.4710E-01	0.5951 0.4197E-01	0.7159 0.5776E-01	0.5400 0.1396	0.3219 0.8111E-01
KURTOSIS	0.6975 0.8980E-01	1.189 0.1091	0.8736 0.1991	0.7685 0.1211	0.5469 0.1954	0.5054 0.8833E-01	0.4948 0.1917	-0.3270E-01 0.9777E-01
SER.COR.	-0.7017E-02 0.2074E-02	0.8290E-03 0.1380E-01	0.1106E-01 0.2763E-01	0.9422E-02 0.1750E-01	0.2186E-01 0.1612E-01	-0.2789E-01 0.3134E-01	0.2505E-01 0.4024E-01	-0.2386E-01 0.5489E-01
QUANTILES								
0.010	-0.3232 0.2791E-02	-0.9654E-01 0.8460E-02	-0.3425E-01 0.4396E-02	-0.8352E-02 0.3528E-02	0.2854E-02 0.2401E-02	0.2246E-01 0.4021E-02	0.3483E-01 0.4350E-02	0.3754E-01 0.2522E-02
0.025	-0.2219 0.5605E-02	-0.5756E-01 0.6655E-02	-0.1091E-01 0.2245E-02	0.8837E-02 0.2649E-02	0.2452E-01 0.5938E-03	0.3733E-01 0.1321E-02	0.4590E-01 0.4799E-02	0.5773E-01 0.1939E-02
0.050	-0.1468 0.3203E-02	-0.2657E-01 0.4689E-02	0.1053E-01 0.7132E-03	0.2797E-01 0.3274E-02	0.3913E-01 0.2014E-02	0.5033E-01 0.2910E-02	0.5607E-01 0.2861E-02	0.6590E-01 0.7316E-03
0.100	-0.7258E-01 0.2018E-02	0.1137E-01 0.3840E-02	0.3464E-01 0.1860E-02	0.4980E-01 0.2945E-02	0.5720E-01 0.8139E-03	0.6648E-01 0.2267E-02	0.6933E-01 0.3078E-02	0.8186E-01 0.2720E-02
0.250	0.2544E-01 0.1678E-02	0.6675E-01 0.3941E-02	0.7936E-01 0.1886E-02	0.8725E-01 0.1215E-02	0.9049E-01 0.1706E-02	0.9236E-01 0.3126E-02	0.9825E-01 0.2507E-02	0.1040 0.2137E-02
0.500	0.1466 0.7099E-03	0.1419 0.2241E-02	0.1424 0.1311E-02	0.1365 0.1791E-02	0.1329 0.9868E-03	0.1313 0.2574E-02	0.1337 0.8158E-04	0.1334 0.3067E-02
0.750	0.3181 0.4639E-03	0.2367 0.1794E-02	0.2153 0.2696E-02	0.1968 0.1351E-02	0.1830 0.5765E-03	0.1759 0.1973E-02	0.1683 0.2782E-02	0.1694 0.5884E-02
0.900	0.4932 0.1815E-02	0.3441 0.2010E-02	0.2927 0.4171E-02	0.2573 0.2794E-02	0.2345 0.6398E-02	0.2286 0.1668E-02	0.2084 0.5122E-02	0.1979 0.3032E-02
0.950	0.6015 0.3477E-02	0.4134 0.6192E-02	0.3463 0.4263E-02	0.2975 0.3286E-02	0.2674 0.3340E-02	0.2591 0.3371E-02	0.2323 0.2814E-02	0.2175 0.6298E-02
0.975	0.6963 0.6570E-02	0.4801 0.7949E-02	0.3979 0.6361E-02	0.3365 0.2089E-02	0.2980 0.3979E-02	0.2933 0.7483E-02	0.2649 0.1142E-01	0.2373 0.4599E-02
0.990	0.8211 0.1052E-01	0.5674 0.9621E-02	0.4532 0.8098E-02	0.3939 0.4790E-02	0.3343 0.2772E-02	0.3236 0.7010E-02	0.2849 0.9559E-02	0.2597 0.7236E-02

MEAN OF REGRESSION ON AVERAGES	0.1390 0.3077E-02	-2.356 1.126	757.6 136.0	-0.4140E+05 6570.	-0.1891E+05 2936.
STD DEV OF REGRESSION	0.8513E-02 0.6128E-03	4.363 0.2746	640.1 30.16	0.3161E+05 1459.	0.1410E+05 666.1
REGRESSION ON VARIANCE	2.299 1.211	-55.25 49.28	808.4 693.7	-4707. 3987.	9333. 7888.

ESTIMATOR: ++Rho(.02911) of SqrtBeta L laplace AR r.v.s, A=.02, L= .4 Using Robust Least Squares estimator (C=4.2)

Figure 56. Summary Statistics, L-Laplace Samples. Robust Least Squares.

SUMMARY STATISTICS (MEAN/STD) 3 SUPER-REPLICATIONS

SUBSAMPLE SIZE	20	60	100	150	200	250	350	500
MEAN	-0.1761 0.2826E-02	-0.4464E-01 0.2248E-02	-0.2191E-01 0.3177E-02	-0.1437E-01 0.2683E-02	-0.7784E-02 0.3007E-02	-0.5379E-02 0.2734E-02	-0.1193E-02 0.1330E-02	-0.2227E-02 0.1023E-02
STD	0.4870 0.1573E-02	0.2560 0.2398E-02	0.1931 0.2903E-02	0.1562 0.2220E-02	0.1362 0.8641E-02	0.1213 0.4474E-02	0.9890E-01 0.2937E-02	0.8557E-01 0.1771E-02
M.S.E	0.5285 0.1993E-02	0.2664 0.2925E-02	0.1998 0.2336E-02	0.1622 0.2553E-02	0.1412 0.2008E-02	0.1261 0.5027E-02	0.1034 0.3197E-02	0.9115E-01 0.1335E-02
SKEWNESS	-0.9088 0.8183E-02	-0.4759 0.2775E-01	-0.3454 0.3367E-01	-0.2904 0.5602E-01	-0.2848 0.6623E-01	-0.1776 0.4684E-01	-0.1783 0.4600E-01	-0.2569 0.6622E-01
KURTOSIS	1.372 0.5605E-01	0.3431 0.5740E-01	0.8291E-01 0.1142	0.1419 0.1515	0.4250E-01 0.1496	-0.4822E-01 0.2557	-0.2861E-01 0.2045	0.6889E-01 0.7272E-01
SER. COR.	-0.7663E-02 0.1298E-01	0.7230E-02 0.2711E-01	-0.1355E-01 0.9084E-02	0.1896E-01 0.1572E-01	0.1920E-02 0.2283E-01	-0.3247E-01 0.5082E-01	-0.1852E-02 0.2133E-01	0.3201E-01 0.4176E-01
QUANTILES								
0.010	-1.600 0.5927E-02	-0.7444 0.8507E-02	-0.5086 0.6054E-02	-0.4226 0.1509E-01	-0.3396 0.1376E-01	-0.3019 0.1250E-01	-0.2358 0.2342E-02	-0.2363 0.5386E-02
0.025	-1.307 0.9456E-02	-0.6056 0.4123E-02	-0.4370 0.1007E-01	-0.3495 0.1376E-01	-0.2953 0.9275E-02	-0.2506 0.1190E-01	-0.1981 0.1201E-01	-0.1913 0.1368E-01
0.050	-1.087 0.9203E-02	-0.4934 0.9596E-02	-0.3634 0.2389E-02	-0.2833 0.2072E-02	-0.2370 0.6664E-02	-0.2106 0.9944E-02	-0.1676 0.6796E-02	-0.1453 0.3627E-02
0.100	-0.8255 0.6722E-02	-0.3798 0.1246E-01	-0.2811 0.3369E-02	-0.2164 0.1101E-02	-0.1856 0.2700E-02	-0.1641 0.1049E-01	-0.1343 0.5204E-02	-0.1142 0.7470E-02
0.250	-0.4519 0.3898E-02	-0.2079 0.4514E-02	-0.1455 0.5428E-02	-0.1119 0.4081E-02	-0.1016 0.3173E-02	-0.8377E-01 0.1023E-01	-0.6919E-01 0.2823E-02	-0.5678E-01 0.2395E-02
0.500	-0.1052 0.3875E-02	-0.2644E-01 0.2267E-02	-0.6689E-02 0.3987E-02	-0.7767E-02 0.3957E-02	-0.2118E-02 0.1841E-02	-0.1359E-02 0.1464E-02	0.4157E-02 0.2075E-02	-0.1941E-02 0.2863E-02
0.750	0.1783 0.7409E-03	0.1378 0.1459E-02	0.1121 0.3724E-02	0.8920E-01 0.2874E-02	0.9232E-01 0.3561E-02	0.8293E-01 0.2810E-02	0.6704E-01 0.1175E-02	0.5847E-01 0.3317E-02
0.900	0.3863 0.8137E-03	0.2665 0.2471E-02	0.2167 0.3096E-02	0.1847 0.2198E-02	0.1594 0.7442E-02	0.1510 0.3771E-02	0.1243 0.2028E-02	0.1055 0.3041E-02
0.950	0.4856 0.3510E-02	0.3386 0.2397E-02	0.2797 0.9182E-02	0.2313 0.8222E-03	0.2002 0.9596E-02	0.1841 0.3863E-02	0.1540 0.6722E-02	0.1358 0.2712E-02
0.975	0.5606 0.2385E-02	0.4015 0.7463E-02	0.3203 0.1191E-01	0.2753 0.7878E-03	0.2438 0.1029E-01	0.2155 0.1020E-01	0.1787 0.9576E-02	0.1570 0.6993E-02
0.990	0.6347 0.6969E-02	0.4575 0.1359E-01	0.3770 0.1455E-01	0.3085 0.9725E-03	0.2825 0.2111E-01	0.2585 0.1153E-01	0.2263 0.2773E-02	0.1853 0.6623E-02

MEAN OF REGRESSION ON AVERAGES	0.2970E-02 0.6870E-02	-1.759 4.637	-108.0 769.9	3103. 0.3904E+05	1149. 0.1750E+05
STD DEV OF REGRESSION	0.1493E-01 0.7525E-03	7.766 0.6230	1175. 121.5	0.5942E+05 6829.	0.2678E+05 3174.
REGRESSION ON VARIANCE	2.658 3.566	39.61 148.6	-589.4 2183.	3805. 0.1312E+05	-7935. 0.2689E+05

ESTIMATOR: **Rho(.02911) of SqrtBeta L laplace AR r.vs., A=.02, L= .4 Using Cressie Estimator

Figure 57. Summary Statistics, L-Laplace Samples. Cressie Estimator.

SUMMARY STATISTICS (MEAN/STD)   3 SUPER-REPLICATIONS

SUBSAMPLE SIZE	20	60	100	150	200	250	350	500
MEAN	0.1430 0.2340E-03	0.2037 0.1369E-02	0.2123 0.4610E-02	0.2246 0.2745E-02	0.2295 0.2938E-02	0.2321 0.5004E-03	0.2339 0.3139E-02	0.2362 0.2435E-02
STD	0.2313 0.1210E-02	0.1500 0.2588E-02	0.1208 0.1297E-02	0.1001 0.8618E-03	0.8854E-01 0.1119E-02	0.7998E-01 0.1359E-02	0.6710E-01 0.1498E-02	0.5746E-01 0.1512E-02
M.S.E	0.2578 0.1076E-02	0.2302 0.7346E-03	0.2195 0.3356E-02	0.2197 0.2826E-02	0.2191 0.3025E-02	0.2182 0.2187E-03	0.2155 0.2548E-02	0.2149 0.1951E-02
SKEWNESS	-0.9472E-02 0.1683E-01	-0.2317E-01 0.2937E-01	0.4351E-01 0.9531E-01	0.1094 0.6289E-01	0.4833E-01 0.2465E-01	0.1221 0.6416E-01	0.1516 0.1128	0.7783E-01 0.1015
KURTOSIS	-0.6672E-01 0.6990E-01	-0.9775E-01 0.5137E-01	-0.1051 0.1576	0.1237 0.4947E-01	0.2014 0.2068	0.6189 0.3462	-0.1105E-01 0.1929	0.1954E-01 0.2595
SER. COR.	-0.8732E-02 0.3565E-02	-0.2095E-02 0.5441E-02	-0.3575E-01 0.2039E-01	-0.4404E-01 0.5811E-02	0.1705E-01 0.7078E-02	0.7705E-02 0.3559E-01	-0.4506E-01 0.2592E-01	-0.6271E-02 0.3335E-02
QUANTILES								
0.010	-0.3841 0.4033E-02	-0.1539 0.1476E-01	-0.5916E-01 0.6130E-02	-0.5015E-02 0.5907E-02	0.2812E-01 0.4066E-02	0.3232E-01 0.7915E-02	0.8205E-01 0.3764E-02	0.1010 0.4065E-02
0.025	-0.3102 0.2882E-02	-0.8595E-01 0.7914E-02	-0.1608E-01 0.7511E-02	0.2852E-01 0.6943E-02	0.5652E-01 0.1975E-02	0.7070E-01 0.4904E-02	0.1083 0.1660E-02	0.1191 0.2926E-02
0.050	-0.2407 0.7844E-03	-0.3846E-01 0.5834E-02	0.1383E-01 0.5580E-02	0.5871E-01 0.3742E-02	0.8898E-01 0.1445E-02	0.1060 0.3249E-02	0.1279 0.3197E-02	0.1397 0.3042E-02
0.100	-0.1560 0.2270E-02	0.1337E-01 0.7382E-02	0.5408E-01 0.6772E-02	0.9673E-01 0.1651E-02	0.1184 0.3562E-02	0.1318 0.2261E-02	0.1497 0.4913E-02	0.1619 0.2828E-02
0.250	-0.1570E-01 0.8207E-03	0.1014 0.3038E-02	0.1282 0.5193E-02	0.1560 0.3176E-02	0.1679 0.1111E-02	0.1771 0.7323E-03	0.1885 0.4974E-02	0.1979 0.4596E-02
0.500	0.1416 0.3574E-03	0.2021 0.2453E-02	0.2136 0.3962E-02	0.2227 0.3299E-02	0.2274 0.4130E-02	0.2354 0.1290E-02	0.2316 0.4116E-02	0.2351 0.3296E-02
0.750	0.3056 0.9947E-03	0.3089 0.8791E-03	0.2962 0.5867E-02	0.2919 0.5904E-02	0.2891 0.3211E-02	0.2843 0.1461E-02	0.2801 0.2981E-02	0.2764 0.1614E-02
0.900	0.4413 0.9417E-03	0.3969 0.2179E-02	0.3666 0.5820E-02	0.3525 0.3092E-02	0.3409 0.5993E-02	0.3322 0.1166E-02	0.3247 0.1352E-02	0.3119 0.3145E-02
0.950	0.5159 0.3052E-02	0.4487 0.5707E-03	0.4103 0.5046E-02	0.3890 0.2226E-02	0.3755 0.4839E-02	0.3623 0.3412E-02	0.3456 0.1636E-02	0.3322 0.9014E-03
0.975	0.5779 0.6158E-02	0.4957 0.7468E-02	0.4500 0.8997E-02	0.4221 0.5539E-02	0.4147 0.5646E-02	0.3912 0.5836E-02	0.3678 0.4665E-02	0.3477 0.2194E-02
0.990	0.6531 0.8688E-02	0.5516 0.2006E-02	0.4953 0.1480E-01	0.4635 0.3860E-02	0.4418 0.2953E-02	0.4279 0.4420E-02	0.4014 0.1358E-01	0.3692 0.1977E-02

MEAN OF REGRESSION ON AVERAGES	0.2347 0.1567E-01	1.868 8.200	-780.0 1221.	0.4302E+05 0.5993E+05	0.1986E+05 0.2657E+05
STD DEV OF REGRESSION	0.9661E-02 0.1456E-02	5.081 0.7253	765.6 94.44	0.3849E+05 4522.	0.1730E+05 2014.
REGRESSION ON VARIANCE	1.782 0.9290	-3.506 37.95	11.63 569.1	-127.2 3546.	365.0 7500.

ESTIMATOR: ++Rho(.02911) of SqrtBeta L laplace AR r.v.s, A=.02, L= .4 Using moment estimator

Figure 58.   Summary Statistics, L-Laplace Samples. Moment Estimator.

SUMMARY STATISTICS (MEAN/STD)   3 SUPER-REPLICATIONS

SUBSAMPLE SIZE	20	60	100	150	200	250	330	500
MEAN	0.1406 0.1322E-02	0.2002 0.1393E-02	0.2189 0.9087E-03	0.2245 0.4851E-03	0.2299 0.4247E-02	0.2319 0.4089E-02	0.2345 0.5845E-03	0.2346 0.2020E-02
STD	0.2340 0.1786E-02	0.1487 0.7683E-03	0.1201 0.8261E-03	0.1004 0.5783E-03	0.8956E-01 0.8787E-03	0.8023E-01 0.2049E-02	0.6935E-01 0.8559E-03	0.5661E-01 0.1561E-02
M.S.E	0.2592 0.1606E-02	0.2267 0.1364E-02	0.2246 0.1199E-02	0.2197 0.1785E-03	0.2199 0.3603E-02	0.2181 0.4547E-02	0.2167 0.8268E-03	0.2132 0.1764E-02
SKEWNESS	0.9924E-02 0.2629E-01	-0.1773E-01 0.5128E-01	0.3042E-01 0.3323E-01	0.9539E-01 0.2149E-01	0.8646E-01 0.6879E-01	0.2251E-01 0.6545E-01	0.2055E-02 0.4984E-01	0.1222 0.2926E-01
KURTOSIS	0.1649 0.2493	-0.7432E-01 0.1203E-01	-0.8312E-01 0.2939E-01	0.2925 0.2144	0.1148 0.1809	-0.2595E-01 0.8618E-01	-0.1303 0.1479	-0.2522 0.4382E-01
SER.COR.	0.5842E-02 0.5277E-02	0.1497E-01 0.1892E-01	0.9456E-02 0.1038E-02	-0.6970E-02 0.1526E-01	-0.2056E-01 0.2036E-01	-0.5468E-02 0.3315E-02	-0.1789E-02 0.2479E-01	-0.3567E-01 0.2680E-01
QUANTILES								
0.010	-0.3982 0.5452E-02	-0.1442 0.5769E-02	-0.5441E-01 0.1211E-02	-0.5544E-02 0.3130E-02	0.2960E-01 0.6614E-02	0.4289E-01 0.8928E-02	0.7816E-01 0.3083E-02	0.1025 0.1114E-01
0.025	-0.3152 0.5661E-02	-0.8679E-01 0.5745E-02	-0.1324E-01 0.3810E-02	0.2626E-01 0.2786E-02	0.5486E-01 0.8062E-02	0.6794E-01 0.7943E-02	0.1002 0.3362E-02	0.1232 0.5215E-02
0.050	-0.2434 0.3718E-02	-0.4162E-01 0.1550E-02	0.1745E-01 0.1264E-02	0.6149E-01 0.3265E-02	0.8821E-01 0.5447E-02	0.9895E-01 0.1510E-02	0.1192 0.2402E-02	0.1456 0.4007E-02
0.100	-0.1590 0.4660E-02	0.8070E-02 0.4241E-03	0.6357E-01 0.1960E-02	0.9781E-01 0.4613E-03	0.1162 0.4154E-02	0.1311 0.1148E-02	0.1408 0.2322E-02	0.1628 0.1728E-02
0.250	-0.1984E-01 0.2134E-02	0.9870E-01 0.3433E-02	0.1364 0.7951E-03	0.1560 0.1022E-02	0.1693 0.4518E-02	0.1778 0.4377E-02	0.1878 0.1730E-02	0.1950 0.1066E-02
0.500	0.1400 0.3139E-02	0.1979 0.2783E-02	0.2193 0.1890E-02	0.2237 0.8679E-03	0.2275 0.4253E-02	0.2315 0.4636E-02	0.2346 0.8258E-03	0.2331 0.2634E-02
0.750	0.3032 0.1002E-02	0.3042 0.2994E-02	0.3017 0.1323E-02	0.2905 0.1460E-02	0.2892 0.6597E-02	0.2850 0.5168E-02	0.2839 0.1989E-02	0.2752 0.3313E-02
0.900	0.4421 0.1376E-02	0.3891 0.2461E-02	0.3707 0.4224E-02	0.3502 0.2514E-02	0.3470 0.5893E-02	0.3317 0.4835E-02	0.3224 0.3936E-02	0.3080 0.1624E-02
0.950	0.5162 0.2674E-02	0.4421 0.3565E-02	0.4193 0.3630E-02	0.3876 0.3189E-02	0.3814 0.4062E-02	0.3685 0.7970E-02	0.3469 0.4954E-02	0.3293 0.2592E-02
0.975	0.5840 0.2749E-02	0.4848 0.3031E-02	0.4595 0.4550E-02	0.4351 0.3054E-02	0.4106 0.1586E-02	0.3897 0.1224E-01	0.3675 0.3098E-02	0.3537 0.2467E-02
0.990	0.6647 0.8034E-02	0.5519 0.5819E-02	0.5019 0.3853E-02	0.4792 0.2953E-02	0.4523 0.5512E-02	0.4198 0.9159E-02	0.4067 0.1200E-01	0.3727 0.8780E-02

MEAN OF REGRESSION ON AVERAGES	0.2390 0.4738E-03	-1.776 1.491	-18.25 283.8	-1464. 0.1469E+05	-1804. 6612.
STD DEV OF REGRESSION .	0.1126E-01 0.1432E-02	5.362 0.8504	812.1 119.1	0.4031E+05 5286.	0.1803E+05 2224.
REGRESSION ON VARIANCE	0.8098 0.7878	40.30 36.03	-642.0 551.4	3759. 3364.	-7460. 6921.

ESTIMATOR: ++Rho(.02911) of SqrtBeta L laplace AR r.vs., A=.02, L= .4 Using Priestly Estimator

Figure 59. Summary Statistics, L-Laplace Samples. Priestley Estimator.

VI. CONCLUSIONS

A. SIMTBED.

These types of statistical simulations would be largely infeasible without some sort of aids to the user. With routines like SIMTBED, these simulations can be interesting investigations. Granted, SIMTBED can be difficult to understand for the beginning simulator. But, once familiarity is gained all sorts of investigations and simulations are within the realm of the user. Now, we have increased the capability of SIMTBED by providing the user the capability to use his personal computer and color output devices in statistical simulations, which is often more convenient and cost effective. As technology proceeds, these types of programs will attract more attention and produce more for the user community.

B. SERIAL CORRELATION ESTIMATION.

These simulation results do not, in the opinion of this author, provide any evidence to indicate that the more esoteric and analytically intractable estimators of serial correlation are any better, with regard to producing the "correct" results, than the well-understood estimators the standard statistical theory has produced. In fact, they may be worse. This is demonstrated by the behavior of the Cressie estimator and the robust regression approach versus the familiar moment and less familiar MLE estimators. Further, if the user interested in serial correlation has additional information about his samples, he can, with these results, determine the behavior of these estimators on his samples. In order to get good estimates of serial correlation, the following advice is offered:

- Use the largest possible sample size. If possible, it is desirable to be outside the small-sample bias effects.
- Try and ascertain the distributional properties of the sample/population. If it is distinctly non-Normal, then recognize there will be a corresponding lack of accuracy in the estimate.
- If possible, obtain multiple samples. Multiple estimates give better overall indications of the correlation.

C. FURTHER EXPERIMENTATION.

It would be interesting to study the "GM-estimate" of Denby and Martin [Ref. 9, p. 141]. Other non-Normal distributions can also be investigated. Given

sufficient time, these simulations can be carried further, in terms of sub-sample sizes, and number of super-replications. Finally, there are probably other esoteric estimators of serial correlation in the literature which could be studied, as this is an important field in which to conduct research.

APPENDIX A. USER'S GUIDE TO SIMTBED, A SIMULATION TEST BED

A. OVERVIEW.

The simulation test bed program used in this thesis, SIMTBED, is designed to aid in the assessment of the distributional properties of any user-supplied statistical estimator based on i.i.d random variables or on time series. It is a FORTRAN subroutine, and provides analysis of estimators at various sample sizes. It controls the user-provided generating routines so that the estimator can be computed many times to study its distribution, moments (through the 4th central moment), and the variability of the estimates of these values. A regression model for the mean and variance can also be constructed. In general, any statistical simulation provides voluminous output, and this approach is designed to compactify the output and make the important features apparent. Graphic displays of the behavior of the user estimators is accomplished using line printer graphics, and the system is designed for the IBM personal computer family of computers. Output consists of a series of plots for each estimator (box plots, percentile plots, normalized quantile plots), summary tables for super- replications, a combined boxplot graph for all estimators, and optionally bivariate histograms for pairs of up to three estimators. If known values of the estimators are provided, the mean squared error from these known values can be obtained.

1. History.

SIMTBED began in 1981 with a mainframe version by Lewis, Orav, and Uribe. The first personal computer version was presented in a thesis by Hans-Walter Drueg, based on the regression adjusted graphics and estimation methodology of Lewis and Heidelberger. It has subsequently been moved to the IBM PC, and modifications include:

- A capability for super-replications to estimate the variability of the estimates of the quantiles, moments, and regression parameters.
- A restart capability for building super-replication simulations over time using files.
- Use of files to capture super-replication output for further processing by other systems.
- Bivariate histograms.
- Most recently, the use of color to compare the estimators with each other in the same plot.

This work reflects changes in data processing technology, and is largely the result of work by P.A.W. Lewis, E.J. Orav, and L. Uribe, while the most recent additions to the system were added as part of this thesis by the author. The remainder of this appendix consists of a discussion of the features of SIMTBED, and how it is used.

B. FEATURES OF SIMTBED.

In order to use SIMTBED, the user must supply the driver, a program which sets the parameters (listed below) and calls SIMTBED, and the estimator routines, which take a random number seed and sample size, and generate one result which is the estimator, based on the sample size. These are referred to as sub-sample sizes, for the following reason.

1. Sample Sizes.

SIMTBED takes a global sample size, N, (5000, for example), and then sections it according to various subsample sizes NE(), and calls the estimator routine(s) the appropriate number of times. In this way multiple evaluations of the estimator are obtained. If the subsample size were 100, and N were 5000, then 50 evaluations of the estimator, based on a sample size of 100 would be obtained. This process can be replicated up to 100 times, through M, the number of replications, so for example, if N is 5000, and NE is 50, and M is 20, then 2000 evaluations of the estimator will be generated. Up to eight different subsample sizes can be specified, and will be used for each of up to five estimators. These evaluations of the estimator are stored in an array, and SIMTBED will obtain statistics from them. The array must hold at least $(N \times M)/NE(1)$, where NE(1) is the first, and smallest subsample size. The subsample sizes are in increasing order, which is conducive to the graphing process.

2. SIMTBED Estimator Statistics.

SMTB computes the mean of the stream of estimators, for each subsample size. Also, the standard deviation is computed using n-1 weighting. The standard error of the mean is computed, as $\sigma/\sqrt{n}$. The mean squared error (MSE) is optionally estimated using the formula

$$\hat{\text{MSE}} = \sqrt{(\hat{\text{VAR}} + (\hat{\mu} - \text{VMSE})^2)} \qquad (A-1)$$

where VMSE is the known input value of the estimator, again for each subsample size. The coefficients of skewness and kurtosis are also computed, using unbiased expressions.

3. Graphs.

a. Boxplots.

A boxplot graph is generated for each estimator, depicting boxplots for each subsample size, showing the mean, median, inter-quartile range, and outlier information. The vertical scale can be set by the user, or SIMTBED will scale the graph according to the largest values encountered. The graphs can be fixed to all have the same scale, or each can be scaled individually. The boxplots can be reduced. That is, the outliers are reported as a numeric count at the tail of the boxplot instead of representing each by a separate symbol. This is often necessary for highly variable estimators. If super-replications are being performed, up to three such plots for each estimator can be obtained.

b. Quantiles.

A plot of eleven quantiles is prepared next. They are depicted in the plot, as well as tabulated below the graph. The lag one serial correlation (moment estimator) is also listed. A normalized quantile plot is also available.

c. Other Plots.

If requested, bivariate histograms for the pairwise combinations of the first three estimators can be obtained to graphically assess relationships between the different estimators. One plot is prepared for all the subsample sizes, giving the possibility of 24 histograms if the full complement of subsample sizes is prepared. The bivariate histogram also lists four measures of association: covariance, correlation coefficient, Spearman's rank correlation coefficient, and Kendall's tau coefficient. Five tests for equidistribution are included: Kolmogorov-Smirnov, Wald-Wolfowitz, Mann-Whitney, Wilcoxson, and Siegel-Tukey tests. The overall univariate statistics are also reproduced with these plots: mean, median, variance, standard deviation, range, etc.

d. Combined Estimator Boxplots.

Finally, a graph of selected boxplots for all estimators together can be obtained, so the relations between them at various sample sizes can be better studied. If a color printer is available, each estimator will have a different color. Up to three of the subsample sizes can be plotted, for up to five estimators, all on the same graph. If regression is being performed, the asymptote lines will be included, in the appropriate color for each estimator. These regression lines are merely reproductions of the full regression SIMTBED performed when the estimators were simulated, and are based on the original SIMTBED regression. No additional simulation or computation is required to produce these combined plots.

4. Regression.

The regression performed by SIMTBED follows the methodology developed by Heidelberger and Lewis (see the body of the thesis, Chapter 3, Section B.). Regression is performed on the mean of estimates using the data from the subsample sizes according to the equation:

$$E[\theta_m] = \theta + \left(\frac{a_1}{m}\right) + \left(\frac{a_2}{m^2}\right) + \left(\frac{a_3}{m^3}\right) + \ldots \qquad (A-2)$$

the degreee of regression is the number of terms of the equation used. In equation (2), θ is the regression estimate of the estimator. This regression is performed for each replication (M), and with these independent data points, the variance and standard deviation of the coefficients of equation (2) can be estimated. These equation coefficients are reported with the standard boxplot graphs SIMTBED produces.

The regression for the variance of the estimator is performed once, using all M*N data points, according to the equation

$$\mathrm{Var}[\theta_m] = \left(\frac{b_1}{m}\right) + \left(\frac{b_2}{m^{1.5}}\right) + \left(\frac{b_3}{m^2}\right) + \left(\frac{b_4}{m^{2.5}}\right) + \ldots \qquad (A-3)$$

These coefficients are also reported in the plot summary.

5. Super-replications.

The simulation process just described can be wholly repeated many times, referred to as super-replications. This makes it possible to get precision information, in the form of standard deviation, for all the statistics SIMTBED computes, instead of just the regression adjusted estimate of the mean of the estimator distribution. With super-replications, averages and standard deviations are available for all the moments, the serial correlation of the replications, and all eleven quantiles for all the subsample sizes, and the regression coefficients as well. If specified, individual run boxplot graphs and quantile plots can be obtained for the first three super-replications, or only for the first. These super-replication summary statistics can be stored in a file in tabular form for further processing by any number of other means, or for archival purposes.

Since the performance of super-replications can be quite time-intensive, SIMTBED has the capability to write the parameters and summary data to a file for super-replications so far performed, and will then restart from where it left off, producing

more super-replications on the same simulation. In this way, the precision needed can be achieved piecewise, at the convenience of the user.

6. Random Number Generation.

Since statistical simulation is dependent on the production of pseudo-random numbers, SIMTBED makes available routines for generating several common distributions of random variables. Among these are uniform, normal, Laplace, Cauchy, gamma, Pareto, beta, geometric, Poisson, and binomial random variables. These random number generators are based on the well tested uniform random number generator LLRANDOM II. These random number generators are linked with the SIMTBED subroutines after compilation, and the user can thus use any linker/library utility to insert or remove any of these routines, or add new random number generators (i.e. IMSL) at will.

C. USE OF SIMTBED.

To use SIMTBED you must write a FORTRAN program which has a CALL statement to the SIMTBED subroutine. This is called a driver program, and your driver must do several things for SIMTBED to operate properly. An example driver is included as figure 1. The driver must do the following:

- Initialize and set appropriate values for the arguments of SIMTBED (see Table 1). This is done through the REAL, INTEGER, DATA, and direct assignment FORTRAN statements.
- Declare the scratch array of the correct size for SIMTBED to use. This size is determined as N * M / NE(1), since NE(1) is the smallest sample size, and will result in the largest number of evaluations of your estimator.
- Declare as EXTERNAL the names of the estimator routines provided to SIMTBED.

1. SIMTBED Arguments.

The argument types for SIMTBED are listed below, along with their function and possible values. In general, the names used here are not required, but they must match in variable type and position in the calling statement, and the arrays must be of the correct dimension. For example, NCOLRNDX is not required to be named as such, but it must be an INTEGER(3) array, and be in the proper order in the argument list of the calling statement.

2. Calling Statement.

The calling statement to invoke SIMTBED is:

```
CALL SIMTBED(ISEED1,ISEED2,ISEED3,ISEED4,ISEED5,Y,N,M,NE,
*L,D,NSR,RG,SEI,SVS,YMIN,YMAX,NEST,GEST1,TTL1,GEST2,TTL2,
*GEST3,TTL3,GEST4,TTL4,GEST5,TTL5,IFIL,NPRT,MSE,VMSE,IPR,
*VMX1,VMX2,VMX3,VMX4,VMX5,IBIV,RSTRT,ICOLOR,IBWPRT,NCPRT,
* NCOLRNDX)
```

3. Estimator Routines.

The user must also provide a number (up to five) of estimator subroutines to SIMTBED, for generating the random numbers and the estimator, based on the sample size. The argument list of the estimator routine must have the following structure (the name is up to the user, of course):

```
SUBROUTINE GEST1(ISEED,N,EVAL)
```

where GEST1 is the declared EXTERNAL name, ISEED is a REAL*8 (double precision) number for the random number seed, N is an INTEGER sample size to use, and EVAL is a REAL*4 result, the estimator evaluated based on sample size N. Within the routine the user is free to do anything necessary to generate the random process and obtain the estimator (could even be non-random nature). It is common to have calls to random number generators and some follow-on calculations (for example, obtaining the fourth moment of standard normal random numbers). It is often of interest to determine how many times this user routine will be called. The value N will be each NE(I) in sequence, so the number of calls to this subroutine will be N * M / NE(I), for each subsample size. This is why the Y array is required to be at least a certain size. It hold's the results from the calls to this routine. Notice there is no provision for other parameters to be passed to the estimator routine. If the user wishes to parameterize the estimator routines through the driver program (useful for complicated and related estimators), COMMON storage is available for this purpose. SIMTBED does not use any COMMON storage.

4. Files.

SIMTBED has the ability to use three files in addition to the console (UNIT 5). These are for normal output (UNIT 6), restart operations (UNIT 2), and super-replication recording results (UNIT 1). Each of these files is used in a different way. They are all created with the OPEN statement. The UNIT 6 file will contain carriage control characters for visual display devices (printers, CRTs), and can be a file, or an actual unit. We recommend using a file to prevent a printer error from causing a program interrupt and loss of data. If a file is used it must be printed with respect given to

the carriage control characters, or the plots will not be correct (the DOS PRINT command suffices nicely). For example

OPEN(06,FILE='RESULTS.TXT',FORM='FORMATTED')

where ACCESS of SEQUENTIAL is the default and not specified. Also, no error handling is present in this example. The file used for output of super-replications is also of type FORMATTED, but is written to UNIT 1, and has no carriage control characters imbedded. The result is a normal ASCII file that can be read by any text editor. The file used for restart is different, however. Using UNIT 2, this file is UNFORMATTED, and can only be read by another FORTRAN program. Since type FORMATTED is the default for sequential files, the user must give this specific declaration in the OPEN statement, or an error will occur. For example,

OPEN(02,FILE='START.DAT',FORM='UNFORMATTED',ACCESS=

'SEQUENTIAL') will cause the correct file type to be created. The data in this file is written in internal storage form, and is inaccessible to the user.

5. Using Random Numbers.

The random number generators available in SIMTBED are based on the uniform random number LLRANDOMII, as mentioned earlier. What will be listed here is the CALL statement and arguments required to generate a random variable stream, and the form of the variables generated. For all the calls, DSEED is a REAL*8 seed, N is the number of variables generated, and X is a real array of size at least N, containing the random numbers after the call.

- For uniform random numbers, use CALL LRNDPC(DSEED,X,N) to obtain uniform [0, 1] random numbers.
- For normal random numbers, use CALL LNORPC(DSEED,X,N) to obtain standard normal random numbers.
- For Laplace random numbers, use CALL LLAPPC(DSEED,X,N) to obtain Laplace random numbers with mean 0 and standard deviation SQRT(2).
- For Cauchy random numbers, use CALL LCHYPC(DSEED,A,N) to obtain Cauchy random number with location 0 and scale 1
- For Gamma random numbers, use CALL LGAMPC(DSEED,X,N,K) to obtain Gamma random numbers with shape K and scale 1. Transform to Gamma(k,b) by dividing by b.
- Pareto random numbers are available by the statement CALL LPARPC(DSEED,X,N,A) where A is the shape parameter of the Pareto distribution.

- Beta random numbers are obtained with CALL LBETPC(DSEED,X,N,P,Q) where P and Q are the parameters of the Beta distribution.
- Geometric (discrete) random variables are available with CALL LGEOPC(DSEED,A,N,P) where P is the parameter of the geometric distribution.
- Generate Poisson random variables with CALL LPOIPC(DSEED,A,N,U) where U is the mean of the desired distribution. The method used to generate these variables uses a table lookup, and requires some front end 'overhead' to set up. For best efficiency, generate as many variables as possible at one time.
- To generate Binomial random numbers, CALL LBINPC(DSEED,X,N,NIND,P) is required. NIND is an INTEGER, for the parameter number of trials. P is the p parameter for the binomial distribution.
- For random variables other than those listed here, or that cannot be obtained through some transformation, the user must provide the routine to generate them, usually starting with a call to LRNDPC.

6. Running SIMTBED.

In general, to run a driver with SIMTBED, the driver must be compiled and linked with the SIMTBED library. For most compilers there is an option for large arrays. This should be used if the dimensions of the Y array are large. The reader is referred to the body of the thesis for examples of SIMTBED output.

D. SIMTBED ARGUMENTS AND PARAMETERS.

1. ISEED1, ISEED2, ISEED3, ISEED4, ISEED5. DOUBLE PRECISION (REAL*8) reals. These are the seeds used to start the random number streams used by SIMTBED. Using the same seed will produce the same results. Use the same number for the seed to get the same random numbers for each estimator.
2. Y. REAL*4 array. The scratch array used by SIMTBED to carry the results of your estimator when called.
3. N. INTEGER. The "global" sample size.
4. M. INTEGER. The number of replications of each subsample size to perform. M cannot exceed 100.
5. NE. INTEGER array of size 8. The subsample sizes for SIMTBED to use. Must be in ascending order. If less than 8 are used, pad with zeros.
6. L. INTEGER. The number of subsample sizes to use. Must be between 1 and 8.
7. D. INTEGER. The degree of regression (number of terms in regression equation). Can be from 1 to 6. If there are not enough subsamples (D < = L - 1) to establish the regression, SIMTBED will lower the degree of regression specified in the call. Using D = 0 will suppress the regression.
8. NSR. INTEGER. Number of super-replications to perform. Must be at least one. If only one super-replication is used, no summary will be generated.

9. RG. INTEGER. Reduced Graphics. 1 = Yes. 0 = No. In reduced graphics, SIMTBED will put the number of outliers at the end of the boxplot instead of plotting each one. This makes the scale of the plot closer to the interquartile range, and thus easier to interpret for highly variable estimators.

10. SEI. INTEGER. Scale Estimators Individually. 1 = Yes. 0 = No. When set to zero, plots for all estimators will have the same scale (the max of the set of estimators). This is required for color combined plots, so if color is selected, this flag will be reset.

11. SVS. INTEGER. Set Vertical Scale. 1 = Yes. 0 = No. If SVS = 1, the user will set the vertical scale with the values YMIN and YMAX. Otherwise, SIMTBED determines these values.

12. YMIN. REAL. Minimum y axis value for SVS = 1.

13. YMAX. REAL. Maximum y axis value for SVS = 1.

14. NEST. INTEGER. Number of Estimators. Must be between 1 and 5. The appropriate arguments for the number of estimators must be defined. Otherwise, dummy arguments can be used (ISEED, GEST, TTL, VMX).

15. GEST1, GEST2, GEST3, GEST4, GEST5. EXTERNAL. These are the names of the estimator routines, passed to SIMTBED in the calling statement. For unused estimators, duplicate the defined names.

16. TTL1, TTL2, TTL3, TTL4, TTL5. CHARACTER*120 (max). A label of the estimator routine, to be placed on the plots.

17. IFIL. INTEGER. 1 = Yes. 0 = No. Yes will write the super-replication statistics to the file declared on UNIT 1, for further processing. This is a formatted ASCII file.

18. NPRT. INTEGER. Print Detail. 1 = Yes. 0 = No. Yes will print full plots for the first three super-replications, and normalized quantile plots in addition to the standard quantile plots. No will print full plots only for the first super- replication. Regardless, the information for the color combined plots comes from the first super-replication.

19. MSE. Mean Squared Error. INTEGER. 1 = Yes. 0 = No. Yes will compute the mean squared error of the estimator at each sample size from the user input known values. No ignores the user input known values.

20. VMSE. Vector of Mean Squared Error. REAL array (8,5).
The known values of each estimator at each subsample size, for SIMTBED to compare with the generated results, based on MSE flag. Initialize to zeros for unused portions of this array.

21. IPR. Print Percentile Plots. 1 = Yes. 0 = No. Yes will cause SIMTBED to count the frequency of occurrence where the evaluated estimator is less than each of four specified values (the percentiles), for each subsample size. These are then converted to probabilities, and a plot is produced.

22. VMX1, VMX2, VMX3, VMX4, VXM5. REAL array (8,4) each. These are user supplied values for the percentile plot calculations. SIMTBED will obtain the frequency that the estimator is less than each value and prepare a percentile plot.

23. IBIV. INTEGER. Bivariate Histograms. 1 = Yes. 0 = No. Yes causes bivariate histograms to be produced for each combination of the first three estimators, at each subsample size.

24. RSTRT. Restart. INTEGER. 1 = Yes. 0 = No. Yes results in a SIMTBED write to the file on UNIT 2 of the super- replication final results, so that the simulation can be continued from this point later. For subsequent runs, Yes causes the file on UNIT 2 to be read, and the parameters checked for agreement. Processing continues with the next super-replication. For example, if 3 is used for NSR, then using 8 for the next run for NST will cause 5 more super- replications to be performed. Using a value for NSR for a later run less than that used on an earlier run causes a message to be displayed and processing stops.

25. ICOLOR. INTEGER. Color Combined Printing. 1 = Yes. 0 = No. Yes causes combined boxplots of all the estimators to be produced. The subsample sizes used are determined by another array. SEI is set to 0. Printer control codes to set colors for standard IBM dot matrix printers are used to print each estimator in a different color. On non-color printers these codes are ignored.

26. IBWPRT. INTEGER. Standard Printing. 1 = Yes. 0 = No. Yes causes the standard printing of boxplots for each estimator in sequence. No suspends this printing for brevity. Bivariate histograms and super-replications summary plots will still be generated. This is used in conjuction with the ICOLOR flag to produce output which will consist of *only* the color combined plot. This way, if the experimenter in only interested in obtaining this plot, he will not have to print all the previously generated plots to obtain it. (This actually came about as an aid during development of the color features.)

27. NCPRT. INTEGER. Number of clusters to print. Must be between 1 and 3. This is the number of subsample sizes for which to print combined boxplots.

28. NCOLRNDX. INTEGER array, size 3. Each entry must be between 1 and 8, in increasing order. Do not duplicate entries. This specifies which of the subsample sizes are used in the combined plots. For example, DATA NCOLRNDX/2,3,4/ will cause SIMTBED to use the second, third, and fourth subsample sizes in the combined plots.

This concludes the discussion of SIMTBED.

APPENDIX B. METHOD USED TO PROGRAM COLOR COMBINED BOXPLOTS IN SIMTBED

A. INTRODUCTION.

This appendix describes the programming approach I used to incorporate the features we wanted into SIMTBED, which took approximately three months of analysis of the SIMTBED program and its approach to statistical simulation. There were four basic aspects involved in incorporating color printing into SIMTBED:

- Understand the architecture and organization of the existing SIMTBED package.
- Understand the method of producing graphics with line printers and dot matrix printers (line printer graphics).
- Understand how color printers are controlled (changing their colors, etc.).
- Determine the approach to placing the color boxplots together to produce the complete plot. Spacing, choice of color, and graph features to be included were all considerations.

Each of these will be discussed in turn.

B. SIMTBED ARCHITECTURE.

SIMTBED, version 13, has 21 subroutines, including two added by this author to control the color dot matrix printers. Most of the subroutines are involved with computing statistics, regression, percentiles, and other numerical interests, and so are not of direct interest here. The starting subroutine, SIMTBED, calls the subroutine PRST once for each estimator being processed, and PRST is the routine that conducts the sectioning, calls the generator routines, obtains the statistics, and prepares the plots. This routine is where the color plotting preparation has to occur, although it will be printed in SIMTBED, since the color plot is not complete until all the estimators have been processed. PRST makes use of two important subroutines in constructing plots: BOXPRT, which puts a boxplot in a specified place in a plot array (given the appropriate statistics, of course), and NUMPRT, which puts an integer value into the plot array in the specified location. The color plotting routine will use both of these.

C. LINE PRINTER GRAPHICS.

Using character printing devices to generate plots is not a complicated concept. The line print (or dot matrix printer) is normally a character oriented device, and can be used

to create plots. The difficulty lies in determining the position of the points to be plotted. Once the position is determined, the appropriate character is simply printed. The approach to this method is to use a character matrix, a two-dimensional array of characters. The row is then the x axis, and the column is the y axis. Then, all the (x, y) pairs of points are scaled such that they run between zero and the dimension of the array. In SIMTBED, the dimensions are 132 spaces across and 50 vertical spaces. The dot matrix printer is used in compressed type mode, to make maximum use of the limited resolution it has compared to other graphic devices. For these purposes, line printer graphics work quite well.

The color plotting approach to line printer graphics uses the same approach, except there is a three dimensional character array: the first two dimensions represent the row and column of the plot, and the third dimension represents the color being plotted. Thus, there is one complete character array for each color to be plotted. When a character is to be plotted in a specified color, it is stored in that 'sheet' of the character array. Then, when printing is to occur, each line is printed the number of times corresponding to the number of colors before moving on to the next line.

D. COLOR PRINTERS.

I found that there is in fact a good degree of standardization in the commands used by different color printers. These commands consist of special escape code sequences to change the printer color (the escape character, the lower case 'r',and a numeric color code). This suggested a simple subroutine that would accept a number representing the color, and would set the printer (FORTRAN UNIT 6) to that color, using the escape code sequence. The other subroutine added merely initialized the three-dimensional color character array to all blanks.

E. DETERMINING THE COLOR PLOTTING.

Having determined the SIMTBED approach to line printer graphics, and the approach to the new color printers, it remained to choose a good way to display the color plots. How many subsample sizes? How should they be spaced? How much of the regression lines could be included before the plot became too cluttered? These issues were addressed through experimentation. Along the way, it was easily possible to increase the number of estimators SIMTBED could process from three to five. With five estimators, we found that more than three subsample sizes could result in twenty boxplots being plotted together in one plot. This was the limit. We restricted the number of subsample sizes for color combined boxplots to three, and with five estimators this proved to be

reasonable, if the subsample sizes are spread out. If they are not, but are closely arranged, then the scaling of the regression line, when plotted, will be incorrect. Because of this, and the clutter in the plot for different regression lines for different estimators, we removed the regression line from the color plots. The asymptote was kept, though, as it did not clutter the graphs terribly. This is the new current version of SIMTBED for the personal computer.

APPENDIX C. SAMPLE SIMTBED DRIVER ROUTINES

```
C     SIMTBED DRIVER FOR NORMAL AR(1) RANDOM VARIABLES                   RST
C     ---------------------------- SMTB12 -------------------------------RST
      REAL*4  Y(6500),YMIN, YMAX, V(8),RHO
      CHARACTER*120 T1,T2,T3,T4,T5                                       RST
      REAL*8 IX1,IX2,IX3,IX4,IX5,IX
      INTEGER  N, M, NE(8), L, D, RG, SEI, SVS, NEST,NCOLRNDX(3)         RST
      EXTERNAL NAR3,NCRES1,NPRIES,NROBUS
      REAL VMSE(8,5), VMX1(8,4),VMX2(8,4),VMX3(8,4),VMX4(8,4),VMX5(8,4) RST
      COMMON RHO
C                                                                        RST
      DATA  N/6000/
      DATA  M/ 20/
      DATA  NE/ 20,60,100,150,200,250,350,500/
      DATA  L/ 8/
      DATA  D/ 4/                                                        RST
      DATA  RG/ 1/                                                       RST
      DATA  SEI/ 0/                                                      RST
      DATA  SVS/ 0/                                                      RST
      DATA  YMIN/ 0./                                                    RST
      DATA  YMAX/ 0./                                                    RST
      DATA  IX/ 88771.D0/                                                RST
      DATA  IFILE /1/
      DATA  NPRT  /0/                                                    RST
      DATA  MSE   /1/                                                    RST
      DATA  VMSE  /32*.9,8*0/
      DATA  IPR   /0/
      DATA  IBIV  /0/
      DATA  IRSTR /1/                                                    RST
      DATA  T3/ '**Rho of N(0,4) r.v.s, rho=0.9 using moment estimator'/
      DATA  T2/ '**Rho of N(0,4) r.v.s, rho=0.9 using robust Least Squar
     *es estimator'/
      DATA  T4/ '**Rho of N(0,4) r.v.s, rho=0.9 using Cressie estimator
     C '/
      DATA  T5/ '**Rho of N(0,4) r.v.s, rho=0.9 using Priestly Estimator
     *'/
      DATA  ICOLOR/1/ IBWPRT/1/ NCPRT/3/NCOLRNDX/1,2,7/
      OPEN(06,FILE='NA1.OUT',ERR=999,IOSTAT=IER)
      OPEN(05,FILE='CON ',ERR=999,IOSTAT=IER)
      OPEN(02,FILE='NA1.RST',ERR=999,IOSTAT=IER,FORM='UNFORMATTED',
     C ACCESS='SEQUENTIAL')
      OPEN(01,FILE='NA1.DAT',ERR=999,IOSTAT=IER,FORM='FORMATTED',
     C ACCESS='SEQUENTIAL')
C --- GENERATOR PARAMETERS                                               RST
      NEST=4
      NSR=8
      RHO=0.9
C                                                                        RST
      IX1=IX                                                             RST
      IX2=IX                                                             RST
      IX3=IX                                                             RST
      IX4=IX
```

```
      IX5=IX
C
C ------------------------------------                                 RST
C
      CALL SIMTBED(IX1,IX2,IX3,IX4,IX5,Y,N,M,NE,L,D,NSR,RG,SEI,SVS,
     * YMIN,YMAX,NEST,NROBUS,T2,NCRES1,T4,NAR3,T3,NPRIES,T5,NAR3,T3,
     * IFILE
     * ,NPRT,MSE,VMSE,IPR,VMX1,VMX2,VMX3,VMX4,VMX5,IBIV,IRSTR,ICOLOR,
     *    IBWPRT,NCPRT,NCOLRNDX)
      STOP
C
 999  CONTINUE
      WRITE(6,*) '****** ERROR OPENING FILE 1, 2 or 6 '
      END                                                              RST
```

NUMBER OF WARNINGS IN PROGRAM UNIT: 0
NUMBER OF ERRORS   IN PROGRAM UNIT: 0

```
      INCLUDE 'AR1.FOR'
      SUBROUTINE AR1(IX,X,N)
C THIS SUBROUTINE GENERATES THE NORMAL AR1 PROCESS, WITH CORR COEF
C RHO, MEAN 0, AND STDDEV 1
      REAL*8 IX
      REAL*4 X(*),RHO
      COMMON RHO
      CALL LNORPC(IX,X,N+1)
      X(1)=RHO*X(N+1)/SQRT(1-RHO*RHO) + X(1)*SQRT(1-RHO*RHO)
C     WRITE(5,*)'X(1): ',X(1)
      DO 100 I=2,N                                                     RST
        X(I)=RHO*X(I-1) + X(I)*SQRT(1-RHO*RHO)
  100 CONTINUE                                                         RST
      RETURN
      END
```

NUMBER OF WARNINGS IN PROGRAM UNIT: 0
NUMBER OF ERRORS   IN PROGRAM UNIT: 0

```
      INCLUDE 'NAR3.FOR'
      SUBROUTINE NAR3(IX,N,EVAL)
C  LAG 1 SERIAL CORRELATION OF NORMAL (0,1) R.V.S , RHO=0.9
C  USING XBAR TO ESTIMATE MU
C  RHO OF THE AR1 PROCESS IS PASSED TO THE
C  GENERATOR AR1 THROUGH COMMON STORAGE
      REAL*8 IX
      REAL*4 X(600),MEAN
      COMMON RHO
      CALL AR1(IX,X,N)
      SUM=0
      SUM1=0
      DO 1 I=1,N
1     SUM=SUM+X(I)
      MEAN=SUM/N
      SUM=0
      DO 2 I=2,N
      SUM=SUM+(X(I)-MEAN)*(X(I-1)-MEAN)
2     SUM1=SUM1+(X(I)-MEAN)**2
```

```
      EVAL=SUM/SUM1
C     WRITE(5,*)'SER.COR = ',EVAL
      RETURN
      END

NUMBER OF WARNINGS IN PROGRAM UNIT:  0
NUMBER OF ERRORS   IN PROGRAM UNIT:  0

      INCLUDE 'NCRES1.FOR'
      SUBROUTINE NCRES1(IX,N,EVAL)
C  COMPUTES THE LAG 1 CORR COEFF ESTIMATOR USING THE CRESSIE FORMULA
C  THE NORMAL AR1 PROCESS HAS RHO PASSED BY
C  COMMON STORAGE.
C  USES FUNCTION AMEDI TO GET THE
C  MEDIAN.
      INTEGER N
      REAL*8 IX
      REAL*4 X(600),MEAN,MEDIAN
      COMMON RHO
      CALL AR1(IX,X,N)
      SUM1=0
      SUM2=0
      MEDIAN=AMEDI(X,N)
C     WRITE(5,*)'MEDIAN = ',MEDIAN
C     WRITE(5,*)'X(I) ',(X(I),I=1,N)
      DO 1 I=1,N-1
      SUM1=SUM1+SQRT(ABS(X(I)-X(I+1)))
1     IF(ABS(X(I)-MEDIAN).GT.0)SUM2=SUM2+SQRT(ABS(X(I)-MEDIAN))
      EVAL=1-(((SUM1/SUM2)**4)/2)
C     WRITE(5,*)'CRESS = ',EVAL
      RETURN
      END

NUMBER OF WARNINGS IN PROGRAM UNIT:  0
NUMBER OF ERRORS   IN PROGRAM UNIT:  0

      FUNCTION AMEDI(X,N)
C THIS ROUTINE USES A SHELLSORT AND INDEXES INTO THE REAL ARRAY X TO FIND
C THE MEDIAN OF X WITHOUT REARRANGING IT.
C THE INDEXES ARE USED AS INTEGER*2 TO CONSERVE STORAGE.
      REAL*4 X(*)
      INTEGER*2 I,INDX(600)
      INTEGER N
      DO 1 I=1,N
1     INDX(I)=I
C     WRITE(5,*)'UNSORTED LIST: ',(X(INDX(I)),I=1,N)
      CALL SRTNDX(X,N,INDX)
C     WRITE(5,*)'SORTED LIST: ',(X(INDX(I)),I=1,N)
      IF(MOD(N,2).EQ.0)THEN
         AMEDI=(X(INDX(N/2))+X(INDX(N/2+1)))/2
      ELSE
         AMEDI=X(INDX(N/2+1))
      ENDIF
      RETURN
      END
```

```
NUMBER OF WARNINGS IN PROGRAM UNIT: 0
NUMBER OF ERRORS   IN PROGRAM UNIT: 0
```

```
        SUBROUTINE SRTNDX(Y,N,INDX)
  C   INPLACE SORT USING SHELL ALGORITHM ********                          SMT
  C  USES INDEXES TO SORT BY TO PRESERVE ORIGINAL ORDER OF Y ARRAY
        REAL Y(*)
        INTEGER*2 GAP,INDX(*),ITEMP
        INTEGER N
        LOGICAL EXCH                                                       SMT
  C                                                                        SMT
        GAP=(N/2)                                                          SMT
     5  IF (.NOT.(GAP.NE.0)) GO TO 500                                     SMT
    10     CONTINUE                                                        SMT
              EXCH=.TRUE.                                                  SMT
              K=N-GAP                                                      SMT
              DO 200 I=1,K                                                 SMT
              KK=I+GAP                                                     SMT
              IF(.NOT.(Y(INDX(I)).GT.Y(INDX(KK)))) GO TO 100
                 ITEMP=INDX(I)
                 INDX(I)=INDX(KK)
                    INDX(KK)=ITEMP
                    EXCH=.FALSE.
   100        CONTINUE
   200        CONTINUE                                                     SMT
           IF (.NOT.(EXCH)) GO TO 10                                       SMT
           GAP=(GAP/2)                                                     SMT
        GO TO 5                                                            SMT
   500  CONTINUE                                                           SMT
        RETURN                                                             SMT
        END                                                                SMT
```

```
NUMBER OF WARNINGS IN PROGRAM UNIT: 0
NUMBER OF ERRORS   IN PROGRAM UNIT: 0
```

```
        INCLUDE 'NPRIES.FOR'
        SUBROUTINE NPRIES(IX,N,EVAL)
  C  COMPUTES THE LAG 1 CORR COEFF ESTIMATOR USING THE PRIESTLY FORMULA
  C  THE NORMAL AR1 PROCESS HAS RHO PASSED BY
  C  COMMON STORAGE.
        REAL*8 IX
        REAL*4 X(600),MEAN,A,B,XBAR1,XBAR2,MUHAT,
       *SUM1,SUM2
        COMMON RHO
        CALL AR1(IX,X,N)
        A=0
        B=0
        XBAR1=0
        XBAR2=0
        SUM1=0
        SUM2=0
        DO 1 I=1,N-1
        A=A+X(I)*X(I)
        B=B+X(I)*X(I+1)
        XBAR1=XBAR1+X(I)
  1     XBAR2=XBAR2+X(I+1)
```

```
C     WRITE(5,*)'I: ',I,' X(I)',X(I),'A: ',A,'B: ',B,'XBAR1:',XBAR1,'XBA
C    *R2:',XBAR2
      XBAR1=XBAR1/(N-1)
      XBAR2=XBAR2/(N-1)
      MUHAT=(A*XBAR2-B*XBAR1)/((A-B)+((N-1)*XBAR1*(XBAR2-XBAR1)))
      DO 2 I=1,N-1
      SUM1=SUM1+(X(I)-MUHAT)*(X(I+1)-MUHAT)
2     SUM2=SUM2+(X(I)-MUHAT)**2
      EVAL=SUM1/SUM2
C     WRITE(5,*)'SUM1:',SUM1,'SUM2:',SUM2
C     WRITE(5,*)'PREISTLY MUHAT',MUHAT
C     WRITE(5,*)'XBAR1: ',XBAR1,' XBAR2: ',XBAR2
C     WRITE(5,*)'A: ',A,' B: ',B,' N: ',N
C     WRITE(5,*)'X>>>>> ',(X(I),I=1,N)
C     WRITE(5,*)'PRIESTLY RHO: ',EVAL
      RETURN
      END
```

```
NUMBER OF WARNINGS IN PROGRAM UNIT: 0
NUMBER OF ERRORS   IN PROGRAM UNIT: 0
```

```
      INCLUDE 'NROBUS.FOR'
      SUBROUTINE NROBUS(SEED,N,EVAL)
C THIS ROUTINE GENERATES A STREAM OF R.V.S, AND THEN PERFORMS ROBUST
C  REGRESSION ON THEM TO ESTIMATE RHO.
C THE CURRENT STREAM IS AR(1) NORMAL R.V.S.
      REAL*8 SEED
      REAL*4 X(600),Y(600),B(1),XX(1,1),XXI(1,1),XY(1,1)
      REAL*4 RMED1,RMED2,RMED3,SR1,SR2,SR3,SY,CR,W(600),U(600)
      REAL*4 WX(600,1),WY(600),EVAL
      INTEGER N,I,IN
      COMMON RHO
      DATA CY/1.0/CR/4.2/CY/1.0/
      DATA NN/1/IPAS/5/
      IX2=1
      CALL AR1(SEED,X,N)
C  CONSTRUCT X AND Y VECTORS FROM X ARRAY
      DO 1 I=1,N-1
1     Y(I)=X(I+1)
      IN=N-1
      IX1=IN
      CALL ROBREG(X,Y,B,IN,NN,IX1,IX2,CR,XX,XXI,XY,W,U,WX,WY,IPAS)
      EVAL=B(1)
C     WRITE(5,*)'EVAL: ',EVAL
      RETURN
      END
```

```
NUMBER OF WARNINGS IN PROGRAM UNIT: 0
NUMBER OF ERRORS   IN PROGRAM UNIT: 0
```

```
      SUBROUTINE ROBREG(X,Y,B,M,N,IX1,IX2,C,XX,XXI,XY,W,U,WX,WY,IPAS)
C  ROBUST REGRESSION ON Y=X*B                                          RRE
C  IPAS=MAX # REGRESSION PASSES. 1 FOR STD (I.E. NON-ROBUST) REGRESSION RRE
C  X=M BY N MATRIX CONTAINED IN AN ARRAY OF DIM(IX1,IX2)               RRE
C  Y=M-VECTOR CONTAINED IN AN ARRAY OF DIM(IX1)                        RRE
C  B=N-VECTOR CONTAINED IN AN ARRAY OF DIM(IX2)                        RRE
```

```
C  XX,XXI=WORK ARRAYS OF DIM(IX2,IX2)                                   RRE
C  W,U,WY=WORK ARRAYS OF DIM(IX1)                                       RRE
C  WX=WORK MATRIX OF DIM(IX1,IX2)                                       RRE
C  XY=WORK ARRAY OF DIM(IX2)                                            RRE
C  WK=WORK ARRAY OF DIM(N**2 + 3*N)    OR LARGER                        RRE
C  C=CONSTANT. USUALLY = 6.                                             RRE
C  CRITERION FOR STOPPING IS WHEN VARIATION IN CONST.TERM < 1%          RRE
C  IT USES SEVERAL SUBROUTINES FROM IMSL FOR MATRIX INVERSION,          RRE
C     MULTIPLICATION, ETC. ALSO USES PXSORT TO SORT ARRAY.              RRE
      REAL*4 X(IX1,IX2),XX(IX2,IX2),XXI(IX2,IX2)
      REAL*4 Y(IX1),WK(600)
      REAL*4 B(IX2),W(IX1),XY(IX2),BB,WX(IX1,IX2),WY(IX1)
      REAL*4 U(IX1),C,S,AM,R,SS,SUM1
      NPAS=0                                                            RRE
      ZERO=0                                                            RRE
C     SUPPRESS ERROR MESSAGES: UNDERFLOW AND IMSL                       RRE
C     CALL ERRSET(208,300,-1,1)                                         RRE
C     CALL UERSET(0,LEVOLD)                                             RRE
      AM=(M+1.)/2.                                                      RRE
      IM=AM                                                             RRE
      R=AM-IM                                                           RRE
      B(1)=-999999.                                                     RRE
      DO 5 I=1 ,M                                                       RRE
       WY(I)=Y(I)                                                       RRE
       DO 6 J=1,N                                                       RRE
        WX(I,J)=X(I,J)                                                  RRE
    6  CONTINUE                                                         RRE
    5 CONTINUE                                                          RRE
C                                                                       RRE
   10 CONTINUE                                                          RRE
      NPAS=NPAS+1                                                       RRE
C     CALL VMULFM(WX,WX,M,N,N,IX1,IX1,XX,IX2,IER)                       RRE
C  VMULFM PERFORMS X TRANSPOSE X. WE CAN DO IT WITH A DO LOOP, SINCE
C IN THIS CASE X IS A VECTOR.  RESULT GOES IN XX (DIM 1,1)

      XX(1,1)=0
      DO 200 I=1,M
  200 XX(1,1)=XX(1,1)+WX(I,1)**2
      IER=0
C     IF (IER.GT.0) WRITE (6,109) IER                                   RRE
  109 FORMAT('ERROR:',2I10)                                             RRE
C     CALL VMULFM(WX,WY,M,N,1,IX1,IX1,XY,IX2,IER)                       RRE
C  VMULFM PERFORMS MATRIX MULTIPLICATION.  WE WILL DO IT WITH A DO LOOP
C SINCE X AND Y ARE VECTORS. RESULT GOES IN XY (DIM 1,1).
      XY(1)=0
      DO 201 I=1,M
  201 XY(1)=XY(1)+WX(I,1)*WY(I)
      IER=0
C     IF (IER.GT.0) WRITE (6,109) IER                                   RRE
C     CALL LINV2F(XX,N,IX2,XXI,ZERO,WK,IER)                             RRE
C  LINV2F FINDS THE INVERSE OF A MATRIX (X TRANSPOSE X).  WE WILL DO IT
C WITH A DO LOOP, SINCE X IS A VECTOR IN THIS CASE. RESULT INTO XXI.

      XXI(1,1)=1/XX(1,1)

C     IF (IER.GT.0) WRITE (6,109) IER                                   RRE
```

```
      BB=B(1)                                                           RRE
C     CALL VMULFF(XXI,XY,N,N,1,IX2,IX2,B,IX2,IER)                       RRE
C  PERFORMING X TRANSPOSE X INVERSE TIMES X TRANSPOSE Y.  RESULT INTO
C B (DIM 1).  DONE WITHOUT A SUB CALL IN THIS CASE, SINCE WE'RE
C DEALING WITH VECTORS.
      B(1)=XY(1)*XXI(1,1)


C     IF (IER.GT.0) WRITE (6,109) IER                                   RRE
C     WRITE(6,101)(B(I),I=1,N)                                          RRE
  101 FORMAT(5F20.10)                                                   RRE
      IF(NPAS .GE. IPAS) GO TO 99                                       RRE
      IF (ABS(B(1)-BB) .LT. .01*ABS(BB)) GO TO 99                       RRE
C     CALL VMULFF(X,B,M,N,1,IX1,IX2,W,IX1,IER)                          RRE
C YHAT COMES FROM X TIMES B, DONE WITH A DO LOOP SINCE WE HAVE VECTORS.
      DO 202 I=1,M
202   W(I)=X(I,1)*B(1)

C     IF (IER.GT.0) WRITE (6,109) IER                                   RRE
C  COMPUTE RESIDUALS AND MEDIAN OF ABSOLUTE RESIDUALS                   RRE
      DO 20 I=1,M                                                       RRE
      W(I)=Y(I)-W(I)                                                    RRE
   20 U(I)=ABS(W(I))                                                    RRE
      CALL PXSORT(U,1,M)                                                RRE
C     WRITE(6, 499) (U(I),I=1,M)                                        RRE
  499 FORMAT(6F20.10)                                                   RRE
  105 FORMAT(8F10.7)                                                    RRE
      S=U(IM) + R*(U(IM+1)-U(IM))                                       RRE
      S=C*C*S*S                                                         RRE
      SS=S**.5                                                          RRE
C     WRITE(6,199) SS                                                   RRE
  199 FORMAT('     CS:',F20.10)                                         RRE
      DO 40 I=1,M                                                       RRE
      W(I)=W(I)*W(I)/S                                                  RRE
      IF (W(I) .LT. 1.) GO TO 25                                        RRE
C     FACTOR IS 0. IN THIS CASE                                         RRE
      WY(I)=0.                                                          RRE
      DO 31 J=1,N                                                       RRE
        WX(I,J)=0.                                                      RRE
   31 CONTINUE                                                          RRE
      GO TO 40                                                          RRE
   25 W(I)=(1-W(I))                                                     RRE
      DO 35 J=1,N                                                       RRE
   35 WX(I,J)=X(I,J)*W(I)                                               RRE
      WY(I)=Y(I)*W(I)                                                   RRE
   40 CONTINUE                                                          RRE
      GO TO 10                                                          RRE
   99 CONTINUE                                                          RRE
      RETURN                                                            RRE
      END                                                               RRE

NUMBER OF WARNINGS IN PROGRAM UNIT: 0
NUMBER OF ERRORS   IN PROGRAM UNIT: 0

C                                                                       PXS
C     SUBROUTINE PXSORT  (M1)                                           PXS
```

```
C                                                                      PXS
C     PURPOSE                                                          PXS
C                                                                      PXS
C        SUBROUTINE PXSORT IS INTENDED TO REARRANGE AN ARRAY OF REAL*4 PXS
C     DATA INTO ASCENDING ORDER BETWEEN TWO SPECIFIED INDICES.         PXS
C                                                                      PXS
C     CALLING SEQUENCE                                                 PXS
C                                                                      PXS
C        CALL PXSORT(A, II, JJ)                                        PXS
C                                                                      PXS
C     ARGUMENTS                                                        PXS
C                                                                      PXS
C        A     A SINGLE DIMENSIONED ARRAY OF REAL*4 DATA TO BE         PXS
C              SORTED INTO ASCENDING ORDER. DIMENSIONED TO AT LEAST JJ.PXS
C                                                                      PXS
C        II    THE STARTING INDEX FOR THE ORDERING OF A,               PXS
C              E.G., II=1 IF THE WHOLE ARRAY IS TO BE SORTED.          PXS
C                                                                      PXS
C        JJ    THE ENDING INDEX FOR ORDERING A.                        PXS
C                                                                      PXS
C     USAGE                                                            PXS
C                                                                      PXS
C        THE ARRAY A WILL BE SORTED INTO INCREASING ORDER SO THAT      PXS
C     A(I) < A(I+1) FOR I = II,II+1,...,JJ-2,JJ-1.                     PXS
C                                                                      PXS
C        PXSORT WILL ONLY SORT REAL*4 DATA, NOT REAL*8, INTEGER, OR    PXS
C     ALPHANUMERIC DATA.                                               PXS
C                                                                      PXS
C     SUBROUTINES REQUIRED                                             PXS
C                                                                      PXS
C        NONE.                                                         PXS
C                                                                      PXS
C     METHOD                                                           PXS
C                                                                      PXS
C        SINGLETON'S VERSION OF THE PARTITION EXCHANGE SORT IS         PXS
C     USED.  THE PROGRAM IS ESSENTIALLY COPIED FROM THE ASSOCIATION    PXS
C     FOR COMPUTING MACHINERY'S ALGORITHM 247, "SORTING WITH           PXS
C     MINIMAL STORAGE."                                                PXS
C                                                                      PXS
C     PROGRAMMER:  IMPLEMENTED AT NPS BY D.W. ROBINSON                 PXS
C                                                                      PXS
C     DATE:        APR 74                                              PXS
C                                                                      PXS
C     -----------------------------------------------------------------PXS
C                                                                      PXS
      SUBROUTINE PXSORT(A,II,JJ)                                       PXS
C                                                                      PXS
      DIMENSION A(JJ),IU(16),IL(16)                                    PXS
      M=1                                                              PXS
      I=II                                                             PXS
      J=JJ                                                             PXS
    5 IF(I .GE. J)GO TO 70                                             PXS
   10 K=I                                                              PXS
      IJ=(I+J)/2                                                       PXS
      T=A(IJ)                                                          PXS
      IF(A(I) .LE. T) GO TO 20                                         PXS
```

```
      A(IJ)=A(I)                                                        PXS
      A(I)=T                                                            PXS
      T=A(IJ)                                                           PXS
   20 L=J                                                               PXS
      IF(A(J) .GE. T) GO TO 40                                          PXS
      A(IJ)=A(J)                                                        PXS
      A(J)=T                                                            PXS
      T=A(IJ)                                                           PXS
      IF(A(I) .LE. T) GO TO 40                                          PXS
      A(IJ)=A(I)                                                        PXS
      A(I)=T                                                            PXS
      T=A(IJ)                                                           PXS
      GO TO 40                                                          PXS
   30 TT = A(L)                                                         PXS
      A(L) = A(K)                                                       PXS
      A(K)=TT                                                           PXS
   40 L=L-1                                                             PXS
      IF(A(L) .GT. T) GO TO 40                                          PXS
   50 K=K+1                                                             PXS
      IF(A(K) .LT. T) GO TO 50                                          PXS
      IF(K .LE. L) GO TO 30                                             PXS
      IF(L-I .LE. J-K) GO TO 60                                         PXS
      IL(M)=I                                                           PXS
      IU(M)=L                                                           PXS
      I=K                                                               PXS
      M=M+1                                                             PXS
      GO TO 80                                                          PXS
   60 IL(M)=K                                                           PXS
      IU(M)=J                                                           PXS
      J=L                                                               PXS
      M=M+1                                                             PXS
      GO TO 80                                                          PXS
   70 M=M-1                                                             PXS
      IF(M .EQ. 0) RETURN                                               PXS
      I=IL(M)                                                           PXS
      J=IU(M)                                                           PXS
   80 IF(J-I .GE. 11)GO TO 10                                           PXS
      IF(I .EQ. II) GO TO 5                                             PXS
      I=I-1                                                             PXS
   90 I=I+1                                                             PXS
      IF(I .EQ. J) GO TO 70                                             PXS
      IF(A(I) .LE. A(I+1)) GO TO 90                                     PXS
      T = A(I+1)                                                        PXS
      K=I                                                               PXS
  100 A(K+1)=A(K)                                                       PXS
      K=K-1                                                             PXS
      IF(T .LT. A(K)) GO TO 100                                         PXS
      A(K+1)=T                                                          PXS
      GO TO 90                                                          PXS
      END                                                               PXS

NUMBER OF WARNINGS IN PROGRAM UNIT: 0
NUMBER OF ERRORS   IN PROGRAM UNIT: 0
NUMBER OF WARNINGS IN COMPILATION : 0
NUMBER OF ERRORS   IN COMPILATION : 0
```

```
C     SIMTBED DRIVER FOR BETA-L-LAPLACE TRANSFORM AR PROCESS          RST
C     ---------------------------- SMTB12 ---------------------------RST
      REAL*4  Y(6500),YMIN, YMAX, V(8),RHO,LL,ALPHA
      CHARACTER*120 T1,T2,T3,T4,T5                                     RST
      REAL*8 IX1,IX2,IX3,IX4,IX5,IX
      INTEGER  N, M, NE(8), L, D, RG, SEI, SVS, NEST,NCOLRNDX(3)      RST
      EXTERNAL BMOM,BCRES1,BBBB,ROBUST
      REAL VMSE(8,5), VMX1(8,4),VMX2(8,4),VMX3(8,4),VMX4(8,4),VMX5(8,4) RST
      COMMON ALPHA,LL
C                                                                      RST
      DATA  N/6000/
      DATA  M/  20/
      DATA  NE/ 20,60,100,150,200,250,350,500/
      DATA  L/ 8/
      DATA  D/ 4/                                                      RST
      DATA  RG/ 1/                                                     RST
      DATA  SEI/ 0/                                                    RST
      DATA  SVS/ 0/                                                    RST
      DATA  YMIN/ 0./                                                  RST
      DATA  YMAX/ 0./                                                  RST
      DATA  IX/ 88771.D0/                                              RST
      DATA  IFILE /1/
      DATA  NPRT  /1/                                                  RST
      DATA  MSE   /1/                                                  RST
      DATA  VMSE  /32*0.8963,8*0/
      DATA  IPR   /0/                                                  RST
      DATA  IBIV  /0/
      DATA  IRSTR /1/                                                  RST
      DATA  T2/ '***Rho(.8963) of SqrtBeta L laplace AR r.v.s, A=.84, L=
     * .95 Using Robust Least Squares estimator (C=4.2)'/
      DATA  T3/ '***Rho(.8963) of SqrtBeta L laplace AR r.v.s, A=.84, L=
     * .95 Using moment estimator'/
      DATA  T4/ '***Rho(.8963) of SqrtBeta L laplace AR r.vs., A=.84, L=
     * .95 Using Cressie Estimator'/
      DATA  T5/ '***Rho(.8963) of SqrtBeta L laplace AR r.vs., A=.84, L=
     * .95 Using Priestly Estimator'/
      DATA  ICOLOR/1/ IBWPRT/1/ NCPRT/3/NCOLRNDX/1,2,7/
      OPEN(06,FILE='bf1.OUT',ERR=999,IOSTAT=IER)
      OPEN(05,FILE='CON ',ERR=999,IOSTAT=IER)
      OPEN(02,FILE='bf1.RST',ERR=999,IOSTAT=IER,FORM='UNFORMATTED',
     C ACCESS='SEQUENTIAL')
      OPEN(01,FILE='Bf1.DAT',FORM='FORMATTED',ACCESS='SEQUENTIAL')
C --- GENERATOR PARAMETERS                                             RST
      NEST=4
      NSR=3
      LL= .95
      ALPHA=0.84
C                                                                      RST
      IX1=IX                                                           RST
      IX2=IX                                                           RST
      IX3=IX                                                           RST
      IX4=IX
      IX5=IX
C
```

```
C ----------------------------------                                  RST
C
      CALL SIMTBED(IX1,IX2,IX3,IX4,IX5,Y,N,M,NE,L,D,NSR,RG,SEI,SVS,
     * YMIN,YMAX,NEST,robust,t2,BCRES1,T4,BMOM,T3,BBBB,T5,BMOM,T3,IFILE
     * ,NPRT,MSE,VMSE,IPR,VMX1,VMX2,VMX3,VMX4,VMX5,IBIV,IRSTR,ICOLOR,
     *    IBWPRT,NCPRT,NCOLRNDX)
      STOP
C
 999  CONTINUE
      WRITE(6,*) '****** ERROR OPENING FILE 1, 2 or 6 '
      END                                                             RST

NUMBER OF WARNINGS IN PROGRAM UNIT: 0
NUMBER OF ERRORS   IN PROGRAM UNIT: 0

      INCLUDE 'BELAR1.for'
      SUBROUTINE BELAR1(SEED,ALPHA,L,N,X)
C THIS SUBROUTINE GENERATES CORRELATED L LAPLACE R.V.S USING THE
C BETA SQUARE ROOT TRANSFORMATION AND THE L LAPLACE DISTRIBUTION
C  THE L LAPLACE DISTRIBUTION IS GENERATED AS THE DIFFERENCE OF
C  2 GAMMA(L,1) RANDOM VARIABLES.
      REAL*8 SEED
      REAL*4 ALPHA,L,X(*),Y(5100),A(5100),B(5100),E,F
      INTEGER N
C FIRST GENERATE THE L LAPLACE STREAM OF R.V.S
C  THIS WAS FOR USING IMSL SSTAT18A RANDOM NUMBER GENERATORS
C     CALL RNSET(12345678)
      CALL LGAMPC(SEED,X,N,L)
      CALL LGAMPC(SEED,Y,N,L)
      DO 10 I=1,N
 10   Y(I)=X(I)-Y(I)
C  NOW GENERATE THE BETA R.V.S (2 STREAMS)
      CALL LBETPC(SEED,A,N,L*ALPHA,L*(1-ALPHA))
      CALL LBETPC(SEED,A,N,L*(1-ALPHA),L*ALPHA)
C     WRITE(5,*)(A(I),I=1,N)
C     WRITE(5,*)(B(I),I=1,N)
C A(I) IS NOW BETA(L*ALPHA,L*(1-ALPHA))
C B(I) IS NOW BETA(L*(1-ALPHA),L*ALPHA)
      X(1)=Y(1)
      DO 12 I=2,N
C     WRITE(5,50)'A',A(I),'B',B(I),'I',I
 50   FORMAT(' ',A,E15.6,A,E15.6,A,I4)
C CHECK FOR UNDERFLOW BEFORE SQRT IS DONE
      IF(A(I).LE.1E-7) THEN
C     WRITE(5,*)'A(I)',A(I)
        A(I)=1E-7
        E=A(I)**0.5
      ELSE
C     WRITE(5,*)'ROOTING A...',I
      E=A(I)**0.5
      ENDIF
      IF(B(I).LE.1E-7) THEN
C     WRITE(5,*)'B(I)',B(I)
        B(I)=1E-7
```

```
           F=B(I)**0.5
          ELSE
 C        WRITE(5,*)'ROOTING B...',I
          F=B(I)**0.5
          ENDIF

 C   A MINUS SIGN AT THE START OF LINE 12 INDICATES WE ARE CONTSTRUCTING
 C   NEGATIVE CORRELATIONS.

 12       X(I)=-X(I-1)*E+Y(I)*F
          RETURN
          END
```

NUMBER OF WARNINGS IN PROGRAM UNIT: 0
NUMBER OF ERRORS   IN PROGRAM UNIT: 0

```
          INCLUDE 'BMOM.for'
          SUBROUTINE BMOM(IX,N,EVAL)
 C   LAG 1 SERIAL CORRELATION OF BETA L LAPLACE R.V.S , RHO=0.7996
 C   USING XBAR TO ESTIMATE MU
 C   L AND ALPHA OF THE BETA AR PROCESS ARE PASSED TO THE
 C   GENERATOR BELAR1 THROUGH COMMON STORAGE
          REAL*8 IX
          REAL*4 X(600),MEAN,ALPHA,L
          COMMON ALPHA,L
          CALL BELAR1(IX,ALPHA,L,N,X)
          SUM=0
          SUM1=0
          DO 1 I=1,N
 1        SUM=SUM+X(I)
          MEAN=SUM/N
          SUM=0
          DO 2 I=2,N
          SUM=SUM+(X(I)-MEAN)*(X(I-1)-MEAN)
 2        SUM1=SUM1+(X(I)-MEAN)**2
          EVAL=SUM/SUM1
 C        WRITE(5,*)'SER.COR = ',EVAL
          RETURN
          END
```

NUMBER OF WARNINGS IN PROGRAM UNIT: 0
NUMBER OF ERRORS   IN PROGRAM UNIT: 0

```
          INCLUDE 'BBBB.for'
          SUBROUTINE BBBB(IX,N,EVAL)
 C   COMPUTES THE LAG 1 CORR COEFF ESTIMATOR USING THE PRIESTLY FORMULA
 C   THE BETA LAPLACE PROCESS HAS ALPHA AND L PASSED BY
 C   COMMON STORAGE.
          REAL*8 IX
          REAL*4 X(600),MEAN,MEANINPU,STDINPU,A,B,XBAR1,XBAR2,MUHAT
          REAL SUM1,SUM2,ALPHA,L
          COMMON ALPHA,L
 C        WRITE(5,*)'ENTERED BPRIES N: ',N,L,ALPHA
          CALL BELAR1(IX,ALPHA,L,N,X)
          A=0
          B=0
```

```
      XBAR1=0
      XBAR2=0
      SUM1=0
      SUM2=0
      DO 1 I=1,N-1
      A=A+X(I)*X(I)
      B=B+X(I)*X(I+1)
      XBAR1=XBAR1+X(I)
1     XBAR2=XBAR2+X(I+1)
C     WRITE(5,*)'I: ',I,' X(I)',X(I),'A: ',A,'B: ',B,'XBAR1:',XBAR1,'XBA
C    *R2:',XBAR2
      XBAR1=XBAR1/(N-1)
      XBAR2=XBAR2/(N-1)
      MUHAT=(A*XBAR2-B*XBAR1)/((A-B)+((N-1)*XBAR1*(XBAR2-XBAR1)))
      DO 2 I=1,N-1
      SUM1=SUM1+(X(I)-MUHAT)*(X(I+1)-MUHAT)
2     SUM2=SUM2+(X(I)-MUHAT)**2
      EVAL=SUM1/SUM2
C     WRITE(5,*)'SUM1:',SUM1,'SUM2:',SUM2
C     WRITE(5,*)'PREISTLY MUHAT',MUHAT
C     WRITE(5,*)'XBAR1: ',XBAR1,' XBAR2: ',XBAR2
C     WRITE(5,*)'A: ',A,' B: ',B,' N: ',N
C     WRITE(5,*)'X>>>>> ',(X(I),I=1,N)
C     WRITE(5,*)'PRIESTLY RHO: ',EVAL
      RETURN
      END
```

```
NUMBER OF WARNINGS IN PROGRAM UNIT: 0
NUMBER OF ERRORS   IN PROGRAM UNIT: 0
```

```
      INCLUDE 'ROBUST.for'
      SUBROUTINE ROBUST(SEED,N,EVAL)
C THIS ROUTINE GENERATES A STREAM OF R.V.S, AND THEN PERFORMS ROBUST
C  REGRESSION ON THEM TO ESTIMATE RHO.
C THE CURRENT STREAM IS AR(1) NORMAL R.V.S.
      REAL*8 SEED
      REAL*4 X(600),Y(600),B(1),XX(1,1),XXI(1,1),XY(1,1)
      REAL*4 RMED1,RMED2,RMED3,SR1,SR2,SR3,SY,CR,W(600),U(600)
      REAL*4 WX(600,1),WY(600),EVAL
      INTEGER N,I,IN
      COMMON ALPHA,L
      DATA CY/1.0/CR/4.2/CY/1.0/
      DATA NN/1/IPAS/5/
      IX2=1
      CALL BELAR1(SEED,ALPHA,L,N,X)
C  CONSTRUCT X AND Y VECTORS FROM X ARRAY
      DO 1 I=1,N-1
1     Y(I)=X(I+1)
      IN=N-1
      IX1=IN
      CALL ROBREG(X,Y,B,IN,NN,IX1,IX2,CR,XX,XXI,XY,W,U,WX,WY,IPAS)
      EVAL=B(1)
C     WRITE(5,*)'EVAL: ',EVAL
      RETURN
      END
```

```
NUMBER OF WARNINGS IN PROGRAM UNIT:  0
NUMBER OF ERRORS   IN PROGRAM UNIT:  0

          SUBROUTINE ROBREG(X,Y,B,M,N,IX1,IX2,C,XX,XXI,XY,W,U,WX,WY,IPAS)
 C   ROBUST REGRESSION ON Y=X*B                                                    RRE
 C   IPAS=MAX # REGRESSION PASSES. 1 FOR STD (I.E. NON-ROBUST) REGRESSION          RRE
 C   X=M BY N MATRIX CONTAINED IN AN ARRAY OF DIM(IX1,IX2)                         RRE
 C   Y=M-VECTOR CONTAINED IN AN ARRAY OF DIM(IX1)                                  RRE
 C   B=N-VECTOR CONTAINED IN AN ARRAY OF DIM(IX2)                                  RRE
 C   XX,XXI=WORK ARRAYS OF DIM(IX2,IX2)                                            RRE
 C   W,U,WY=WORK ARRAYS OF DIM(IX1)                                                RRE
 C   WX=WORK MATRIX OF DIM(IX1,IX2)                                                RRE
 C   XY=WORK ARRAY OF DIM(IX2)                                                     RRE
 C   WK=WORK ARRAY OF DIM(N**2 + 3*N)    OR LARGER                                 RRE
 C   C=CONSTANT. USUALLY = 6.                                                      RRE




 C   CRITERION FOR STOPPING IS WHEN VARIATION IN CONST.TERM < 1%                   RRE
 C   IT USES SEVERAL SUBROUTINES FROM IMSL FOR MATRIX INVERSION,                   RRE
 C      MULTIPLICATION, ETC. ALSO USES PXSORT TO SORT ARRAY.                       RRE
        REAL*4 X(IX1,IX2),XX(IX2,IX2),XXI(IX2,IX2)
        REAL*4 Y(IX1),WK(600)
        REAL*4 B(IX2),W(IX1),XY(IX2),BB,WX(IX1,IX2),WY(IX1)
        REAL*4 U(IX1),C,S,AM,R,SS,SUM1
        NPAS=0                                                                     RRE
        ZERO=0                                                                     RRE
 C      SUPPRESS ERROR MESSAGES: UNDERFLOW AND IMSL                                RRE
 C      CALL ERRSET(208,300,-1,1)                                                  RRE
 C      CALL UERSET(0,LEVOLD)                                                      RRE
        AM=(M+1.)/2.                                                               RRE
        IM=AM                                                                      RRE
        R=AM-IM                                                                    RRE
        B(1)=-999999.                                                              RRE
        DO 5 I=1 ,M                                                                RRE
         WY(I)=Y(I)                                                                RRE
         DO 6 J=1,N                                                                RRE
          WX(I,J)=X(I,J)                                                           RRE
      6  CONTINUE                                                                  RRE
      5 CONTINUE                                                                   RRE
 C                                                                                 RRE
     10 CONTINUE                                                                   RRE
        NPAS=NPAS+1                                                                RRE
 C      CALL VMULFM(WX,WX,M,N,N,IX1,IX1,XX,IX2,IER)                                RRE
 C   VMULFM PERFORMS X TRANSPOSE X. WE CAN DO IT WITH A DO LOOP, SINCE
 C  IN THIS CASE X IS A VECTOR.  RESULT GOES IN XX (DIM 1,1)

        XX(1,1)=0
        DO 200 I=1,M
   200  XX(1,1)=XX(1,1)+WX(I,1)**2
        IER=0
 C      IF (IER.GT.0) WRITE (6,109) IER                                            RRE
    109 FORMAT('ERROR:',2I10)                                                      RRE
 C      CALL VMULFM(WX,WY,M,N,1,IX1,IX1,XY,IX2,IER)                                RRE
 C   VMULFM PERFORMS MATRIX MULTIPLICATION.  WE WILL DO IT WITH A DO LOOP
```

```
C SINCE X AND Y ARE VECTORS. RESULT GOES IN XY (DIM 1,1).
      XY(1)=0
      DO 201 I=1,M
201   XY(1)=XY(1)+WX(I,1)*WY(I)
      IER=0
C     IF (IER.GT.0) WRITE (6,109) IER                                   RRE
C     CALL LINV2F(XX,N,IX2,XXI,ZERO,WK,IER)                             RRE
C  LINV2F FINDS THE INVERSE OF A MATRIX (X TRANSPOSE X).  WE WILL DO IT
C WITH A DO LOOP, SINCE X IS A VECTOR IN THIS CASE. RESULT INTO XXI.

      XXI(1,1)=1/XX(1,1)

C     IF (IER.GT.0) WRITE (6,109) IER                                   RRE
      BB=B(1)                                                           RRE
C     CALL VMULFF(XXI,XY,N,N,1,IX2,IX2,B,IX2,IER)                       RRE
C  PERFORMING X TRANSPOSE X INVERSE TIMES X TRANSPOSE Y.  RESULT INTO
C B (DIM 1).  DONE WITHOUT A SUB CALL IN THIS CASE, SINCE WE'RE
C DEALING WITH VECTORS.
      B(1)=XY(1)*XXI(1,1)


C     IF (IER.GT.0) WRITE (6,109) IER                                   RRE
C     WRITE(6,101)(B(I),I=1,N)                                          RRE
  101 FORMAT(5F20.10)                                                   RRE
      IF(NPAS .GE. IPAS) GO TO 99                                       RRE
      IF (ABS(B(1)-BB) .LT. .01*ABS(BB)) GO TO 99                       RRE
C     CALL VMULFF(X,B,M,N,1,IX1,IX2,W,IX1,IER)                          RRE
C YHAT COMES FROM X TIMES B, DONE WITH A DO LOOP SINCE WE HAVE VECTORS.
      DO 202 I=1,M
202   W(I)=X(I,1)*B(1)

C     IF (IER.GT.0) WRITE (6,109) IER                                   RRE
C  COMPUTE RESIDUALS AND MEDIAN OF ABSOLUTE RESIDUALS                   RRE
      DO 20 I=1,M                                                       RRE
      W(I)=Y(I)-W(I)                                                    RRE
   20 U(I)=ABS(W(I))                                                    RRE
      CALL PXSORT(U,1,M)                                                RRE
C     WRITE(6, 499) (U(I),I=1,M)                                        RRE
  499 FORMAT(6F20.10)                                                   RRE
  105 FORMAT(8F10.7)                                                    RRE
      S=U(IM) + R*(U(IM+1)-U(IM))                                       RRE
      S=C*C*S*S                                                         RRE
      IF(S.LT.1E-07) THEN
c     WRITE(5,*)'S: IN ROBREG: ',S,'(SET 1 1E-07) U(IM)',U(IM),'R',
c    *     R
           S=1.E-07
           ss=1.e-14
      ELSE
      SS=S**.5                                                          RRE
      ENDIF
  199 FORMAT('    CS:',F20.10)                                          RRE
      DO 40 I=1,M                                                       RRE
      W(I)=W(I)*W(I)/S                                                  RRE
      IF (W(I) .LT. 1.) GO TO 25                                        RRE
C     FACTOR IS 0. IN THIS CASE                                         RRE
      WY(I)=0.                                                          RRE
```

```
      DO 31 J=1,N                                                       RRE
        WX(I,J)=0.                                                      RRE
   31 CONTINUE                                                          RRE
      GO TO 40                                                          RRE
   25 W(I)=(1-W(I))                                                     RRE
      DO 35 J=1,N                                                       RRE
   35 WX(I,J)=X(I,J)*W(I)                                               RRE
      WY(I)=Y(I)*W(I)                                                   RRE
   40 CONTINUE                                                          RRE
      GO TO 10                                                          RRE
   99 CONTINUE                                                          RRE
      RETURN                                                            RRE
      END                                                               RRE
```

NUMBER OF WARNINGS IN PROGRAM UNIT: 0
NUMBER OF ERRORS   IN PROGRAM UNIT: 0

```
C                                                                       PXS
C     SUBROUTINE PXSORT  (M1)                                           PXS
C                                                                       PXS
C     PURPOSE                                                           PXS
C                                                                       PXS
C         SUBROUTINE PXSORT IS INTENDED TO REARRANGE AN ARRAY OF REAL*4 PXS
C     DATA INTO ASCENDING ORDER BETWEEN TWO SPECIFIED INDICES.          PXS
C                                                                       PXS
C     CALLING SEQUENCE                                                  PXS
C                                                                       PXS
C         CALL PXSORT(A, II, JJ)                                        PXS
C                                                                       PXS
C     ARGUMENTS                                                         PXS
C                                                                       PXS
C         A     A SINGLE DIMENSIONED ARRAY OF REAL*4 DATA TO BE         PXS
C               SORTED INTO ASCENDING ORDER. DIMENSIONED TO AT LEAST JJ.PXS
C                                                                       PXS
C         II    THE STARTING INDEX FOR THE ORDERING OF A,               PXS
C               E.G. , II=1 IF THE WHOLE ARRAY IS TO BE SORTED.         PXS
C                                                                       PXS
C         JJ    THE ENDING INDEX FOR ORDERING A.                        PXS
C                                                                       PXS
C     USAGE                                                             PXS
C                                                                       PXS
C         THE ARRAY A WILL BE SORTED INTO INCREASING ORDER SO THAT      PXS
C     A(I) < A(I+1) FOR I = II,II+1,...,JJ-2,JJ-1.                      PXS
C                                                                       PXS
C         PXSORT WILL ONLY SORT REAL*4 DATA, NOT REAL*8, INTEGER, OR    PXS
C     ALPHANUMERIC DATA.                                                PXS
C                                                                       PXS
C     SUBROUTINES REQUIRED                                              PXS
C                                                                       PXS
C         NONE.                                                         PXS
C                                                                       PXS
C     METHOD                                                            PXS
C                                                                       PXS
C         SINGLETON'S VERSION OF THE PARTITION EXCHANGE SORT IS         PXS
C     USED.  THE PROGRAM IS ESSENTIALLY COPIED FROM THE ASSOCIATION     PXS
C     FOR COMPUTING MACHINERY'S ALGORITHM 247, "SORTING WITH            PXS
```

```
C      MINIMAL STORAGE."                                                  PXS
C                                                                         PXS
C      PROGRAMMER:  IMPLEMENTED AT NPS BY D.W. ROBINSON                   PXS
C                                                                         PXS
C      DATE:        APR 74                                                PXS
C                                                                         PXS
C      -------------------------------------------------------------------PXS
C                                                                         PXS
       SUBROUTINE PXSORT(A,II,JJ)                                         PXS
C                                                                         PXS
       DIMENSION A(JJ),IU(16),IL(16)                                      PXS
       M=1                                                                PXS
       I=II                                                               PXS
       J=JJ                                                               PXS
     5 IF(I .GE. J)GO TO 70                                               PXS
    10 K=I                                                                PXS
       IJ=(I+J)/2                                                         PXS
       T=A(IJ)                                                            PXS
       IF(A(I) .LE. T) GO TO 20                                           PXS
       A(IJ)=A(I)                                                         PXS
       A(I)=T                                                             PXS
       T=A(IJ)                                                            PXS
    20 L=J                                                                PXS
       IF(A(J) .GE. T) GO TO 40                                           PXS
       A(IJ)=A(J)                                                         PXS
       A(J)=T                                                             PXS
       T=A(IJ)                                                            PXS
       IF(A(I) .LE. T) GO TO 40                                           PXS
       A(IJ)=A(I)                                                         PXS
       A(I)=T                                                             PXS
       T=A(IJ)                                                            PXS
       GO TO 40                                                           PXS
    30 TT = A(L)                                                          PXS
       A(L) = A(K)                                                        PXS
       A(K)=TT                                                            PXS
    40 L=L-1                                                              PXS
       IF(A(L) .GT. T) GO TO 40                                           PXS
    50 K=K+1                                                              PXS
       IF(A(K) .LT. T) GO TO 50                                           PXS
       IF(K .LE. L) GO TO 30                                              PXS
       IF(L-I .LE. J-K) GO TO 60                                          PXS
       IL(M)=I                                                            PXS
       IU(M)=L                                                            PXS
       I=K                                                                PXS
       M=M+1                                                              PXS
       GO TO 80                                                           PXS
    60 IL(M)=K                                                            PXS
       IU(M)=J                                                            PXS
       J=L                                                                PXS
       M=M+1                                                              PXS
       GO TO 80                                                           PXS
    70 M=M-1                                                              PXS
       IF(M .EQ. 0) RETURN                                                PXS
       I=IL(M)                                                            PXS
       J=IU(M)                                                            PXS
    80 IF(J-I .GE. 11)GO TO 10                                            PXS
```

```
      IF(I .EQ. II) GO TO 5                                          PXS
      I=I-1                                                          PXS
   90 I=I+1                                                          PXS
      IF(I .EQ. J) GO TO 70                                          PXS
      IF(A(I) .LE. A(I+1)) GO TO 90                                  PXS
      T = A(I+1)                                                     PXS
      K=I                                                            PXS
  100 A(K+1)=A(K)                                                    PXS
      K=K-1                                                          PXS
      IF(T .LT. A(K)) GO TO 100                                      PXS
      A(K+1)=T                                                       PXS
      GO TO 90                                                       PXS
      END                                                            PXS
```

```
NUMBER OF WARNINGS IN PROGRAM UNIT: 0
NUMBER OF ERRORS   IN PROGRAM UNIT: 0
```

```
      INCLUDE 'BCRES1.for'
      SUBROUTINE BCRES1(IX,N,EVAL)
C  COMPUTES THE LAG 1 CORR COEFF ESTIMATOR USING THE CRESSIE FORMULA
C  THE BETA LAPLACE PROCESS HAS ALPHA AND LPASSED BY
C  COMMON STORAGE.
C  USES FUNCTION AMEDI (WHICH USES HEAPSORT AND HEAPIFY) TO GET THE
C  MEDIAN.
      REAL*8 IX
      REAL*4 X(600),MEAN,MEANINPU,MEDIAN,STDINPU,ALPHA,L
      COMMON ALPHA,L
      CALL BELAR1(IX,ALPHA,L,N,X)
      SUM1=0
      SUM2=0
      MEDIAN=AMEDI(X,N)
C     WRITE(5,*)'MEDIAN = ',MEDIAN
C     WRITE(5,*)'X(I) ',(X(I),I=1,N)
      DO 1 I=1,N-1
      SUM1=SUM1+SQRT(ABS(X(I)-X(I+1)))
1     IF(ABS(X(I)-MEDIAN).GT.0)SUM2=SUM2+SQRT(ABS(X(I)-MEDIAN))
      EVAL=1-(((SUM1/SUM2)**4)/2)
C     WRITE(5,*)'CRESS = ',EVAL
      RETURN
      END
```

```
NUMBER OF WARNINGS IN PROGRAM UNIT: 0
NUMBER OF ERRORS   IN PROGRAM UNIT: 0
```

```
      FUNCTION AMEDI(X,N)
C THIS ROUTINE USES A SHELLSORT AND INDEXES INTO THE REAL ARRAY X TO FIND
C THE MEDIAN OF X WITHOUT REARRANGING IT.
C THE INDEXES ARE USED AS INTEGER*2 TO CONSERVE STORAGE.
      REAL*4 X(*)
      INTEGER*2 I,INDX(600)
      INTEGER N
      DO 1 I=1,N
1     INDX(I)=I
C     WRITE(5,*)'UNSORTED LIST: ',(X(INDX(I)),I=1,N)
      CALL SRTNDX(X,N,INDX)
C     WRITE(5,*)'SORTED LIST: ',(X(INDX(I)),I=1,N)
```

```
      IF(MOD(N,2).EQ.0)THEN
         AMEDI=(X(INDX(N/2))+X(INDX(N/2+1)))/2
      ELSE
         AMEDI=X(INDX(N/2+1))
      ENDIF
      RETURN
      END

NUMBER OF WARNINGS IN PROGRAM UNIT:  0
NUMBER OF ERRORS   IN PROGRAM UNIT:  0

      SUBROUTINE SRTNDX(Y,N,INDX)
C   INPLACE SORT USING SHELL ALGORITHM ********                          SMT
C  USES INDEXES TO SORT BY TO PRESERVE ORIGINAL ORDER OF Y ARRAY
      REAL Y(*)
      INTEGER*2 GAP,INDX(*),ITEMP
      INTEGER N
      LOGICAL EXCH                                                      SMT
C                                                                       SMT
      GAP=(N/2)                                                         SMT
    5 IF (.NOT.(GAP.NE.0)) GO TO 500                                    SMT
   10    CONTINUE                                                       SMT
            EXCH=.TRUE.                                                 SMT
            K=N-GAP                                                     SMT
            DO 200 I=1,K                                                SMT
            KK=I+GAP                                                    SMT
            IF(.NOT.(Y(INDX(I)).GT.Y(INDX(KK)))) GO TO 100
               ITEMP=INDX(I)
               INDX(I)=INDX(KK)
                  INDX(KK)=ITEMP
                  EXCH=.FALSE.
  100       CONTINUE
  200       CONTINUE                                                    SMT
         IF (.NOT.(EXCH)) GO TO 10                                      SMT
         GAP=(GAP/2)                                                    SMT
      GO TO 5                                                           SMT
  500 CONTINUE                                                          SMT
      RETURN                                                            SMT
      END                                                               SMT

NUMBER OF WARNINGS IN PROGRAM UNIT:  0
NUMBER OF ERRORS   IN PROGRAM UNIT:  0
NUMBER OF WARNINGS IN COMPILATION :  0
NUMBER OF ERRORS   IN COMPILATION :  0
```

APPENDIX D. SMTBED SOURCE CODE (VERSION 13)

```
Source File: SMTB13.FOR          Options: /b /l /x          07/18/88 20:50:28

C  UPDATED 05-10-84 BY LCU TO ADD SERIAL CORRELATION                     SMT
C  UPDATED 07-16-84 BY LCU TO MULTIPLE RUN AND SUMMARY STATISTICS PAGE   SMT
C  UPDATED 07-30-84 BY LCU TO ADD NORMALIZED QUANTILES                   SMT
C  UPDATED 10-25-84 BY LCU TO CONVERT TO FORTRAN 77                      SMT
C  UPDATED 08-12-85 BY LCU TO ALLOW DEGREE REGRESSION UP TO 6 ON VAR.    SMT
C  UPDATED 04-04-86 BY LCU TO SAVE STATS. FROM EACH SUPER REPL. ON FILE SMT
C  UPDATED 05-12-86 BY LCU TO COMPUTE MEAN SQUARE ERROR                  SMT
C  UPDATED 09-09-86 BY LCU TO DO PERCENTILE PLOTS                        SMT
C  UPDATED 09-29-86 BY LCU TO DO BIVARIATE HISTOGRAMS. BUG FIX 02-09-87 SMT
C  UPDATED 06-10-87 BY LCU TO ADD RESTART FEATURE                        SMT
C  UPDATE 12/87 BY RLY FOR COLOR PRINTER SUPPORT OF COMBINED BOXPLOTS
C  UPDATE 4/88 BY RLY FOR 5 ESTIMATORS
C                                                                        SMT
C  PURPOSE    TO GENERATE REGRESSION ADJUSTED ESTIMATES AND BOX PLOTS    SMT
C             OF ESTIMATES OF AN INPUT RAW DATA SERIES X CONTAING M      SMT
C             (REPLICATIONS) OF N VALUES EACH.  UP TO 3 ESTIMATING       SMT
C             FUNCTIONS CAN BE USED.  THE GRAPHS CAN ALL BE OF THE SAME SMT
C             SCALE OR SCALED INDIVIDUALLY.                              SMT
C   *         THE SERIES X IS GENERATED FROM INSIDE SIMTB BY USER        SMT
C   *         PROVIDED FUNCTION                                          SMT
C                                                                        SMT
C     DESCRIPTION OF PARAMETERS                                          SMT
C                                                                        SMT
C *   GEST1, GEST2, GEST3,GEST4,GEST5:
C             USER SUBROUTINES THAT WILL GENERATE THE DATA,SMT00220
C *           AND THEN COMPUTE AN ESTIMATE FROM IT.                      SMT
C *           THE ACTUAL NAMES USED NAMES MUST BE DECLARED AS EXTERNAL   SMT
C *           IN THE CALLING PROGRAM.  CALLS MUST BE AS FOLLOWS:         SMT
C *               CALL SUBROUTINE_NAME(IX,NX,EVAL)                       SMT
C *           WHERE:                                                     SMT
C *               IX=SEED (INPUT AND OUTPUT ARGUMENT)                    SMT
C *               NX=NO. OF DATA POINTS TO TO GENERATE                   SMT
C *               EVAL=VALUE OF ESTIMATOR APPLIED TO THE NX DATA POINTS SMT
C *           NOTICE THAT PROGRAM ASSUMES THREE SUBROUTINES IN THE       SMT
C *           ARGUMENTS.  WHEN USING LESS THAN 3 ESTIMATORS FILL THE     SMT
C *           ARGUMENT POSITIONS WITH DUMMY VARIABLES OR JUST REPEAT     SMT
C *           THE LAST ONE.                                              SMT
C                                                                        SMT
C *   ISEED1, ISEED2, ISEED3,ISEED4, ISEED5:
C                             SEEDS FOR DATA GENERATORS 1, 2 & 3.        SMT
C *           THE SEEDS ARE UPDATED (ADVANCED) UPON RETURN FROM SIMTB    SMT
C                                                                        SMT
C *   Y       WORK ARRAY OF SIZE >= M*N/NE(1))                           SMT
C                                                                        SMT
C     N       NUMBER OF DATA ELEMENTS PER SECTION (N IS SAMPLE SIZE).    SMT
C                                                                        SMT
C     M       NUMBER OF SECTIONS (REPLICATIONS).                         SMT
C             M CANNOT EXCEED 100                                        SMT
C                                                                        SMT
```

```
C     NE        INTEGER ARRAY OF SIZE 8 CONTAINING SUBSAMPLE SIZES FOR N.  SMT
C               THE VALUES OF NE MUST BE FROM SMALLEST TO LARGEST.         SMT
C               NO ELEMENT OF THE ARRAY NE CAN BE GREATER THAN N.          SMT
C                                                                          SMT
C     L         NUMBER OF SUBSAMPLE SIZES FROM NE(8) THAT WILL BE USED TO  SMT
C               SECTION N.  MUST BE BETWEEN 2 AND 8.                       SMT
C               IT IS ALSO THE NUMBER OF BOXPLOTS THAT WILL BE PRODUCED.   SMT
C                                                                          SMT
C     D         DEGREE OF REGRESSION FOR MEAN AND VARIANCE REGRESSIONS.    SMT
C               D WILL BE REDUCED BY SIMTB IF THE SAMPLE IS NOT LARGE      SMT
C               ENOUGH. D MUST BE 1 TO 6. D=0 WILL IGNORE REGRESSIONS.     SMT
C                                                                          SMT
C     NSR     NO. OF RUNS TO REPEAT SIMTB (>=1) WITH A PAGE OF             SMT
C               SUMMARY STATISTCS AT THE END WHEN >1.                      SMT
C                                                                          SMT
C               *** SCALING ***                                            SMT
C                 SCALING IS ACCOMPLISHED BY TAKING THE SMALLEST AND THE   SMT
C                 LARGEST ESTIMATE VALUES FROM ALL ESTIMATING FUNCTIONS    SMT
C                 EVALUATED ON THE SHORTEST SECTION LENGTH ( NE(1) ).      SMT
C                    THE SEI PARAMETER ALLOWS THE USER TO SCALE THE GRAPHSSMT
C                 OF EACH ESTIMATOR INDIVIDUALLY OR TO SCALE THEM ALL TO   SMT
C                 THE SAME SCALE.  SCALING ALL TO THE SAME SCALE IS        SMT
C                 ACCOMPLISHED BY TAKING THE MINIMUM AND MAXIMUM  ESTIMATESMT
C                 FROM ALL THE ESTIMATORS USING NE(1) SUBSAMPLE SIZE.      SMT
C                    THE RG PARAMETER ALLOWS THE USER TO RREDUCE THE VER-  SMT
C                 TICAL SCALE TO; THE UPPER QUARTILE DISTANCE + 1.5 TIMES  SMT
C                 INTERQUARTILE DISTANCE AS THE MAX VALUE AND THE LOWER    SMT
C                 QUARTILE - 1.5 TIMES THE INTERQUARTILE DISTANCE AS THE   SMT
C                 MIN VALUE.  THE INTERQUARTILE DISTANCE IS COMPUTED FROM  SMT
C                 THE SAMPLE OF ESTIMATES FROM THE NE(1) SUBSAMPLE SIZE.   SMT
C                 IF THERE ARE NO ESTIMATES OUTSIDE THESE MIN AND MAX      SMT
C                 VALUES THEN THE SCALE IS TO THE FIRST VALUE WITHIN.      SMT
C                 IF THERE ARE ESTIMATES OUTSIDE THESE LIMITS THEN THEY    SMT
C                 ARE COUNTED AND THE NUMBER PRINTED AT THE ENDS OF THE    SMT
C                 BOX PLOTS.                                               SMT
C                    THE SVS PARAMET ALLOWS THE USER TO SET THE VERTICAL   SMT
C                 SCALE. WHEN THE VERTICAL SCALE IS SET THE SEI PARAMETER  SMT
C                 IS IGNORED AND THE VERTICAL SCALE BECOMES YMIN AND YMAX. SMT
C                                                                          SMT
C                                                                          SMT
C     RG        RG=0 DO NOT REDUCE THE VERTICAL SCALE OF THE GRAPHS.       SMT
C               RG=1 REDUCE GRAPHICS VERTICAL SCALE TO UPPER (LOWER)       SMT
C                    QUARTILE + (-) INTERQUARTILE DISTANCE.                SMT
C                                                                          SMT
C     SEI       SEI=0 DO NOT SCALE ESTIMATORS' GRAPHS INDIVIDUALLY.        SMT
C               SEI=1 SCALE ESTIMATORS' GRAPHS INDIVIDUALLY.               SMT
C                                                                          SMT
C     SVS       SVS=0  PROGRAM WILL CALCULATE VERTICAL SCALE.              SMT
C               SVS=1  USER SETS VERTICAL SCALE TO YMIN AND YMAX.          SMT
C                                                                          SMT
C     YMIN      LOW VALUE OF VERTICAL SCALE. SET BY USER WHEN SVS=1        SMT
C                                                                          SMT
C     YMAX      HIGH VALUE OF VERTICAL SCALE. SET BY USER WHEN SVS=1       SMT
C                                                                          SMT
C     NEST      NUMBER OF ESTIMATORS THAT WILL BE USED TO CALCULATE        SMT
C               STATISTICAL PARAMETER FROM X DATA.                         SMT
```

```
C                 NEST MUST BE 1,2,3,4 OR 5.
C                                                                   SMT
C      TTL1       TITLES ASSOCIATED WITH EACH ESTIMATOR (1 THRU 5).  A MAX  SMT
C      TTL2       OF 120 CHARACTERS CAN BE USED TO DESCRIBE EACH ESTIMATOR. SMT
C      TTL3       EACH TITLE MUST BE DECLARED AS CHARACTER*120 STRINGS      SMT
C      TTL4
C      TTL5
C                                                                   SMT
C      IFIL       0 - DO NOT RECORD STATS. FROM EACH SUPER REPL. ON FILE    SMT
C                 1 - RECORD STATISTICS ON FILE UNIT 1 FOR FURTHER PROCESSNGSMT
C                 UNIT 1 FILE IS SEQUENTIAL, FORMATTED.  THE DATA PLACED    SMT
C                 IN THE FILE INCLUDES LABELS                               SMT
C                                                                   SMT
C      NPRT       0 - PRINT DETAIL FOR UP TO 3 SUPER REPLICATIONS           SMT
C                     AND PRINT NORMALIZED QUANTILE PLOTS (STANDARD SETTING)SMT
C                 1 - PRINT ONLY 1 SUPER REPL.  AND NO NORMALIZED QUANTILE  SMT
C                                                                   SMT
C      MSE        0 - DO NOT PRINT MEAN SQUARED ERRORS USING VMSE MATRIX    SMT
C                 1 - PRINT MEAN SQUARED ERRORS WITH VMSE MATRIX            SMT
C                                                                   SMT
C      VMSE       MATRIX 8X5 WITH KNOWN MEANS ON EACH COL.FOR EACH ESTIMATORSMT
C                                                                   SMT
C      IPR        0 - DO NOT PRINT PERCENTILE PLOTS                         SMT
C                 1 - PRINT PERCENTILE PLOTS                                SMT
C                                                                   SMT
C      VMX1       3 MATRICES 8X4 EACH WITH VALUES FOR PERCENTILE PLOTS      SMT
C      VMX2         EACH REPRESENTING 8 VALUES FOR EACH SECTION SIZE        SMT
C      VMX3         (IF L<8 PAD WITH ZEROES). 4 SUCH SETS MUST BE ENTERED   SMT
C      VMX4         FOR EACH MATRIX                                         SMT
C      VMX5
C                                                                   SMT
C      IBIV       0 - DO NOT PRINT BIVARIATE HISTOGRAMS                     SMT
C                 1 - PRINT BIVARIATE HISTOGRAMS                            SMT
C                                                                   SMT
C      RSTRT      0 - THIS IS NOT A RESTART RUN.                            SMT
C                 1 - THIS IS A RESTART RUN.  AT THE END SAVE RESTART       SMT
C                     VALUES ON UNIT 2.  AT THE BEGINNING READ UNIT 2       SMT
C                     AND CONTINUE EXECUTION AT THE SUPER-REPLICATION       SMT
C                     WHERE IT WAS LEFT OFF.  IF NO FILE EXISTS FOR         SMT
C                     FOR UNIT 2 IT WILL ASSUME THIS IS THE FIRST RUN.      SMT
C                     UNIT 2 SHOULD BE OPENED BY THE CALLING PROGRAM IF     SMT
C                     A DIFFERENT NAME OF THE FORTRAN DEFAULT IS TO BE      SMT
C                     USED.  ON VM/CMS A FILEDEF WILL DO THE SAME.          SMT
C                                                                   SMT
C                     THE RESTART FILE IS TYPE SEQUENTIAL UNFORMATTED, AND  SMT
C                     CAN ONLY BE READ BY A FORTRAN PROGRAM.  USE UNIT 2    SMT
C                     FOR STATISTICS FILE.  DEFAULT UNFORMATTED RECORD      SMT
C                     LENGTH OF 1024 BYTES IS ADEQUATE.
C                                                                   SMT
C      ICOLOR     0 - NO COLOR PRINTOUT
C                 1 - PRINT COLOR COMBINATION OF ESTIMATORS FOR THE
C                     ESTIMATORS FOR UP TO THREE SAMPLE SIZES (NEED AT
C                     LEAST 2 TO MAKE SENSE).  THIS IS IN ADDITION TO
C                     THE REGULAR PRINTOUTS, WHICH ARE FLAGGED SEPARATELY.
C
C      IBWPRT     0 - DO NOT PRINT THE STANDARD PRINTOUTS.
```

```
C            1 - PRINT THE STANDARD PRINTOUTS.
C
C
C            NOTE:  IF BOTH ICOLOR AND IBWPRT ARE ZERO, NO PRINTOUTS
C                   OCCUR.
C
C      NCPRT - NUMBER OF SAMPLE SIZES FOR WHICH TO PRINT COLOR BOXPLOT
C              COMBINATIONS.  MUST BE 1,2, OR 3.
C
C      NCOLRNDX( ) - ARRAY INDEX TO NE(*); USE THESE SAMPLE SIZES FOR COLOR
C                    BOXPLOTS
C
C
C
C*************************************************************************SMT
C                                                                        SMT
      SUBROUTINE SMTBED(ISEED1,ISEED2,ISEED3,ISEED4,ISEED5,
     * Y,N,M,NE,L,D,NSR,RG,SEI,SVS,YMIN,YMAX,NEST,GEST1,TTL1,GEST2,TTL2,
     * GEST3,TTL3,GEST4,TTL4,GEST5,TTL5,IFIL,NPRT,
     * MSE,VMSE,IPR,VMX1,VMX2,VMX3,VMX4,VMX5,
     * IBIV,RSTRT,ICOLOR,IBWPRT,NCPRT,NCOLRNDX)
      PARAMETER (NBROWS=500)
      real*8  ISEED1,ISEED2,ISEED3,ISEED4,ISEED5, ISEED
      CHARACTER*1 COLRPLT(122,50,5),COLRXIS(122,2),BLK
      CHARACTER*120 TTL1, TTL2,TTL3,TTL4,TTL5                            SMT
      CHARACTER*8 LABEL(7)
      REAL Y(1),GV(2), VMSE(8,5),BA(7),BS(7),BV(7),V(7)                  SMT
      REAL VMX1(8,4), VMX2(8,4), VMX3(8,4),COLRSTAT(8,7,5)               SMT
      REAL VMX4(8,4),VMX5(8,4)
      REAL BIV1(NBROWS,8),BIV2(NBROWS,8),BIV3(NBROWS,8),BIV4(NBROWS,8)   SMT
      REAL TX(NBROWS), TY(NBROWS),DLH(4)                                 SMT
      INTEGER NE(8),RG,SEI,SVS,SM, IFIL                                  SMT
      INTEGER D,L,NEST,TEST, IBPTR(8)                                    SMT
      INTEGER RSTRT, NNE(8), NCOLRNDX(3)                                 SMT
      REAL VYMAX(5),VYMIN(5)                                             SMT
      REAL*8 GST1(8,7,2), GPV1(8,11,2)                                   SMT
      REAL*8 GST2(8,7,2), GPV2(8,11,2)                                   SMT
      REAL*8 GST3(8,7,2), GPV3(8,11,2)                                   SMT
      REAL*8 GST4(8,7,2), GPV4(8,11,2)                                   SMT
      REAL*8 GST5(8,7,2), GPV5(8,11,2)                                   SMT
      REAL*8 GBA1(7,2), GBV1(7,2), GBS1(7,2), GVV1(7,2)                  SMT
      REAL*8 GBA2(7,2), GBV2(7,2), GBS2(7,2), GVV2(7,2)                  SMT
      REAL*8 GBA3(7,2), GBV3(7,2), GBS3(7,2), GVV3(7,2)                  SMT
      REAL*8 GBA4(7,2), GBV4(7,2), GBS4(7,2), GVV4(7,2)                  SMT
      REAL*8 GBA5(7,2), GBV5(7,2), GBS5(7,2), GVV5(7,2)                  SMT
      DATA GST1/112*0.D0/, GPV1/176*0.D0/                                SMT
      DATA GST2/112*0.D0/, GPV2/176*0.D0/                                SMT
      DATA GST3/112*0.D0/, GPV3/176*0.D0/                                SMT
      DATA GST4/112*0.D0/, GPV4/176*0.D0/                                SMT
      DATA GST5/112*0.D0/, GPV5/176*0.D0/                                SMT
      DATA GBA1/14*0.D0/, GBV1/14*0.D0/, GBS1/14*0.D0/, GVV1/14*0.D0/    SMT
      DATA GBA2/14*0.D0/, GBV2/14*0.D0/, GBS2/14*0.D0/, GVV2/14*0.D0/    SMT
      DATA GBA3/14*0.D0/, GBV3/14*0.D0/, GBS3/14*0.D0/, GVV3/14*0.D0/    SMT
      DATA GBA4/14*0.D0/, GBV4/14*0.D0/, GBS4/14*0.D0/, GVV4/14*0.D0/    SMT
      DATA GBA5/14*0.D0/, GBV5/14*0.D0/, GBS5/14*0.D0/, GVV5/14*0.D0/    SMT
      DATA BLK/' '/
```

```
C                                                                       
C --- SAVE BASIC RUN PARAMETERS & TITLES ON STATISTICS FILE              
      IF(IFIL .EQ. 1)WRITE(1,560)N,M,NE,L,D,NSR,NEST,TTL1,TTL2,TTL3,    
     *TTL4,TTL5
560   FORMAT('SAMPLE SIZE:    ',I10,'REPLICATIONS:   ',I10,
     * /'SUBSAMPLE SIZES:'/8I10/'L =   ',I10,'DEGREE REG:     ',I10,'SUPER
     *-REPS:     ',I10,/'NO. ESTIM.:     ',I10,/5(A120/))
C                                                                       
C --- DETERMINE CASE OF RESTART.  IRST=0 NO RESTART                     
C                                 IRST=1 IS 1ST RUN (SAVE VALUES AT END)
C                                 IRST=2 IS CONTINUATION OF PREVIOUS RUN
      INSR=1                                                            
      WRITE(5,555)
555   format(' SMTBED - A Simulation Test Bed (Version 13.0, April 1988)
     *'/' (PC version)....     START OF RUN')
C  IF THERE IS ONLY ONE ESTIMATOR, CAN'T COMBINE FOR COLOR OUTPUT
      IF(NEST.EQ.1)ICOLOR=0
      IRST=RSTRT                                                        
      IF(RSTRT .EQ. 0) GO TO 9004                                       
       REWIND 2                                                         
       READ(2,END=9003) NN,MM,NNE,LL,IDD,NNSR,NNEST,NSEI,               
     *                ISEED1,ISEED2,ISEED3, ISEED4,ISEED5,VYMIN,VYMAX   
       IF(NN.NE.N .OR. MM.NE.M .OR. LL.NE.L .OR. IDD.NE.D .OR.          
     *    NSR.LE.NNSR .OR. NSEI.NE.SEI .OR. NNEST.NE.NEST) GO TO 9002   
       DO 9001 I=1,8                                                    
        IF(NNE(I).NE.NE(I)) GO TO 9002                                  
 9001  CONTINUE                                                         
       IRST=2                                                           
C
C  THESE READ STATEMENTS ARE BROKEN UP SO THE DEFAULT UNFORMATTED
C  RECORD LENGTH WILL HOLD THE DATA, AND THE USER DOESN'T HAVE TO
C  WORRY WITH RUNTIME PARAMETERS ON HIS COMPILED PROGRAM
C
       READ(2,END=9006)((GPV1(I,J,1),I=1,8),J=1,11)
       READ(2,END=9006)((GPV1(I,J,2),I=1,8),J=1,11)
       READ(2,END=9006)GST1
       READ(2,END=9006)GBA1,GBV1,GBS1,GVV1                              

       IF(NEST .GE. 2) THEN
           READ(2,END=9006)((GPV2(I,J,1),I=1,8),J=1,11)
           READ(2,END=9006)((GPV2(I,J,2),I=1,8),J=1,11)
           READ(2,END=9006)GST2
           READ(2,END=9006)GBA2,GBV2,GBS2,GVV2                          
       ENDIF
       IF(NEST .GE. 3) THEN
           READ(2,END=9006)((GPV3(I,J,1),I=1,8),J=1,11)
           READ(2,END=9006)((GPV3(I,J,2),I=1,8),J=1,11)
           READ(2,END=9006)GST3
           READ(2,END=9006)GBA3,GBV3,GBS3,GVV3                          
       ENDIF
       IF(NEST .GE. 4) THEN
           READ(2,END=9006)((GPV4(I,J,1),I=1,8),J=1,11)
           READ(2,END=9006)((GPV4(I,J,2),I=1,8),J=1,11)
           READ(2,END=9006)GST4
           READ(2,END=9006)GBA4,GBV4,GBS4,GVV4                          
       ENDIF
```

```
         IF(NEST .GE. 5) THEN
             READ(2,END=9006)((GPV5(I,J,1),I=1,8),J=1,11)
             READ(2,END=9006)((GPV5(I,J,2),I=1,8),J=1,11)
             READ(2,END=9006)GST5
             READ(2,END=9006)GBA5,GBV5,GBS5,GVV5                          SMT
         ENDIF
C ---    FIX VERTICAL SCALE TO USE YMIN,YMAX COMPUTED IN PREVIOUS RUN     SMT
         SVS=1                                                            SMT
C ---    START SUPER-REPLICATIONS COUNT FOLLOWING WHERE LEFT OFF          SMT
         INSR=NNSR+1                                                      SMT
         GO TO 50                                                         SMT
  9002   CONTINUE                                                         SMT
C ---    ERROR IN A RESTART RUN.  NEW PARMS NOT COMPATIBLE WITH OLD ONES  SMT
         WRITE(5,*) 'ARGUMENTS FOR RESTART DO NOT AGREE WITH VALUES ',    SMT
     *       'FOUND ON FILE UNIT 2. THEY ARE:'                            SMT
         WRITE(5,*) 'N,M,L,D,NSR,NEST,SEI=',NN,MM,LL,IDD,NNSR,NNEST,NSEI  SMT
         WRITE(5,*) 'NE:', (NNE(I), I=1,LL)                               SMT
         STOP                                                             SMT
C ---    TABLES NOT PRESENT FOR ALL ESTIMATORS                            SMT
  9006   CONTINUE                                                         SMT
         WRITE(5,*) ' *** ERROR: RESTART FILE DOES NOT HAVE TABLES FOR',  SMT
     *                  ' ALL ESTIMATORS'                                 SMT
         STOP                                                             SMT
C                                                                         SMT
  9003   CONTINUE                                                         SMT
C ---    RESTART FILE NOT EXISTENT.  ASSUME 1ST RUN                       SMT
         IRST=1                                                           SMT
  9004  CONTINUE                                                          SMT
C                                                                         SMT
C  SET SEI TO 0 FOR NO INDIVIDUAL SCALING IF WE ARE DOING COLOR PLOTS
C
        IF(ICOLOR.EQ.1)SEI=0
C
        IF(SVS.EQ.1) THEN                                                 SMT
           DO 9011 I=1,5                                                  SMT
            VYMAX(I)=YMAX                                                 SMT
            VYMIN(I)=YMIN                                                 SMT
  9011     CONTINUE                                                       SMT
        ENDIF                                                             SMT
        SM=NE(1)                                                          SMT
        MN = M*N                                                          SMT
        LT=L-1                                                            SMT
        IF(LT.EQ.0) GO TO 13                                              SMT
        DO 11 I=1,LT                                                      SMT
           I1=I+1                                                         SMT
           IF(NE(I).GT.NE(I1))WRITE(6,110)                                SMT
    11  CONTINUE                                                          SMT
    13  TEST=0                                                            SMT
        IF(NEST.EQ.1 .OR.  NEST.EQ.2 .OR.  NEST.EQ.3.OR.NEST.EQ.4.OR.
     *  NEST.EQ.5) GO TO 2                                                SMT
         WRITE(5,106)                                                     SMT
         TEST=1                                                           SMT
C 2     IF(M.GE.1.AND.M.LE.100) GO TO 3                                   SMT
         WRITE(5,104)                                                     SMT
         TEST=1                                                           SMT
  3     IF(L.GE.1.AND.L.LE.8) GO TO 4                                     SMT
```

```
       WRITE(5,103)                                                     SMT
       TEST=1                                                           SMT
 4    IF(D.LE.6) GO TO 5                                                SMT
       WRITE(5,108)                                                     SMT
       TEST=1                                                           SMT
 5    CONTINUE                                                          SMT
      IF(NE(L) .LT. N) GO TO 6                                          SMT
       WRITE(5,107)                                                     SMT
       TEST=1                                                           SMT
 6    CONTINUE                                                          SMT
      IF(NSR .GE. 1) GO TO 7                                            SMT
       WRITE(5,111)                                                     SMT
       TEST=1                                                           SMT
 7    CONTINUE                                                          SMT
      IF (TEST.NE.0) GO TO 8001                                         SMT
C                                                                       SMT
C     BYPASS SCALE CALCULATION WHEN FIXED                               SMT
      IF (SVS .EQ. 1) GO TO 50                                          SMT
C     ******************************************************************SMT
C     * COMPUTE SCALE FOR EACH ESTIMATOR                                SMT
C     ******************************************************************SMT
      ISEED=ISEED1                                                      SMT
      DO 10 IK=1,1                                                      SMT
C      FIND VERTICAL SCALE FOR 1ST ESTIMATOR.                           SMT
       CALL SECEST(GEST1,ISEED,N,M,NE(IK),Y,KP)                         SMT
       IF(RG.EQ.1) CALL DELETO(Y,KP,VYMAX(1),VYMIN(1))                  SMT
       IF(RG.NE.1) CALL MAXMIN(Y,KP,VYMAX(1),VYMIN(1))                  SMT
       YMIN=VYMIN(1)                                                    SMT
       YMAX=VYMAX(1)                                                    SMT
   10 CONTINUE                                                          SMT
C                                                                       SMT
C     FIND VERTICAL SCALE FOR 2ND ESTIMATOR.                            SMT
      IF(NEST .LT. 2) GO TO 40                                          SMT
      ISEED=ISEED2                                                      SMT
      DO 20 IK=1,1                                                      SMT
       CALL SECEST(GEST2,ISEED,N,M,NE(IK),Y,KP)                         SMT
       IF(RG.EQ.1) CALL DELETO(Y,KP,VYMAX(2),VYMIN(2))                  SMT
       IF(RG.NE.1) CALL MAXMIN(Y,KP,VYMAX(2),VYMIN(2))                  SMT
       YMIN=AMIN1(YMIN, VYMIN(2))                                       SMT
       YMAX=AMAX1(YMAX, VYMAX(2))                                       SMT
   20 CONTINUE                                                          SMT
C                                                                       SMT
C     FIND VERTICAL SCALE FOR 3RD ESTIMATOR.                            SMT
      IF(NEST .LT. 3) GO TO 40                                          SMT
      ISEED=ISEED3                                                      SMT
      DO 30 IK=1,1                                                      SMT
       CALL SECEST(GEST3,ISEED,N,M,NE(IK),Y,KP)                         SMT
       IF(RG.EQ.1) CALL DELETO(Y,KP,VYMAX(3),VYMIN(3))                  SMT
       IF(RG.NE.1) CALL MAXMIN(Y,KP,VYMAX(3),VYMIN(3))                  SMT
       YMIN=AMIN1(YMIN, VYMIN(3))                                       SMT
       YMAX=AMAX1(YMAX, VYMAX(3))                                       SMT
   30 CONTINUE                                                          SMT
C                                                                       SMT
C     FIND VERTICAL SCALE FOR 4TH ESTIMATOR.                            SMT
      IF(NEST .LT. 4) GO TO 40                                          SMT
      ISEED=ISEED4                                                      SMT
```

```
      DO 31 IK=1,1                                                      SMT
       CALL SECEST(GEST4,ISEED,N,M,NE(IK),Y,KP)                         SMT
       IF(RG.EQ.1) CALL DELETO(Y,KP,VYMAX(4),VYMIN(4))                  SMT
       IF(RG.NE.1) CALL MAXMIN(Y,KP,VYMAX(4),VYMIN(4))                  SMT
       YMIN=AMIN1(YMIN, VYMIN(4))                                       SMT
       YMAX=AMAX1(YMAX, VYMAX(4))                                       SMT
   31 CONTINUE                                                          SMT
C
C     FIND VERTICAL SCALE FOR 5TH ESTIMATOR.                            SMT
      IF(NEST .LT. 5) GO TO 40                                          SMT
      ISEED=ISEED5                                                      SMT
      DO 32 IK=1,1                                                      SMT
       CALL SECEST(GEST5,ISEED,N,M,NE(IK),Y,KP)                         SMT
       IF(RG.EQ.1) CALL DELETO(Y,KP,VYMAX(5),VYMIN(5))                  SMT
       IF(RG.NE.1) CALL MAXMIN(Y,KP,VYMAX(5),VYMIN(5))                  SMT
       YMIN=AMIN1(YMIN, VYMIN(5))                                       SMT
       YMAX=AMAX1(YMAX, VYMAX(5))                                       SMT
   32 CONTINUE                                                          SMT
   40 CONTINUE                                                          SMT
C --- WHEN SCALE IS SAME FOR ALL USE WIDEST OF ALL THREE SCALES FOUND   SMT
      IF(SEI .EQ. 0) THEN                                               SMT
         DO 41 IK=1,5                                                   SMT
          VYMIN(IK)=YMIN                                                SMT
          VYMAX(IK)=YMAX                                                SMT
   41    CONTINUE                                                       SMT
      ENDIF                                                             SMT
C                                                                       SMT
C                                                                       SMT
C     PROCESS BOXPLOTS USING VERTICAL SCALE AS DETERMINED               SMT
C     ONE CALL FOR EACH ESTIMATOR USED.                                 SMT
   50 CONTINUE                                                          SMT
C  CLEAR COLOR PLOT ARRAY
      CALL CLRCOLR(COLRPLT)                                             SMT
      GV(1)=VYMIN(1)                                                    SMT
      GV(2)=VYMAX(1)                                                    SMT
      WRITE(5,552)
552   FORMAT(/'PROCESSING ESTIMATOR # 1')
      CALL PRST(GEST1,ISEED1,N,M,NE,L,RG,D,VYMIN(1),VYMAX(1),           SMT
     * Y,GV,TTL1,NSR,IFIL, NPRT,MSE,VMSE(1,1), IPR,VMX1,                SMT
     * IBIV,IBPTR,BIV1, INSR,GPV1,GST1,GBA1,GBV1,GBS1,GVV1,
     * IBWPRT,NCPRT,NCOLRNDX,COLRXIS,1,NEST,                            SMT
     * COLRPLT,COLRSTAT,BA,BV,BS,V,DLH,VSCALE,IWIDTH,LABEL)
C                                                                       SMT
      IF (NEST. LT. 2) GO TO 80                                         SMT
      WRITE(5,553)
553   FORMAT(/'PROCESSING ESTIMATOR #2')
      CALL PRST(GEST2,ISEED2,N,M,NE,L,RG,D,VYMIN(2),VYMAX(2),           SMT
     * Y,GV,TTL2,NSR,IFIL, NPRT,MSE,VMSE(1,2), IPR,VMX2,                SMT
     * IBIV,IBPTR,BIV2, INSR,GPV2,GST2,GBA2,GBV2,GBS2,GVV2,             SMT
     * IBWPRT,NCPRT,NCOLRNDX,COLRXIS,2,NEST,
     * COLRPLT,COLRSTAT,BA,BV,BS,V,DLH,VSCALE,IWIDTH,LABEL)
C                                                                       SMT
      IF (NEST. LT. 3) GO TO 80                                         SMT
      WRITE(5,554)
554   FORMAT(/'PROCESSING ESTIMATOR # 3')
      CALL PRST(GEST3,ISEED3,N,M,NE,L,RG,D,VYMIN(3),VYMAX(3),           SMT
```

```
     * Y,GV,TTL3,NSR,IFIL, NPRT,MSE,VMSE(1,3), IPR,VMX3,                         SMT
     * IBIV,IBPTR,BIV3, INSR,GPV3,GST3,GBA3,GBV3,GBS3,GVV3,                      SMT
     * IBWPRT,NCPRT,NCOLRNDX,COLRXIS,3,NEST,
     * COLRPLT,COLRSTAT,BA,BV,BS,V,DLH,VSCALE,IWIDTH,LABEL)
C                                                                                SMT
      IF (NEST. LT. 4) GO TO 80                                                  SMT
      WRITE(5,556)
556   FORMAT(/'PROCESSING ESTIMATOR # 4')
      CALL PRST(GEST4,ISEED4,N,M,NE,L,RG,D,VYMIN(4),VYMAX(4),                    SMT
     * Y,GV,TTL4,NSR,IFIL, NPRT,MSE,VMSE(1,4), IPR,VMX4,                         SMT
     * IBIV,IBPTR,BIV4, INSR,GPV4,GST4,GBA4,GBV4,GBS4,GVV4,                      SMT
     * IBWPRT,NCPRT,NCOLRNDX,COLRXIS,4,NEST,
     * COLRPLT,COLRSTAT,BA,BV,BS,V,DLH,VSCALE,IWIDTH,LABEL)
C                                                                                SMT
      IF (NEST. LT. 5) GO TO 80                                                  SMT
      WRITE(5,557)
557   FORMAT(/'PROCESSING ESTIMATOR # 5')
      CALL PRST(GEST5,ISEED5,N,M,NE,L,RG,D,VYMIN(5),VYMAX(5),                    SMT
     * Y,GV,TTL5,NSR,IFIL, NPRT,MSE,VMSE(1,5), IPR,VMX5,                         SMT
     * IBIV,IBPTR,BIV4, INSR,GPV5,GST5,GBA5,GBV5,GBS5,GVV5,                      SMT
     * IBWPRT,NCPRT,NCOLRNDX,COLRXIS,5,NEST,
     * COLRPLT,COLRSTAT,BA,BV,BS,V,DLH,VSCALE,IWIDTH,LABEL)
C                                                                                SMT
   80 CONTINUE                                                                   SMT
C
C  IF COLOR FLAG NOT SET, SKIP COLOR OUTPUT SECTION
C  IF WE ARE ON SECOND STAGE OF A RESTART RUN, DON'T DO THE COLOR PART.
      IF(ICOLOR.EQ.0.OR.IRST.EQ.2)GOTO 340
C
C  OUTPUT SECTION FOR COLOR COMBINED BOXPLOTS
C  OUTPUT SECTION FOR COLOR COMBINED BOXPLOTS
C  OUTPUT SECTION FOR COLOR COMBINED BOXPLOTS
C  OUTPUT SECTION FOR COLOR COMBINED BOXPLOTS
C
C
      WRITE(6,325)N,M,D
325   FORMAT('1',5X,'SMTB COMBINED ESTIMATOR BOXPLOTS:   SAMPLE SIZE:'
     * ,I7,' NO. REPLICATIONS: ',I4,' DEG REGRESSION: ',I2/' ',131('-'))
      DO 303 K=50,1,-1
      IF(MOD(K,5).NE.0)GO TO 304
        YLABEL=(K-DLH(2))/VSCALE+YMIN
        WRITE(6,622)YLABEL
622     FORMAT(' ',G8.2,'-')
        CALL SETCOLR(1)
        WRITE(6,322)(COLRPLT(I,K,1),I=1,IWIDTH)
        WRITE(6,322)(COLRPLT(I,K,1),I=1,IWIDTH)
        CALL SETCOLR(0)
        WRITE(6,364)
        CALL SETCOLR(2)
        WRITE(6,322)(COLRPLT(I,K,2),I=1,IWIDTH)
        WRITE(6,322)(COLRPLT(I,K,2),I=1,IWIDTH)
322     FORMAT('+',9X,122A1)
        CALL SETCOLR(6)
        WRITE(6,322)(COLRPLT(I,K,3),I=1,IWIDTH)
        WRITE(6,322)(COLRPLT(I,K,3),I=1,IWIDTH)
        CALL SETCOLR(3)
```

```
          WRITE(6,322)(COLRPLT(I,K,4),I=1,IWIDTH)
          WRITE(6,322)(COLRPLT(I,K,4),I=1,IWIDTH)
          CALL SETCOLR(0)
          WRITE(6,322)(COLRPLT(I,K,5),I=1,IWIDTH)
          WRITE(6,322)(COLRPLT(I,K,5),I=1,IWIDTH)
          GOTO 303
 304    CONTINUE
        WRITE(6,363)
 363    FORMAT(9X,'|')
        CALL SETCOLR(1)
        WRITE(6,322)(COLRPLT(I,K,1),I=1,IWIDTH)
        WRITE(6,322)(COLRPLT(I,K,1),I=1,IWIDTH)
        CALL SETCOLR(0)
        WRITE(6,364)
 364    FORMAT('+',131X,'|')
        CALL SETCOLR(2)
        WRITE(6,322)(COLRPLT(I,K,2),I=1,IWIDTH)
        WRITE(6,322)(COLRPLT(I,K,2),I=1,IWIDTH)
        CALL SETCOLR(6)
        WRITE(6,322)(COLRPLT(I,K,3),I=1,IWIDTH)
        WRITE(6,322)(COLRPLT(I,K,3),I=1,IWIDTH)
        CALL SETCOLR(3)
        WRITE(6,322)(COLRPLT(I,K,4),I=1,IWIDTH)
        WRITE(6,322)(COLRPLT(I,K,4),I=1,IWIDTH)
        CALL SETCOLR(0)
        WRITE(6,322)(COLRPLT(I,K,5),I=1,IWIDTH)
        WRITE(6,322)(COLRPLT(I,K,5),I=1,IWIDTH)
 303    CONTINUE
        CALL SETCOLR(0)
        WRITE(6,326)(COLRXIS(I,1),I=1,IWIDTH)
 326    FORMAT(9X,'+',122A1,'+')
        WRITE(6,327)(COLRXIS(I,2),I=1,IWIDTH)
 327    FORMAT(10X,122A1)
        DO 390 J=1,NEST
        WRITE(6,370)(NE(NCOLRNDX(I)),I=1,NCPRT)
 370    FORMAT('0SUBSAMPLE SIZE: ',18X,3(I8,6X))
        WRITE(6,371)J,(COLRSTAT(NCOLRNDX(I),1,J),I=1,NCPRT)
 371    FORMAT(' ESTIMATOR ',I1,':    MEAN: ',8X,3G14.4)
        WRITE(6,372)(COLRSTAT(NCOLRNDX(I),2,J),I=1,NCPRT)
 372    FORMAT(' ',15X,'STD:   ',8X,3G14.4)
        WRITE(6,373)(COLRSTAT(NCOLRNDX(I),3,J),I=1,NCPRT)
 373    FORMAT(' ',15X,'STD MEAN:',5X,3G14.4)
 390    CONTINUE
        IF(NEST.GE.1)THEN
          WRITE(6,374)
 374    FORMAT('0ESTIMATOR 1:')
          CALL SETCOLR(1)
          WRITE(6,375)TTL1
          WRITE(6,375)TTL1
 375    FORMAT('+',14X,A120)
          CALL SETCOLR(0)
        ENDIF
        IF(NEST.GE.2) THEN
          WRITE(6,376)
        CALL SETCOLR(0)
 376    FORMAT(' ESTIMATOR 2:')
```

```
        CALL SETCOLR(2)
        WRITE(6,375)TTL2
        WRITE(6,375)TTL2
        CALL SETCOLR(0)
      ENDIF
      IF(NEST.GE.3) THEN
        WRITE(6,377)
377   FORMAT(' ESTIMATOR 3:')
        CALL SETCOLR(6)
        WRITE(6,375)TTL3
        WRITE(6,375)TTL3
        CALL SETCOLR(0)
      ENDIF
      IF(NEST.GE.4) THEN
         WRITE(6,378)
378   FORMAT(' ESTIMATOR 4:')
         CALL SETCOLR(3)
         WRITE(6,375)TTL4
         WRITE(6,375)TTL4
         CALL SETCOLR(0)
      ENDIF
      IF(NEST.GE.5) THEN
         WRITE(6,379)
379   FORMAT(' ESTIMATOR 5:')
         CALL SETCOLR(0)
         WRITE(6,375)TTL5
         WRITE(6,375)TTL5
      ENDIF
C THIS IS THE LAST LINE OF OUTPUT.  IF NO COLOR, PUT IN ON LAST SHEET.
C  END UP HERE IF DOING RESTART, OR NO COLOR OUTPUT.
340   WRITE(6,122) GV                                                   SMT
C --- SAVE RESTART INFORMATION                                          SMT
      IF(IRST .GT.  0) THEN                                             SMT
       REWIND 2                                                         SMT
C  THESE WRITE STATEMENTS ARE BROKEN UP SO THE DEFAULT UNFORMATTED
C  RECORD LENGTH WILL HOLD THE DATA.  THE USER IS THEN UNCONCERNED WITH
C  RUNTIME PARAMETERS TO CHANGE THE RECORD LENGTH (R/M FORTRAN)
C
      WRITE(2)N,M,NE,L,D,NSR,NEST,SEI,ISEED1,ISEED2,ISEED3,ISEED4,
     * ISEED5,VYMIN,VYMAX                                               SMT
       WRITE(2)((GPV1(I,J,1),I=1,8),J=1,11)
       WRITE(2)((GPV1(I,J,2),I=1,8),J=1,11)
       WRITE(2)GST1
       WRITE(2)GBA1,GBV1,GBS1,GVV1                                      SMT
       IF(NEST .GE.  2)  THEN
           WRITE(2)((GPV2(I,J,1),I=1,8),J=1,11)
           WRITE(2)((GPV2(I,J,2),I=1,8),J=1,11)
           WRITE(2)GST2
           WRITE(2)GBA2,GBV2,GBS2,GVV2                                  SMT
       ENDIF
       IF(NEST .GE.  3) THEN
           WRITE(2)((GPV3(I,J,1),I=1,8),J=1,11)
           WRITE(2)((GPV3(I,J,2),I=1,8),J=1,11)
           WRITE(2)GST3
           WRITE(2)GBA3,GBV3,GBS3,GVV3                                  SMT
       ENDIF
```

```
       IF(NEST .GE. 4) THEN
           WRITE(2)((GPV4(I,J,1),I=1,8),J=1,11)
           WRITE(2)((GPV4(I,J,2),I=1,8),J=1,11)
           WRITE(2)GST4
           WRITE(2)GBA4,GBV4,GBS4,GVV4                                  SMT
       ENDIF
       IF(NEST .GE. 5) THEN
           WRITE(2)((GPV5(I,J,1),I=1,8),J=1,11)
           WRITE(2)((GPV5(I,J,2),I=1,8),J=1,11)
           WRITE(2)GST5
           WRITE(2)GBA5,GBV5,GBS5,GVV5                                  SMT
       ENDIF
       WRITE(6,*) '*** THIS WAS A RESTART RUN. NSR START/END=',INSR,NSR SMT
      ENDIF                                                             SMT
C                                                                       SMT
C --- BIVARIATE HISTOGRAMS                                              SMT
C                                                                       SMT
      IF(IBIV .EQ. 1 .AND. NEST.GT.1) THEN                              SMT
        DO 120 K=1,L                                                    SMT
         IBP=IBPTR(K)                                                   SMT
         DO 201 I=1,IBP                                                 SMT
          TX(I)=BIV1(I,K)                                               SMT
          TY(I)=BIV2(I,K)                                               SMT
 201     CONTINUE                                                       SMT
C                                                                       SMT
         WRITE(6,123) 'BIVARIATE HISTOGRAM FOR ESTIMATORS 1 & 2.',      SMT
     *            ' SECTION SIZE=',NE(K)                                SMT
         CALL BIHSPC(TX,TY, IBP,Y)                                      SMT
         WRITE(6,*) '1 - ', TTL1                                        SMT
         WRITE(6,*) '2 - ', TTL2                                        SMT
C                                                                       SMT
         IF(NEST .LT. 3) GO TO 120                                      SMT
         DO 202 I=1,IBP                                                 SMT
          TX(I)=BIV1(I,K)                                               SMT
          TY(I)=BIV3(I,K)                                               SMT
 202     CONTINUE                                                       SMT
         WRITE(6,123) 'BIVARIATE HISTOGRAM FOR ESTIMATORS 1 & 3.',      SMT
     *            ' SECTION SIZE=',NE(K)                                SMT
         CALL BIHSPC(TX,TY, IBP,Y)                                      SMT
         WRITE(6,*) '1 - ', TTL1                                        SMT
         WRITE(6,*) '3 - ', TTL3                                        SMT
C                                                                       SMT
         DO 203 I=1,IBP                                                 SMT
          TX(I)=BIV2(I,K)                                               SMT
          TY(I)=BIV3(I,K)                                               SMT
 203     CONTINUE                                                       SMT
         WRITE(6,123) 'BIVARIATE HISTOGRAM FOR ESTIMATORS 2 & 3.',      SMT
     *            ' SECTION SIZE=',NE(K)                                SMT
         CALL BIHSPC(TX,TY, IBP,Y)                                      SMT
         WRITE(6,*) '2 - ', TTL2                                        SMT
         WRITE(6,*) '3 - ', TTL3                                        SMT
  120   CONTINUE                                                        SMT
      ENDIF                                                             SMT
C  JUMP TO HERE IF AN ERROR EXISTS IN THE INPUT DATA.  RETURN .
8001  CONTINUE
C                                                                       SMT
```

```
  123 FORMAT('1',A45, A13,I5)                                            SMT
  122 FORMAT(/' *** WIDEST Y VALUES FOUND:  YMIN=',G10.4,                SMT
     *        '  -  YMAX=',G10.4, '   *** SMTB ***')                     SMT
  103 FORMAT(' *** ERROR...L MUST BE AN INTEGER BETWEEN 2 AND 8. ***')   SMT
  104 FORMAT(' *** ERROR...M MUST BE AN INTEGER BETWEEN 1 AND 100. ***')SMT
  106 FORMAT(' *** ERROR...NEST MUST BE 3 OR LESS. ***')                 SMT
  107 FORMAT(' *** ERROR...N/NE(L) MUST BE 1 OR GREATER TO COMPUTE',     SMT
     +' STATISTICS.',/,'   INCREASE N OR DECREASE NE(L). ***')           SMT
  108 FORMAT(' *** ERROR...D MUST BE LESS THAN OR EQUAL TO 3. ***')      SMT
  110 FORMAT(' *** WARNING... NE ARRAY ELLEMENTS ARE NOT IN ORDER OF '/ SMT
     +'     INCREASING SIZE. IF NE(1) IS NOT SMALLEST ELEMENT, SCALING'/SMT
     +'     MAY CAUSE POINTS TO FALL OUTSIDE RANGE OF SCALE.')           SMT
  111 FORMAT(' *** ERROR...NSR MUST NOT BE LESS THAN 1  ***')            SMT
      END                                                                SMT
C                                                                        SMT
      SUBROUTINE PRST(GENEST,ISEED,N,M,NE,L,RG,UD,YMIN,YMAX,Y,           SMT
     * GV,TTL, NSR, IFIL, NPRT,MSE,VMSE, IPR,VMX, IBIV,IBPTR,BIV,        SMT
     * INSR,GPVAL,GSTAT,GBA,GBV,GBS,GVV,IBWPRT,NCPRT,NCOLRNDX,           SMT
     * COLRXIS,IEST,NEST,
     * COLRPLT,COLRSTAT,BA,BV,BS,V,DLH,VSCALE,IWIDTH,LABEL)
      PARAMETER (NBROWS=500)                                             SMT
C
C     REGRESSION ADJUSTED ESTIMATE                                       SMT
C     CALCULATES ESTIMATES FROM USER DATA USING "EST" FUNCTION           SMT
C     PLOTS BASIC OR RETRENCHED GRAPH ON LINE PRINTER                    SMT
C *   GENEST = SUBROUTINE GENERATE THE DATA AND PRODUCE ESTIMATE         SMT
C *   ISEED = SEED FOR GENEST                                            SMT
C     N = NUMBER OF VALUES IN EACH REPLICATION (M*N MUST BE <= 50000)    SMT
C     M = NO. OF REPLICATIONS (MUST BE <= 100)                           SMT
C     Y = USERS VECTOR WITH M CONSECUTIVE BATCHES OF N VALUES EACH       SMT
C     L = NO. OF SECTION SIZES (MUST BE BETWEEN 1 AND 8)                 SMT
C     NE = ARRAY WITH THE L SUBSAMPLE SIZES (MUST BE IN ASCENDING ORDER)SMT
C     UD = DEGREE OF THE REGRESSION (MUST BE <= 6 & <= L-1 )             SMT
C     GV=WIDEST Y VALUES. PRINTED ON LAST PLOT TO AID IN SETTING SCALE   SMT
C     NSR=NUMBER OF REPETITIONS OF THE PLOT (EXTRA SUMMARY PLOT WHEN >1)SMT
C     INSR=START SUPER-REPLICATION. NORMALLY 1 EXCEPT ON RESTART         SMT
C     IEST=NUMBER OF ESTIMATOR WE ARE CURRENTLY DEALING WITH (1,2,OR3)
C     IBWPRT=0-NO STD PRINTOUS - 1 STD PRINTOUTS
C     NCPRT=#OF SAMPLE SIZES FOR WHICH TO PRINT COLOR BOXPLOTS
C     NCOLRNDX=ARRAY INDX TO NE() FOR COLOR BOXPLOTS
C     COLRPLT=CHARACTER 3D ARRAY, 1 SHEET FOR EACH COLOR/ESTIMATOR
C     ICLRLOCX=2D ARRAY, LOCS OF BOXPLOTS IN EACH SHEET
C     COLRXIS=2DARRAY, CHAR. COLOR X AXIS
C
C
      real*8 ISEED
      INTEGER IBIV,IBPTR(8),NCOLRNDX(3),ICLRLOCX(5,3)
      CHARACTER*120 TTL                                                  SMT
      CHARACTER*1 PLOT(122,50), CBAR,BLK,DASH,DOT,SYM(11),XAXIS(122,2), SMT
     * COLRPLT(122,50,5),COLRXIS(122,2),TEMPLOT(122,50)
      CHARACTER*8 LABEL(7)                                               SMT
      DIMENSION  NE(8)                                                   SMT
      INTEGER NB(8),LOCX(8),D1,IWIDTH,UD,D,LT,DT,RG, DA,D1A              SMT
      REAL BIV(NBROWS,8)                                                 SMT
      REAL*4 PVAL(8,11), QVAL(11),VNORM(11), VMSE(8)                     SMT
      REAL*4 DLH(4),Y(1),GV(2), SVAR(8),COLRSTAT(8,7,5)                  SMT
```

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      REAL*4 RH(8,100),STAT(8,7),VT(8),  VMX(8,4),VXCNT(8,4)              SMT
      REAL*8 SUM2,SUM3,SUM4                                               SMT
      REAL*4 RA(8,7),RV(8,7),B(7,100),V(7),BA(7),BV(7),BS(7),RT(8),BT(8)SMT
      INTEGER ISCOL(5)                                                    SMT
      REAL*8 GSTAT(8,7,2), GPVAL(8,11,2)                                  SMT
      REAL*8 XSTAT(8,7,2), XPVAL(8,11,2)                                  SMT
      REAL*8 GBA(7,2), GBV(7,2), GBS(7,2), GVV(7,2)                       SMT
      REAL*8 XBA(7,2), XBV(7,2), XBS(7,2), XVV(7,2)                       SMT
      DATA B/700*0./                                                      SMT
      DATA BLK/' '/, DASH/'-'/, CBAR/'|'/, DOT/'.'/                       SMT
      DATA ISCOL/1,2,4,5,7/                                               SMT
      DATA SYM/'&','$','+','%','Q','M','Q','%','+','$','&'/, NSYM/11/    SMT
      DATA QVAL/.01, .025, .05, .10, .25, .5, .75, .90, .95,.975,.99/    SMT
      DATA VNORM/-2.33,-1.96,-1.645,-1.29,-.67, 0.,                       SMT
     *                 .67,1.29,1.645,1.96,2.33/                          SMT
      DATA ICSPCE/1/
      JCSPCE=(ICSPCE+3)*NEST
      DO 401 I=1,7
401   V(I)=0
      DLH(1)=1.
      DLH(2)=1.
      DLH(3)=122.
      DLH(4)=50.
      LABEL(1)='MEAN'
      LABEL(2)='STD'
      LABEL(3)='STD MEAN'
      LABEL(4)='SKEWNESS'
      LABEL(5)='KURTOSIS'
      LABEL(6)='M.S.E  '
      LABEL(7)='SER.COR.'
C --- CLEAR COUNTS MATRIX FOR PERCENTILE PLOT                             SMT
      DO 1 I=1,8                                                          SMT
       DO 2 J=1,4                                                         SMT
        VXCNT(I,J)=0.                                                     SMT
    2  CONTINUE                                                           SMT
    1 CONTINUE                                                            SMT
      D=MIN0(UD,L-1)                                                      SMT
C --- DEGREE OF REGRESSION FOR AVERAGES LIMITED TO 4                      SMT
      DA=MIN(4, D)                                                        SMT
      D1=D+1                                                              SMT
      D1A=DA+1                                                            SMT
      IX1=8                                                               SMT
      IX2=7                                                               SMT
      MN=M*N                                                              SMT
      IWIDTH= IFIX(DLH(3))                                                SMT
C     BUILD REGRESSION MATRICES FOR AVERAGES AND VARIANCES                SMT
      DO 84 K=1,L                                                         SMT
       DO 86 J=1,D1                                                       SMT
        T=FLOAT(NE(L))/FLOAT(NE(K))                                       SMT
        RA(K,J)=T**(J-1)                                                  SMT
        RV(K,J)=T**(FLOAT(J)*.5)                                          SMT
   86  CONTINUE                                                           SMT
   84 CONTINUE                                                            SMT
C     SET HORIZONTAL XMIN, XMAX                                           SMT
      XMIN=.7*NE(1)                                                       SMT
      XMAX=1.2*NE(L)                                                      SMT
```

```
C --- SCALING FACTORS FOR REGR., QUANTILE PLOT & NORM. QUANTILE (QVSC)  SMT
      VSCALE=(DLH(4)-DLH(2))/(YMAX-YMIN)                                SMT
      HSCALE=(DLH(3)-DLH(1))/(XMAX-XMIN)                                SMT
      QVSC  =(DLH(4)-DLH(2))/(3. - (-3.))                               SMT


      WRITE(5,*)'COMPUTING BOXPLOT LOCATIONS'


C     COMPUTE LOCATION OF BOXPLOTS ALONG X-AXIS                         SMT
      LAST=-1                                                           SMT
      DO 5 K=1,L                                                        SMT
       NB(K)=N/NE(K)                                                    SMT
       LOCX(K)=(NE(K)-XMIN)*HSCALE + DLH(1)+.5                          SMT
       IF(LOCX(K).LT. LAST+4) LOCX(K)=LAST+4                            SMT
       LAST=LOCX(K)                                                     SMT
    5 CONTINUE                                                          SMT
      IF(LOCX(L) .GT. 120) LOCX(L)=120                                  SMT
C COMPUTE LOCATION OF COLOR BOXPLOTS
C WE WILL DO THIS ONCE ONLY. CAN DO ALL ESTIMATOR LOCATIONS FOR COLOR
C USING THE FIRST ESTIMATOR POSITIONS AND OFFSETS.
      IF(IEST.NE.1) GOTO 7

C  ICSPCE IS SPACING FOR COLOR BOXPLOTS (1 ==> 1 SPACE BETWEEN,
C                                        0 ==> ADJACENT)

C FIRST COMPUTE POSITIONS OF SECOND ESTIMATOR (CENTER)

      DO 6 K=1,NCPRT
C
C        USE SAME HORIZONTAL SCALE AS DETERMINED ABOVE FOR BW BOXPLOTS
C        WE ARE COMPUTING THE LOCATIONS OF THE SECOND PLOT OF EACH
C        GROUP, AND WE WILL THEN OFFSET THE OTHERS BASED ON THE SPACING
C        COMPUTE TENTATIVE POSITION BASED ON SCALE, THEN CHECK BOUNDS
          KK=(NE(NCOLRNDX(K))-XMIN)*HSCALE+DLH(1)+.5

C  MINIMUM BOXPLOT POSITION IS 6+ICSPCE, FOR CENTER(SECOND) BOXPLOT
           IF(K.EQ.1)THEN
              IF(KK.GE.6+ICSPCE)ICLRLOCX(2,1)=KK
              IF(KK.LT.6+ICSPCE)ICLRLOCX(2,1)=6+ICSPCE
           ENDIF
C  CHECK THAT THERE IS AT LEAST (3+ICSPCE)*NEST NUMBER OF SPACES
C  BETWEEN EACH GROUP OF PLOTS
C  SO THAT ALL THE ESTIMATORS WILL FIT. THIS IS JCSPCE.

           IF(K.EQ.2) THEN
             IF(KK-ICLRLOCX(2,1).LT.JCSPCE)
     *          ICLRLOCX(2,2)=ICLRLOCX(2,1)+JCSPCE
             IF(KK-ICLRLOCX(2,1).GE.JCSPCE)ICLRLOCX(2,2)=KK
           ENDIF

           IF(K.EQ.3) THEN
              IF(KK-ICLRLOCX(2,2).LT.JCSPCE)
     *       ICLRLOCX(2,3)=ICLRLOCX(2,2)+JCSPCE
              IF(KK-ICLRLOCX(2,2).GE.JCSPCE)ICLRLOCX(2,3)=KK
           ENDIF
```

```
C  NOW OFFSET THE OTHER ESTIMATOR'S LOCATIONS TO EITHER SIDE OF THESE

             IF(NEST.GE.2)ICLRLOCX(1,K)=ICLRLOCX(2,K)-3-ICSPCE
             IF(NEST.GE.3)ICLRLOCX(3,K)=ICLRLOCX(2,K)+3+ICSPCE
             IF(NEST.GE.4)ICLRLOCX(4,K)=ICLRLOCX(2,K)+6+ICSPCE+ICSPCE
             IF(NEST.GE.5)ICLRLOCX(5,K)=ICLRLOCX(2,K)+9+3*ICSPCE
C
C  IF THE POSITION OF THE RIGHTMOST MEMBER OF THE GROUP OF BOXPLOTS
C  EXCEEDS THE RIGHT MARGIN, MOVE THEM ALL BACK AN AMOUNT OF THE
C  THE DISTANCE FROM THE MARGIN, THEN SET THE LAST ONE AT THE MARGIN
C
        IF(ICLRLOCX(NEST,K).GT.120)  THEN
           DO 8 I=1,NEST-1
8          ICLRLOCX(I,K)=ICLRLOCX(I,K)-(ICLRLOCX(5,K)-120)
           ICLRLOCX(NEST,K)=120
        ENDIF
6       CONTINUE
7       CONTINUE
C
C---- LABEL X-AXIS                                                     SMT
        DO 115 I=1,122                                                 SMT
          XAXIS(I,1)=DASH                                              SMT
          XAXIS(I,2)=BLK                                               SMT
          IF(IEST.EQ.1)COLRXIS(I,1)=DASH
          IF(IEST.EQ.1)COLRXIS(I,2)=BLK
    115 CONTINUE                                                       SMT
        DO 130 J=1,L                                                   SMT
         IBPTR(J)=0                                                    SMT
         XAXIS(LOCX(J),1)=CBAR                                         SMT
         IK = NE(J)                                                    SMT
         IX = LOCX(J)                                                  SMT
         CALL NUMPRT(IX,2,IK,XAXIS)                                    SMT
    130 CONTINUE                                                       SMT
C
C  THE X AXIS POSITION MARKER IS PLACED AT THE POSITION OF THE SECOND
C  BOXPLOT FOR COLOR COMBINED PLOTS
C
        IF(IEST.NE.2)GOTO 132
        DO 131 J=1,NCPRT
         COLRXIS(ICLRLOCX(IEST,J),1)=CBAR
         IK=NE(NCOLRNDX(J))
         IX=ICLRLOCX(IEST,J)
         CALL NUMPRT(IX,2,IK,COLRXIS)
131      CONTINUE
C       FIND NO. OF STATISTICS TO BE CALCULATED AT EACH SAMPLE SIZE    SMT
132     L1=0                                                           SMT
        L2=0                                                           SMT
        L3=0                                                           SMT
        DO 21 I=1,L                                                    SMT
         K1 = M*(N/NE(I))                                              SMT
         IF (K1.GE.2) L1=I                                             SMT
         IF (K1.GE.3) L2=I                                             SMT
         IF (K1.GE.4) L3=I                                             SMT
     21 CONTINUE                                                       SMT
        DT=MIN(D1, L1)                                                 SMT
C --- CLEAR GRANDTOTALS FOR SUMMARY PAGE EXCEPT WHEN RESTARTING        SMT
```

```
      IF(INSR .EQ. 1) CALL CLRGT(GBA,GBV,GBS,GVV,GSTAT,GPVAL)           SMT
C                                                                       SMT
C --- OUTER LOOP ADDED 07-16-84 TO DO MULTIPLE RUNS PER NSR PARAMETER   SMT
C                                                                       SMT
      DO 2000 ISREP=INSR,NSR                                            SMT
      WRITE(5,*) 'ESTIMATOR ',IEST,' SUPER-REP #',ISREP,' (OF ',NSR,' )'SMT
C      CLEAR PLOT ARRAY, BW ONLY                                        SMT
       CALL CLRPA(PLOT)                                                 SMT
C
C  INITIALIZE COUNTER FOR COLOR PLOT INDEX
C
       KCOUNT=1
       DO 80 K=1,L                                                      SMT
       WRITE(5,557)NE(K)
557    FORMAT(/'SUBSAMPLE SIZE: ',I5)
       NBK=NB(K)                                                        SMT
C      SECTION & COMPUTE ESTIMATORS FOR SIZE K                          SMT
       CALL SECEST(GENEST,ISEED,N,M,NE(K),Y,KP)                         SMT
C      AVERAGE ESTIMATES OF SIZE NE(K) FOR EACH OF M REPLICATIONS       SMT
       KP=0                                                             SMT
       DO 10 I=1,M                                                      SMT
        RH(K,I)=0.                                                      SMT
        DO 15 J=1,NBK                                                   SMT
         KP=KP+1                                                        SMT
         GV(1)=AMIN1(GV(1), Y(KP))                                      SMT
         GV(2)=AMAX1(GV(2), Y(KP))                                      SMT
         RH(K,I)=RH(K,I)+Y(KP)                                          SMT
   15   CONTINUE                                                        SMT
        RH(K,I)=RH(K,I)/FLOAT(NBK)                                      SMT
   10  CONTINUE                                                         SMT
C      COMPUTE MEAN AND MOMENT ESTIMATES                                SMT
       CALL CMPMOM(Y,KP,VMSE,MSE,K, STAT,SVAR)                          SMT
       DO 180 IM1=1,KP                                                  SMT
C ---   APPEND TO BIVARIATE MATRICES                                    SMT
        IF(IBPTR(K) .LT. NBROWS) THEN                                   SMT
             IBPTR(K)=IBPTR(K)+1                                        SMT
             BIV(IBPTR(K), K)=Y(IM1)                                    SMT
        ENDIF                                                           SMT
C ---   FREQUENCY COUNTS FOR PERCENTILE PLOTS                           SMT
        IF(IPR .EQ. 1) THEN                                             SMT
          DO 181 IK=1,4                                                 SMT
           IF(Y(IM1) .LE. VMX(K,IK)) VXCNT(K,IK)=VXCNT(K,IK) + 1        SMT
  181     CONTINUE                                                      SMT
        ENDIF                                                           SMT
  180  CONTINUE                                                         SMT
       CALL BOXPRT(Y,KP,LOCX(K),PLOT,RG,XMIN,YMIN,XMAX,YMAX,VSCALE)     SMT
C  FOR COLOR BOXPLOTS, IF K, THE INDEX THROUGH ALL THE SAMPLE SIZES,
C  IS EQUAL TO THE INDEX OF SAMPLE SIZES SPECIFIED FOR COLOR PLOTS,
C THEN PREPARE A BOXPLOT FOR THIS SAMPLE SIZE, AT LOCATION DETERMINED
C ABOVE.  MOVE SHEET OF 3D PLOT ARRAY TO TEMP (2D ARRAY), CALL BOXPRT, AND
C  THEN RESTORE SHEET OF 3D ARRAY.
C
C IF KCOUNT GT NCPRT THEN WE HAVE DONE ALL THE SAMPLE SIZES FOR THIS
C  ESTIMATOR
C
      IF(KCOUNT.GT.NCPRT)GOTO 184
```

```
      IF(K.NE.NCOLRNDX(KCOUNT))GOTO 184
C
C FOUND A SAMPLE SIZE, AND NOW WANT A COLOR BOXPLOT
C FIRST CLEAR TEMPLOT ARRAY, AND THEN PUT BOX PLOT IN IT
      CALL CLRPA(TEMPLOT)
C
C  FOR SUPERREPS, ONLY DO COLOR COMBINATION FOR FIRST SUPERREP
      IF(ISREP.GT.1)GOTO 182
      CALL BOXPRT(Y,KP,ICLRLOCX(IEST,KCOUNT),TEMPLOT,RG,XMIN,YMIN,XMAX,
     * YMAX,VSCALE)
      DO 183 KK=1,122
      DO 183 IKK=1,50
183   IF(COLRPLT(KK,IKK,IEST).EQ.BLK)COLRPLT(KK,IKK,IEST)=TEMPLOT(KK,IKK
     C)
182   CONTINUE
C
C
      KCOUNT=KCOUNT+1
C
C ---  STORE QUANTILES FOR 2ND PAGE PLOT                                 SMT
184    DO 18 IQ=1,NSYM                                                   SMT
        PVAL(K,IQ)=PCTL(Y,KP,QVAL(IQ),1)                                 SMT
   18  CONTINUE                                                          SMT
   80  CONTINUE                                                          SMT
C                                                                        SMT
C     IF D1.LT.2 THEN NO REGRESSIONS OR PLOTING CAN BE DONE              SMT
C                                                                        SMT
      IF(D1.LT.2) GO TO 113                                              SMT
      DO 92 K=1,M                                                        SMT
       DO 47 J=1,L                                                       SMT
        RT(J)=RH(J,K)                                                    SMT
  47   CONTINUE                                                          SMT
       CALL RREG(RA,RT,BT,L,D1A,IX1,IX2)                                 SMT
       B(1,K)=BT(1)                                                      SMT
       DO 23 KT=2,D1A                                                    SMT
        B(KT,K)=BT(KT)*NE(L)**(KT-1)                                     SMT
   23  CONTINUE                                                          SMT
   92 CONTINUE                                                           SMT
C                                                                        SMT
C     AVERAGE REGRESSION COEFF. OVER M REPLICATIONS & CALC. VARIANCE     SMT
      CALL COEFF(B,M,D1A, BA,BV,BS,V)                                    SMT
C                                                                        SMT
C     ESTABLISH REGRESSION LINE & ASYMPTOTE                              SMT
C                                                                        SMT
      DO 98 I=3,IWIDTH                                                   SMT
C      MAP I FROM DEVICE SPACE TO USER SPACE                             SMT
       UX=(I-DLH(1))/HSCALE + XMIN                                       SMT
C      COMPUTE THE Y VALUE FROM X AND THE REGRESSION COEFFICIENTS.       SMT
       UY=BA(1)                                                          SMT
       DO 99 J=1,DA                                                      SMT
        UY=UY+BA(J+1)/UX**J                                              SMT
   99  CONTINUE                                                          SMT
C                                                                        SMT
C      MAP THE Y VALUE FROM USER SPACE TO DEVICE SPACE                   SMT
       J=(UY-YMIN)*VSCALE + DLH(2) + .5                                  SMT
       IF(J .LT. 1 .OR. J .GT. 50) GO TO 98                              SMT
```

```
         IF(PLOT(I,J) .EQ. BLK) PLOT(I,J)=DOT                         SMT
C  COMMENTED OUT THE LINES TO PUT REGRESSION LINES IN COLOR PLOT
C        IF(ISREP.EQ.1.AND.COLRPLT(I,J,IEST).EQ.BLK)
C    C       COLRPLT(I,J,IEST)=DOT

   98 CONTINUE                                                         SMT
C                                                                      SMT
C     SCALE ASYMPTOTE, BETAO, AND PLOT ACROSS PLOT.                    SMT
      J=(BA(1)-YMIN)*VSCALE + DLH(2) + .5                              SMT
      IF(J .LT. 1 .OR. J .GT. 50) GO TO 117                            SMT
      DO 120 I=3,IWIDTH                                                SMT
       IF(PLOT(I,J) .EQ. BLK) PLOT(I,J)=DASH                           SMT
       IF(ISREP.EQ.1.AND.COLRPLT(I,J,IEST).EQ.BLK)
     C    COLRPLT(I,J,IEST)=DASH
  120 CONTINUE                                                         SMT
C                                                                      SMT
C     REGRESSION ON VARIANCES FROM EACH SEGMENT WITH A VARIANCE.       SMT
C                                                                      SMT
  117 CONTINUE                                                         SMT
      IF(DT.LT.2) GO TO 113                                            SMT
      DO 48 J=1,L                                                      SMT
C ---    MODIFIED FOR MEAN SQUARE ERROR     08-22-86                   SMT
         VT(J)=SVAR(J)*(NE(J)**0.5)                                    SMT
   48 CONTINUE                                                         SMT
      CALL RREG(RV,VT,V,L1,DT,IX1,IX2)                                 SMT
      DO 77 I=1,DT                                                     SMT
         V(I) = V(I)*NE(L)**(FLOAT(I)/2.)                              SMT
   77 CONTINUE                                                         SMT
C --- ACCUMULATE FOR SUMMARY STATISTICS OVER SUPER-REPLICATIONS        SMT
  113 CONTINUE                                                         SMT
      CALL CMPSUM(BA,BV,BS,V,DT,D1,L,STAT,PVAL,NSYM,                   SMT
     *                            GBA,GBV,GBS,GVV,GSTAT,GPVAL)         SMT
C
C  UPDATE COLOR STATISTIC MATRIX WITH STATS FROM THIS ESTIMATOR
C
       IF(ISREP.EQ.1) THEN
          DO 450 I=1,8
          DO 450 J=1,7
450       COLRSTAT(I,J,IEST)=STAT(I,J)
       ENDIF
C
C --- SAVE STATISTICS FOR THIS SUPER REPLICATION                       SMT
C - USE FORMATTED FILE FOR EASE IN FURTHER PROCESSING.  LABEL THE DATA
C - IN THE FILE
C
      IF(IFIL.EQ.1) THEN
         WRITE(1,458)TTL
458   FORMAT(A120)
         WRITE(1,456)ISREP
456   FORMAT('SUPER-REP #:    ',I10)
         WRITE(1,452)
452   FORMAT('SAMP SIZE MEAN      STD       SKEWNESS  KURTOSIS  SER CORR
     *')
         DO 453 I=1,L
453      WRITE(1,451)NE(I),(STAT(I,ISCOL(J)),J=1,5)                    SMT
         WRITE(1,454)
```

```
 454   FORMAT(/'QUANT. ROWS: 1 EA FOR .01 .025 .05 .1 .25 .5 .75 .9 .95
      *.975 .99')
          DO 457 J=1,11
 457      WRITE(1,455)(PVAL(I,J),I=1,8)
 455   FORMAT(8G10.4)
 451   FORMAT(I10,(5G10.4))
       ENDIF
 C                                                                    SMT
 C --- PRINT ONLY FIRST 3 PLOTS WHEN NPRT=0.  ONLY 1 PLOT OTHERWISE    SMT
       IF(NPRT.NE.1 .AND. ISREP .GT. 3 ) GO TO 2000                   SMT
       IF(NPRT.EQ.1 .AND. ISREP .GT. 1 ) GO TO 2000                   SMT
 C IF BWPRT FLAG IS NOT SET, DONT PRINT THESE STANDARD PRINTOUTS
       IF(IBWPRT.EQ.0)GOTO 2000
 C
 C                                                                    SMT
 C     PLOT   **************                                          SMT
 C                                                                    SMT
       WRITE(6,102)                                                   SMT
       WRITE(6,161) N,M,D                                             SMT
       WRITE(6,101)                                                   SMT
       DO 90 K=50,1,-1                                                SMT
        IF(MOD(K,5).NE.0) GO TO 85                                    SMT
        YLABEL=(K-DLH(2))/VSCALE + YMIN                               SMT
        WRITE(6,103)  YLABEL,(PLOT(I,K),I=1,IWIDTH)                   SMT
        GO TO 90                                                      SMT
   85   CONTINUE                                                      SMT
        WRITE(6,100)  (PLOT(I,K),I=1,IWIDTH)                          SMT
   90 CONTINUE                                                        SMT
      WRITE(6,106) (XAXIS(I,1),I=1,IWIDTH)                            SMT
      WRITE(6,104) (XAXIS(I,2),I=1,IWIDTH)                            SMT
      WRITE(6,156)                                                    SMT
      WRITE(6,146) (NE(I),I=1,L)                                      SMT
      WRITE(6,157) LABEL(1), (STAT(K,1),K=1,L)                        SMT
   11 WRITE(6,157) LABEL(2), (STAT(K,2),K=1,L1)                       SMT
      WRITE(6,157) LABEL(3), (STAT(K,3),K=1,L1)                       SMT
      IF(MSE .EQ. 1) WRITE(6,157) LABEL(6), (STAT(K,6),K=1,L1)        SMT
   12 WRITE(6,158) LABEL(4), (STAT(K,4),K=1,L2)                       SMT
   13 WRITE(6,158) LABEL(5), (STAT(K,5),K=1,L3)                       SMT
      IF(D1.LT.2) GO TO 444                                           SMT
      WRITE(6,151) (BA(I),I=1,D1A)                                    SMT
   14 IF(M.LT.2)GO TO 444                                             SMT
      WRITE(6,152) (BV(I),I=1,D1A)                                    SMT
      WRITE(6,153) (BS(I),I=1,D1A)                                    SMT
  444 IF(DT.LE.1) GO TO 999                                           SMT
      WRITE(6,159) (V(I),I=1,DT)                                      SMT
  999 WRITE(6,162)                                                    SMT
      WRITE(6,108) TTL                                                SMT
      WRITE(6,201) YMIN,YMAX                                          SMT
 C --- PREPARE 2ND PAGE PLOT                                           SMT
 C --- CLEAR PLOT ARRAY                                               SMT
      CALL CLRPA(PLOT)                                                SMT
 C --- ENTER QUANTILES                                                SMT
      DO 220 K=1,L                                                    SMT
       DO 240 IS=1,NSYM                                               SMT
        IQ=(PVAL(K,IS)-YMIN)*VSCALE + 1                               SMT
        IF(IQ .GE. 1 .AND. IQ .LE. 50) PLOT(LOCX(K), IQ)=SYM(IS)      SMT
```

```
  240   CONTINUE                                                          SMT
  220  CONTINUE                                                           SMT
C ---  PRINT 2ND PAGE                                                     SMT
       WRITE(6,202)                                                       SMT
       WRITE(6,161) N,M,D                                                 SMT
       WRITE(6,101)                                                       SMT
       DO 191 J=1,50                                                      SMT
        K=51-J                                                            SMT
        IF(MOD(K,5).NE.0) GO TO 185                                       SMT
        YLABEL=(K-DLH(2))/VSCALE + YMIN                                   SMT
        WRITE(6,103)  YLABEL,(PLOT(I,K),I=1,IWIDTH)                       SMT
        GO TO 191                                                         SMT
   185  CONTINUE                                                          SMT
        WRITE(6,100)  (PLOT(I,K),I=1,IWIDTH)                              SMT
   191 CONTINUE                                                           SMT
       WRITE(6,106) (XAXIS(I,1),I=1,IWIDTH)                               SMT
       WRITE(6,104) (XAXIS(I,2),I=1,IWIDTH)                               SMT
C ---  PRINT QUANTILE AMOUNTS                                             SMT
       WRITE(6,156)                                                       SMT
       WRITE(6,147) (NE(I),I=1,L)                                         SMT
       DO 199 I=1,NSYM                                                    SMT
        WRITE(6,167)  QVAL(I), (PVAL(K,I),K=1,L)                          SMT
   199 CONTINUE                                                           SMT
       WRITE(6,158) LABEL(7), (STAT(K,7),K=1,L1)                          SMT
       WRITE(6,108) TTL                                                   SMT
C ---  WHEN NPRT=1 SUPPRESS NORMALIZED PLOTS                              SMT
       IF(NPRT .EQ. 1) GO TO 2000                                         SMT
C ---                                                                     SMT
C ---  PREPARE 3ND PAGE PLOT                                              SMT
C ---  CLEAR PLOT ARRAY                                                   SMT
       CALL CLRPA(PLOT)                                                   SMT
C ---  NORMALIZE QUANTILES                                                SMT
       DO 1220 K=1,L                                                      SMT
        DO 1240 IS=1,NSYM                                                 SMT
         IF(STAT(K,2).GT.0) PVAL(K,IS)=(PVAL(K,IS)-STAT(K,1))/STAT(K,2)   SMT
         IQ=(PVAL(K,IS)+3.)*QVSC + 1                                      SMT
         IF(IQ .GE. 1 .AND. IQ .LE. 50) PLOT(LOCX(K), IQ)=SYM(IS)         SMT
 1240   CONTINUE                                                          SMT
 1220  CONTINUE                                                           SMT
C ---  ADD RIGHT SIDE NORMAL POINT MARKS                                  SMT
       DO 1260 IS=1,NSYM                                                  SMT
        IQ=(VNORM(IS)+3.)*QVSC + 1                                        SMT
        IF(IQ .GE. 1 .AND. IQ .LE. 50) PLOT(122, IQ)=SYM(IS)              SMT
 1260  CONTINUE                                                           SMT
C ---  PRINT 3RD PAGE                                                     SMT
       WRITE(6,203)                                                       SMT
       WRITE(6,161) N,M,D                                                 SMT
       WRITE(6,101)                                                       SMT
       DO 1191 J=1,50                                                     SMT
        K=51-J                                                            SMT
        IF(MOD(K,5).NE.0) GO TO 1185                                      SMT
        YLABEL=(K-DLH(2))/QVSC - 3.                                       SMT
        WRITE(6,103)  YLABEL,(PLOT(I,K),I=1,IWIDTH)                       SMT
        GO TO 1191                                                        SMT
  1185  CONTINUE                                                          SMT
        WRITE(6,100)  (PLOT(I,K),I=1,IWIDTH)                              SMT
```

```
 1191 CONTINUE                                                        SMT
      WRITE(6,106) (XAXIS(I,1),I=1,IWIDTH)                            SMT
      WRITE(6,104) (XAXIS(I,2),I=1,IWIDTH)                            SMT
C --- PRINT NORMALIZED QUANTILE AMOUNTS                                SMT
      WRITE(6,156)                                                    SMT
      WRITE(6,147) (NE(I),I=1,L)                                      SMT
      DO 1199 I=1,NSYM                                                SMT
       WRITE(6,167)  QVAL(I), (PVAL(K,I),K=1,L)                       SMT
 1199 CONTINUE                                                        SMT
      WRITE(6,108) TTL                                                SMT
C  ********************************                                   SMT
 2000 CONTINUE                                                        SMT
      IF(NSR .EQ. 1) RETURN                                           SMT
      DO 360 I=1,L                                                    SMT
       DO 370 J=1,7                                                   SMT
        XSTAT(I,J,1)=GSTAT(I,J,1)/NSR                                 SMT
        XSTAT(I,J,2)=(GSTAT(I,J,2)-NSR*XSTAT(I,J,1)**2)/              SMT
     *          ((NSR-1)*NSR)                                         SMT
        IF(XSTAT(I,J,2) .LT. 0) XSTAT(I,J,2)=0                        SMT
        XSTAT(I,J,2)=SQRT(XSTAT(I,J,2))                               SMT
  370  CONTINUE                                                       SMT
  360 CONTINUE                                                        SMT
      DO 375 K=1,L                                                    SMT
       DO 378 IS=1,NSYM                                               SMT
        XPVAL(K,IS,1)=GPVAL(K,IS,1)/NSR                               SMT
        XPVAL(K,IS,2)=(GPVAL(K,IS,2)-NSR*XPVAL(K,IS,1)**2)/           SMT
     *          ((NSR-1)*NSR)                                         SMT
        IF(XPVAL(K,IS,2) .GT. 0.) XPVAL(K,IS,2)=SQRT(XPVAL(K,IS,2))   SMT
        IF(XPVAL(K,IS,2) .LT. 0.) XPVAL(K,IS,2)=0.                    SMT
 378   CONTINUE                                                       SMT
 375  CONTINUE                                                        SMT
C ---       $$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$                    SMT
C --- PREPARE SUPER REPLICATIONS QUANTILE PLOT                        SMT
C --- CLEAR PLOT ARRAY                                                SMT
      CALL CLRPA(PLOT)                                                SMT
C --- ENTER QUANTILES                                                 SMT
      DO 2220 K=1,L                                                   SMT
       DO 2240 IS=1,NSYM                                              SMT
        IQ=(XPVAL(K,IS,1)-YMIN)*VSCALE + 1                            SMT
        IF(IQ .GE. 1 .AND. IQ .LE. 50) PLOT(LOCX(K), IQ)=SYM(IS)      SMT
2240   CONTINUE                                                       SMT
2220  CONTINUE                                                        SMT
C --- PRINT SUMMARY QUANTILE PLOT                                     SMT
      WRITE(6,173)                                                    SMT
      WRITE(6,161) N,M,D                                              SMT
      WRITE(6,101)                                                    SMT
      DO 2191 J=1,50                                                  SMT
       K=51-J                                                         SMT
       IF(MOD(K,5).NE.0) GO TO 2185                                   SMT
       YLABEL=(K-DLH(2))/VSCALE + YMIN                                SMT
       WRITE(6,103)  YLABEL,(PLOT(I,K),I=1,IWIDTH)                    SMT
       GO TO 2191                                                     SMT
 2185  CONTINUE                                                       SMT
       WRITE(6,100)  (PLOT(I,K),I=1,IWIDTH)                           SMT
 2191 CONTINUE                                                        SMT
      WRITE(6,106) (XAXIS(I,1),I=1,IWIDTH)                            SMT
```

```
      WRITE(6,104) (XAXIS(I,2),I=1,IWIDTH)                                   SMT
C --- PRINT QUANTILE AMOUNTS                                                 SMT
      WRITE(6,156)                                                           SMT
      WRITE(6,147) (NE(I),I=1,L)                                             SMT
      DO 2199 I=1,NSYM                                                       SMT
       WRITE(6,167)  QVAL(I), (XPVAL(K,I,1),K=1,L)                           SMT
 2199 CONTINUE                                                               SMT
      WRITE(6,158) LABEL(7), (XSTAT(K,7,1),K=1,L1)                           SMT
      WRITE(6,108) TTL                                                       SMT
C ---           $$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$                       SMT
C --- PRINT SUMMARY STATISTICS PAGE                                          SMT
      WRITE(6,171) NSR                                                       SMT
      WRITE(6,172) (NE(I),I=1,L)                                             SMT
      WRITE(6,262)                                                           SMT
      WRITE(6,157) LABEL(1), (XSTAT(K,1,1), K=1,L)                           SMT
      WRITE(6,157) BLK,      (XSTAT(K,1,2), K=1,L)                           SMT
      WRITE(6,262)                                                           SMT
      WRITE(6,157) LABEL(2), (XSTAT(K,2,1), K=1,L1)                          SMT
      WRITE(6,157) BLK,      (XSTAT(K,2,2), K=1,L1)                          SMT
      IF(MSE .NE. 1) GO TO 443                                               SMT
      WRITE(6,262)                                                           SMT
      WRITE(6,157) LABEL(6), (XSTAT(K,6,1), K=1,L1)                          SMT
      WRITE(6,157) BLK,      (XSTAT(K,6,2), K=1,L1)                          SMT
  443 CONTINUE                                                               SMT
      WRITE(6,262)                                                           SMT
      WRITE(6,157) LABEL(4), (XSTAT(K,4,1), K=1,L2)                          SMT
      WRITE(6,157) BLK,      (XSTAT(K,4,2), K=1,L2)                          SMT
      WRITE(6,262)                                                           SMT
      WRITE(6,157) LABEL(5), (XSTAT(K,5,1), K=1,L3)                          SMT
      WRITE(6,157) BLK,      (XSTAT(K,5,2), K=1,L3)                          SMT
      WRITE(6,262)                                                           SMT
      WRITE(6,157) LABEL(7), (XSTAT(K,7,1), K=1,L1)                          SMT
      WRITE(6,157) BLK,      (XSTAT(K,7,2), K=1,L1)                          SMT
C --- PRINT QUANTILE STATISTICS                                              SMT
      WRITE(6,169)                                                           SMT
      DO 445 I=1,NSYM                                                        SMT
       WRITE(6,262)                                                          SMT
       WRITE(6,167)  QVAL(I), (XPVAL(K,I,1),K=1,L)                           SMT
       WRITE(6,168)  BLK,     (XPVAL(K,I,2),K=1,L)                           SMT
  445 CONTINUE                                                               SMT
C                                                                            SMT
      DO 350 I=1,6                                                           SMT
       XBA(I,1)=GBA(I,1)/NSR                                                 SMT
       XBA(I,2)=SQRT((GBA(I,2)-NSR*XBA(I,1)**2)/                             SMT
     *          ((NSR-1)*NSR))                                               SMT
       XBV(I,1)=GBV(I,1)/NSR                                                 SMT
C***   XBV(I,2)=SQRT((GBV(I,2)-NSR*XBV(I,1)**2)/                             SMT
C*** *          ((NSR-1)*NSR))                                               SMT
       XBS(I,1)=GBS(I,1)/NSR                                                 SMT
       XBS(I,2)=SQRT((GBS(I,2)-NSR*XBS(I,1)**2)/                             SMT
     *          ((NSR-1)*NSR))                                               SMT
       XVV(I,1)=GVV(I,1)/NSR                                                 SMT
       XVV(I,2)=SQRT((GVV(I,2)-NSR*XVV(I,1)**2)/                             SMT
     *          ((NSR-1)*NSR))                                               SMT
  350 CONTINUE                                                               SMT
C                                                                            SMT
```

```
      IF(D1 .LT. 2) GO TO 410                                           SMT
      WRITE(6,262)                                                      SMT
      WRITE(6,151) (XBA(I,1),I=1,D1A)                                   SMT
      WRITE(6,176) (XBA(I,2),I=1,D1A)                                   SMT
      IF(M  .LT. 2) GO TO 410                                           SMT
      WRITE(6,262)                                                      SMT
      WRITE(6,153) (XBS(I,1),I=1,D1)                                    SMT
      WRITE(6,176) (XBS(I,2),I=1,D1)                                    SMT
  410 CONTINUE                                                          SMT
      IF(DT .LT. 2) GO TO 420                                           SMT
      WRITE(6,262)                                                      SMT
      WRITE(6,159) (XVV(I,1),I=1,DT)                                    SMT
      WRITE(6,176) (XVV(I,2),I=1,DT)                                    SMT
  420 CONTINUE                                                          SMT
C                                                                       SMT
      WRITE(6,262)                                                      SMT
      WRITE(6,108) TTL                                                  SMT
C --- PERCENTILE PLOT                                                   SMT
      IF(IPR .EQ. 1) CALL PRPLOT(M,N,NSR,L,NE,D,VMX,VXCNT,PLOT,TTL,     SMT
     *    XAXIS,LOCX)                                                   SMT
C                                                                       SMT
  100 FORMAT(9X,'|',122A1,'|')                                          SMT
  101 FORMAT(9X,'+',122('-'),'+')                                       SMT
  102 FORMAT('1')                                                       SMT
  103 FORMAT(' ',G8.2,'-',122A1,'|')                                    SMT
  104 FORMAT(10X,122A1)                                                 SMT
  106 FORMAT(9X,'+',122A1,'+')                                          SMT
  107 FORMAT(10G13.8)                                                   SMT
  108 FORMAT(/,' ESTIMATOR: ',A120)                                     SMT
  151 FORMAT(/,' MEAN OF REGRESSION ON AVERAGES',6G17.4)                SMT
  176 FORMAT(  '                               ',6G17.4)                SMT
  152 FORMAT(' VARIANCE OF REGRESSION',8X,6G17.4)                       SMT
  153 FORMAT(' STD DEV OF REGRESSION',9X,6G17.4)                        SMT
  156 FORMAT(/' SUBSAMPLE')                                             SMT
  146 FORMAT( ' SIZE     ',8(I8,6X),/)                                  SMT
  147 FORMAT( ' SIZE     ',8(I8,6X),/' QUANTILE',/)                     SMT
  167 FORMAT(1X,F5.3,6X,8G14.4)                                         SMT
  168 FORMAT(1X,A8,3X, 8G14.4)                                          SMT
  169 FORMAT(/,' QUANTILES')                                            SMT
  157 FORMAT(1X,A8,3X,8G14.4)                                           SMT
  158 FORMAT(1X,A8,8F14.4)                                              SMT
  159 FORMAT(/,' REGRESSION ON VARIANCE',7X,6G17.4)                     SMT
  161 FORMAT(9X,'SAMPLE SIZE (N):',I5,20X,'NO. OF REPLICATIONS (M):',   SMT
     +I5,18X,'DEGREE OF REGRESSION (D):',I3)                            SMT
  162 FORMAT(/)                                                         SMT
  171 FORMAT('1',30X,'SUMMARY STATISTICS (MEAN/STD)',                   SMT
     *       I8,' SUPER-REPLICATIONS')                                  SMT
  172 FORMAT(/,' SUBSAMPLE',/,' SIZE         ',8(I8,6X),/)              SMT
  173 FORMAT('1',30X,'SUMMARY QUANTILE PLOT FOR ALL SUPER-REPLICATIONS')SMT
  262 FORMAT(' ')                                                       SMT
  201 FORMAT(1X,'VERTICAL SCALE:  YMIN =',F10.4,/,18X,'YMAX =',F10.4/)  SMT
  202 FORMAT('1',40X,'QUANTILE PLOT')                                   SMT
  203 FORMAT('1',40X,'NORMALIZED QUANTILE PLOT')                        SMT
      END
```

```
      SUBROUTINE SETCOLR(I)
      WRITE(6,1)I
1     FORMAT('+?r',I1)
      RETURN
      END

      SUBROUTINE CLRCOLR(COLRPLT)                                       SMT
      CHARACTER COLRPLT(122,50,5)*1                                     SMT
      DO 1 I=1,122
      DO 1 J=1,50
      DO 1 K=1,5
1     COLRPLT(I,J,K) = ' '
      RETURN
      END

C---                                                                    SMT
      SUBROUTINE CMPSUM(BA,BV,BS,V,DT,D1,L,STAT,PVAL,NSYM,              SMT
     *                            GBA,GBV,GBS,GVV,GSTAT,GPVAL)          SMT
C --- ACCUMULATE SUMMARY STATISTICS: GBA,GBV,GBS,GVV,GSTAT,GPVAL        SMT
      REAL*4 STAT(8,7),PVAL(8,11)                                       SMT
      REAL*4 V(7),BA(7),BV(7),BS(7)                                     SMT
      REAL*8 GSTAT(8,7,2), GPVAL(8,11,2)                                SMT
      REAL*8 GBA(7,2), GBV(7,2), GBS(7,2), GVV(7,2)                     SMT
      INTEGER L,DT,D1,NSYM                                              SMT
      IF(D1.LT.2 .OR. DT.LT.2) GO TO 113                                SMT
      DO 300 I=1,6                                                      SMT
       GBA(I,1)=GBA(I,1) + BA(I)                                        SMT
       GBA(I,2)=GBA(I,2) + BA(I)**2                                     SMT
       GBV(I,1)=GBV(I,1) + BV(I)                                        SMT
       GBV(I,2)=GBV(I,2) + BV(I)**2                                     SMT
       GBS(I,1)=GBS(I,1) + BS(I)                                        SMT
       GBS(I,2)=GBS(I,2) + BS(I)**2                                     SMT
       IF(I .LE. DT) THEN                                               SMT
                    GVV(I,1)=GVV(I,1) +  V(I)                           SMT
                    GVV(I,2)=GVV(I,2) +  V(I)**2                        SMT
       ENDIF                                                            SMT
  300 CONTINUE                                                          SMT
  113 CONTINUE                                                          SMT
      DO 310 I=1,L                                                      SMT
       DO 320 J=1,7                                                     SMT
        GSTAT(I,J,1)=GSTAT(I,J,1) + STAT(I,J)                           SMT
        GSTAT(I,J,2)=GSTAT(I,J,2) + STAT(I,J)**2                        SMT
  320  CONTINUE                                                         SMT
  310 CONTINUE                                                          SMT
C --- COMPUTE SUMMARY FOR QUANTILES                                     SMT
      DO 1221 K=1,L                                                     SMT
       DO 1241 IS=1,NSYM                                                SMT
        GPVAL(K,IS,1)=GPVAL(K,IS,1) + PVAL(K,IS)                        SMT
        GPVAL(K,IS,2)=GPVAL(K,IS,2) + PVAL(K,IS)**2                     SMT
 1241  CONTINUE                                                         SMT
 1221 CONTINUE                                                          SMT
      END                                                               SMT

C ---                                                                   SMT
      SUBROUTINE COEFF(B,M,D1A, BA,BV,BS,V)                             SMT
C --- COMPUTE REGRESSION COEFFICIENTS, VARS, ETC: BA,BV,BS,V            SMT
```

```
      REAL*4 V(7),BA(7),BV(7),BS(7), B(7,100)                         SMT
      INTEGER D1A                                                     SMT
      AM=M                                                            SMT
      DO 93 I=1,7                                                     SMT
       BA(I)=0                                                        SMT
       BV(I)=0                                                        SMT
       BS(I)=0                                                        SMT
       V(I)=0                                                         SMT
   93 CONTINUE                                                        SMT
      DO 94 I=1,D1A                                                   SMT
       DO 95 J=1,M                                                    SMT
        BA(I)=BA(I)+B(I,J)                                            SMT
        BV(I)=BV(I)+B(I,J)**2                                         SMT
   95  CONTINUE                                                       SMT
       BA(I)=BA(I)/AM                                                 SMT
       IF(M.EQ.1)GO TO 94                                             SMT
       BV(I)=(BV(I)-AM*BA(I)**2)/(AM*(AM-1.))                         SMT
       IF(BV(I) .LT. 0.) BV(I)=0                                      SMT
       BS(I)=BV(I)**.5                                                SMT
   94 CONTINUE                                                        SMT
      END                                                             SMT

C ---                                                                 SMT
      SUBROUTINE CMPMOM(Y,KP,VMSE,MSE,K, STAT,SVAR)                   SMT
C --- COMPUTE MOMENT ESTIMATES IN STAT(K,*) AND SVAR(K)               SMT
      REAL Y(1), VMSE(8), STAT(8,7), SVAR(8)                          SMT
      REAL*8 XMEAN,DEV,SUM2,SUM3,SUM4                                 SMT
      XMEAN=0.                                                        SMT
      DO 180 IM1=1,KP                                                 SMT
       XMEAN=XMEAN+Y(IM1)                                             SMT
  180 CONTINUE                                                        SMT
      XMEAN=XMEAN/FLOAT(KP)                                           SMT
      SUM2 = 0.0D0                                                    SMT
      SUM3 = 0.0D0                                                    SMT
      SUM4 = 0.0D0                                                    SMT
      DO 190 IP1=1,KP                                                 SMT
       DEV = Y(IP1) - XMEAN                                           SMT
       SUM2 = SUM2 + DEV * DEV                                        SMT
       SUM3 = SUM3 + DEV ** 3                                         SMT
       SUM4 = SUM4 + DEV ** 4                                         SMT
  190 CONTINUE                                                        SMT
C                                                                     SMT
C     CHECK TO INSURE SAMPLE SIZE IS LARGE ENOUGH FOR EACH MOMENT     SMT
C     CLEAR STAT                                                      SMT
      DO 419 IP1=1,7                                                  SMT
       STAT(K,IP1)=0.                                                 SMT
  419 CONTINUE                                                        SMT
      IF (KP.LT.2) GO TO 9                                            SMT
      VAR = SUM2 / (KP - 1.0)                                         SMT
      STDV = SQRT(VAR)                                                SMT
C --- SERIAL CORRELATION                                              SMT
      SERCOR=0.                                                       SMT
      DO 421 IP1=2,KP                                                 SMT
       SERCOR=SERCOR + (Y(IP1-1)-XMEAN)*(Y(IP1)-XMEAN)                SMT
  421 CONTINUE                                                        SMT
      SERCOR=SERCOR/((KP-1)*VAR)                                      SMT
```

```
    7 IF (KP.LT.3) GO TO 9                                          SMT
      SUM3 = SUM3 * KP /( (KP-1.) * (KP-2.))                        SMT
      SKEW = SUM3  / STDV ** 3                                      SMT
    8 IF (KP.LT.4) GO TO 9                                          SMT
      SUM4 = SUM4*((KP-2.)*KP+3.)/((KP-1.)*(KP-2.)*(KP-3.))         SMT
      SUM4 = SUM4 - VAR*VAR*3.*(KP-1.)*(KP+KP-3.)/(KP*(KP-2.)*(KP-3)) SMT
      CKURT = SUM4 / (VAR * VAR) - 3.                               SMT
    9 STAT(K,1)=XMEAN                                               SMT
      STAT(K,2)=STDV                                                SMT
      STAT(K,3)=STDV/SQRT(FLOAT(KP))                                SMT
      STAT(K,4)=SKEW                                                SMT
      STAT(K,5)=CKURT                                               SMT
C --- MEAN SQUARE ERROR     05-25-86                                SMT
      IF(MSE .EQ. 1) THEN                                           SMT
                 STAT(K,6)=SQRT(VAR + (XMEAN-VMSE(K))**2)           SMT
      ELSE                                                          SMT
                 STAT(K,6)=0.                                       SMT
      ENDIF                                                         SMT
      SVAR(K)=VAR                                                   SMT
      STAT(K,7)=SERCOR                                              SMT
      END                                                           SMT

C---                                                                SMT
      SUBROUTINE PRPLOT(M,N,NSR,L,NE,D,VMX,VXCNT,PLOT,TTL,XAXIS,LOCX) SMT
C --- PRINT PERCENTILE PLOTS                        09-09-86        SMT
      REAL*4 VMX(8,4),VXCNT(8,4), VXSTD(8,4)                        SMT
      REAL*4 ULH(4),DLH(4)                                          SMT
      INTEGER NE(8), LOCX(8)                                        SMT
      CHARACTER*120 TTL                                             SMT
      CHARACTER*1 SYM(4), PLOT(122,50), XAXIS(122,2)                SMT
      DATA DLH/1.,1.,122.,50./, SYM/'*','#','@','&'/                SMT
C     CLEAR PLOT ARRAY                                              SMT
      CALL CLRPA(PLOT)                                              SMT
C     SET HORIZONTAL XMIN, XMAX                                     SMT
      ULH(1)=.7*NE(1)                                               SMT
      ULH(3)=1.2*NE(L)                                              SMT
      ULH(2)=0.                                                     SMT
      ULH(4)=1.                                                     SMT
      XMIN=ULH(1)                                                   SMT
      XMAX=ULH(3)                                                   SMT
      YMIN=ULH(2)                                                   SMT
      YMAX=ULH(4)                                                   SMT
      IWIDTH=DLH(3)                                                 SMT
C --- SCALING FACTORS                                               SMT
      VSCALE=(DLH(4)-DLH(2))/(ULH(4)-ULH(2))                        SMT
C                                                                   SMT
C --- CONVERT FREQUENCY COUNTS TO PROBS. FOR PLOT & VARS FOR PRINTING SMT
      DO 10 I=1,8                                                   SMT
       NBK=N/NE(I)                                                  SMT
       AN=NSR*M*NBK                                                 SMT
       DO 20 J=1,4                                                  SMT
        P=VXCNT(I,J)/AN                                             SMT
        VXCNT(I,J)=P                                                SMT
        IQ=(P - ULH(2))*VSCALE + 1                                  SMT
        IQ=MIN(IQ, 50)                                              SMT
        IK=LOCX(I)+J-2                                              SMT
```

```
          DO 21 K=1,IQ                                                  SMT
           PLOT(IK, K)=SYM(J)                                           SMT
   21     CONTINUE                                                      SMT
          VXSTD(I,J)=SQRT(P*(1-P)/AN)                                   SMT
   20    CONTINUE                                                       SMT
   10 CONTINUE                                                          SMT
C --- PRINT PERCENTILE PLOT                                              SMT
      WRITE(6,173) NSR                                                  SMT
      WRITE(6,161) N,M,D                                                SMT
      WRITE(6,101)                                                      SMT
      DO 2191 J=1,50                                                    SMT
       K=51-J                                                           SMT
       IF(MOD(K,5).NE.0) GO TO 2185                                     SMT
       YLABEL=(K-DLH(2))/VSCALE + ULH(2)                                SMT
       WRITE(6,103)  YLABEL,(PLOT(I,K),I=1,IWIDTH)                      SMT
       GO TO 2191                                                       SMT
 2185  CONTINUE                                                         SMT
       WRITE(6,100)  (PLOT(I,K),I=1,IWIDTH)                             SMT
 2191 CONTINUE                                                          SMT
      WRITE(6,106) (XAXIS(I,1),I=1,IWIDTH)                              SMT
      WRITE(6,104) (XAXIS(I,2),I=1,IWIDTH)                              SMT
      WRITE(6,156)                                                      SMT
      WRITE(6,146) (NE(I),I=1,L)                                        SMT
C --- PRINT VARIANCES                                                    SMT
      DO 50 J=1,4                                                       SMT
       WRITE(6,150) J, (VMX(I,J), I=1,L)                                SMT
       WRITE(6,151) (VXCNT(I,J), I=1,L)                                 SMT
       WRITE(6,152) (VXSTD(I,J), I=1,L)                                 SMT
   50 CONTINUE                                                          SMT
      WRITE(6,108) TTL                                                  SMT
  100 FORMAT(9X,'|',122A1,'|')                                          SMT
  101 FORMAT(9X,'+',122('-'),'+')                                       SMT
  102 FORMAT('1')                                                       SMT
  103 FORMAT(' ',G8.2,'-',122A1,'|')                                    SMT
  104 FORMAT(10X,122A1)                                                 SMT
  106 FORMAT(9X,'+',122A1,'+')                                          SMT
  108 FORMAT(/,' ESTIMATOR: ',A120)                                     SMT
  146 FORMAT( ' SIZE      ',8(I8,6X) )                                  SMT
  150 FORMAT(/,' X-VALUE-',I2, 8G15.4)                                  SMT
  151 FORMAT(  ' PERCENTILE', 8G15.4)                                   SMT
  152 FORMAT(  ' ESTIM. STD', 8G15.4)                                   SMT
  156 FORMAT(/' SUBSAMPLE')                                             SMT
  161 FORMAT(9X,'SAMPLE SIZE (N):',I5,20X,'NO. OF REPLICATIONS (M):',   SMT
     +I5,18X,'DEGREE OF REGRESSION (D):',I3)                            SMT
  173 FORMAT('1',30X,'PERCENTILE PLOTS FOR FOUR X-VALUES',I8,           SMT
     *   ' SUPER-REPLICATIONS',/)                                       SMT
      END                                                               SMT

C ------------------------------------------------------------------    SMT
      SUBROUTINE CLRGT(GBA,GBV,GBS,GVV,GSTAT,GPVAL)                     SMT
C --- CLEAR GRANDTOTALS                                                  SMT
      REAL*8 GPVAL(8,11,2)                                              SMT
      REAL*8 GBA(7,2), GBV(7,2), GBS(7,2), GVV(7,2), GSTAT(8,7,2)       SMT
      DO 41 I=1,7                                                       SMT
       DO 42 J=1,2                                                      SMT
        GBA(I,J)=0                                                      SMT
```

```
        GBV(I,J)=0                                                      SMT
        GBS(I,J)=0                                                      SMT
        GVV(I,J)=0                                                      SMT
  42   CONTINUE                                                         SMT
  41  CONTINUE                                                          SMT
      DO 43 I=1,8                                                       SMT
       DO 44 J=1,7                                                      SMT
        GSTAT(I,J,1)=0                                                  SMT
        GSTAT(I,J,2)=0                                                  SMT
  44   CONTINUE                                                         SMT
       DO 46 J=1,11                                                     SMT
        GPVAL(I,J,1)=0                                                  SMT
        GPVAL(I,J,2)=0                                                  SMT
  46   CONTINUE                                                         SMT
  43  CONTINUE                                                          SMT
      END                                                               SMT

C                                                                       SMT
      SUBROUTINE CLRPA(PLOT)                                            SMT
      CHARACTER*1 PLOT(122,50),BLK                                      SMT
      DATA BLK/' '/                                                     SMT
      DO 3 J=1,50                                                       SMT
       DO 4 I=1,122                                                     SMT
        PLOT(I,J)=BLK                                                   SMT
   4   CONTINUE                                                         SMT
   3  CONTINUE                                                          SMT
      END                                                               SMT

C***********************************************************************SMT
C                                                                       SMT
      SUBROUTINE BOXPRT(Y,NY,IX,PLOT,RG,XMIN,YMIN,XMAX,YMAX,VSCALE)     SMT
C     PREPARES BOXPLOT  FROM VECTOR Y (IN 2-D ARRAY PLOT)               SMT
      CHARACTER*1 PLOT(122,50),DASH,CBAR,CROSS,CSTR,CO                  SMT
      REAL Y(NY)                                                        SMT
      INTEGER RG                                                        SMT
      LOGICAL LFLAG                                                     SMT
      DATA DASH/'-'/,CBAR/'|'/,CSTR/'*'/,CROSS/'+'/,CO/'0'/             SMT
      IF(NY .GE. 9) GO TO 5                                             SMT
C     WHEN LESS THAN 9 POINTS JUST SHOW THE POINTS                      SMT
      DO 8 I=1,NY                                                       SMT
       J=(Y(I)-YMIN)*VSCALE + 1.                                        SMT
C      IGNORE VALUE IF IT FALLS OUTSIDE WINDOW                          SMT
       IF(J.GT.50  .OR.  J.LT.1) GO TO 8                                SMT
       PLOT(IX,J)=CO                                                    SMT
    8 CONTINUE                                                          SMT
      SUM=0.                                                            SMT
      DO 88 I=1,NY                                                      SMT
       SUM=SUM+Y(I)                                                     SMT
   88 CONTINUE                                                          SMT
      SUM=SUM/FLOAT(NY)                                                 SMT
      MEAN=(SUM-YMIN)*VSCALE+1                                          SMT
      IF(MEAN.LE.50  .AND.  MEAN.GE.1) PLOT(IX,MEAN)=CSTR               SMT
      GO TO 99                                                          SMT
    5 CONTINUE                                                          SMT
      LFLAG=.FALSE.                                                     SMT
      P25 =PCTL(Y,NY,.25,0)                                             SMT
```

```
      P75 =PCTL(Y,NY,.75,1)                                            SMT
      P50 =PCTL(Y,NY,.50,1)                                            SMT
      IQ1=(P25-YMIN)*VSCALE+1.                                         SMT
      IQ2=(P50-YMIN)*VSCALE+1.                                         SMT
      IQ3=(P75-YMIN)*VSCALE+1.                                         SMT
      XLOW=2*P25-P75                                                   SMT
      ILOW=(XLOW-YMIN)*VSCALE+1.                                       SMT
      XHI=2*P75-P25                                                    SMT
      IHI=(XHI-YMIN)*VSCALE+1.                                         SMT
      CLOW=2.5*P25-1.5*P75                                             SMT
      CHI=2.5*P75-1.5*P25                                              SMT
C     DRAW BOX                                                         SMT
      DO 20 J=IQ1,IQ3                                                  SMT
       IF(J.GT.50  .OR.  J.LT.1) GO TO 20                              SMT
       PLOT(IX-1,J)=CBAR                                               SMT
       PLOT(IX+1,J)=CBAR                                               SMT
   20 CONTINUE                                                         SMT
      IF(IQ1.GT.50  .OR.  IQ1.LT.1) GO TO 21                           SMT
       PLOT(IX-1,IQ1)=DASH                                             SMT
       PLOT(IX+1,IQ1)=DASH                                             SMT
   21 CONTINUE                                                         SMT
      IF(IQ3.GT.50  .OR.  IQ3.LT.1) GO TO 22                           SMT
       PLOT(IX-1,IQ3)=DASH                                             SMT
       PLOT(IX+1,IQ3)=DASH                                             SMT
   22 CONTINUE                                                         SMT
C     DETERMINE IF OUTLIERS ARE TO BE COUNTED AND THE NUMBER PRINTED.  SMT
      IF (RG.EQ.1) GO TO 55                                            SMT
      DO 30 I=1,NY                                                     SMT
       J=(Y(I)-YMIN)*VSCALE+1.                                         SMT
C      IGNORE VALUE IF IT FALLS OUTSIDE WINDOW                         SMT
       IF(J.GT.50  .OR.  J.LT.1) GO TO 30                              SMT
       IF(Y(I).LT.CLOW) PLOT(IX,J)=CSTR                                SMT
       IF(Y(I).LT.CLOW) GO TO 30                                       SMT
       IF(Y(I).GE.CLOW .AND.Y(I).LT.XLOW) PLOT(IX,J)=CO                SMT
       IF(LFLAG .OR. Y(I).LT.XLOW) GO TO 25                            SMT
C      THIS IS THE LOW-CROSS POINTER (1ST POINT GE XLOW)               SMT
       LFLAG=.TRUE.                                                    SMT
       ILX=J                                                           SMT
C      NEXT LINE ENDS UP WITH HI-CROSS POINTER (LAST POINT LE XHI)     SMT
   25  IF(Y(I).LE.XHI) IHX=J                                           SMT
       IF(Y(I).GT.XHI .AND.Y(I).LE.CHI) PLOT(IX,J)=CO                  SMT
       IF(Y(I).GT.CHI) PLOT(IX,J)=CSTR                                 SMT
   30 CONTINUE                                                         SMT
      GO TO 56                                                         SMT
C                                                                      SMT
C     SCALE TO INTERQUARTILE +(-) INTERQUARTILE DISTANCE.              SMT
C                                                                      SMT
   55 II=0                                                             SMT
      III=0                                                            SMT
      DO 31 I=1,NY                                                     SMT
       J=(Y(I)-YMIN)*VSCALE+1.                                         SMT
       IF(Y(I).LT.CLOW) II = II+1                                      SMT
       IF(Y(I).GT.CHI) III=III+1                                       SMT
       IF(J.GT.50  .OR.  J.LT.1) GO TO 31                              SMT
       IF(Y(I).GE.CLOW .AND.Y(I).LT.XLOW) PLOT(IX,J)=CO                SMT
       IF(LFLAG .OR. Y(I).LT.XLOW) GO TO 26                            SMT
```

```
C      THIS IS THE LOW-CROSS POINTER (1ST POINT GE XLOW)                   SMT
        LFLAG=.TRUE.                                                       SMT
        ILX=J                                                              SMT
C      NEXT LINE ENDS UP WITH HI-CROSS POINTER (LAST POINT LE XHI)        SMT
   26   IF(Y(I).LE.XHI) IHX=J                                              SMT
        IF(Y(I).GT.XHI .AND.Y(I).LE.CHI) PLOT(IX,J)=CO                     SMT
   31 CONTINUE                                                             SMT
C                                                                          SMT
C     PRINT NUMBER OF OUTLIERS UNLESS 0.                                   SMT
      DO 36 K=1,2                                                          SMT
      IK=II                                                                SMT
      J=(CLOW-YMIN)*VSCALE + 1                                             SMT
      IF(K.EQ.2)IK=III                                                     SMT
      IF(K.EQ.2)J=(CHI-YMIN)*VSCALE + 1                                    SMT
      IF(J.LT.1) J=1                                                       SMT
      IF(J.GT.50)J=50                                                      SMT
      IF (IK.EQ.0) GO TO 36                                                SMT
      CALL NUMPRT(IX,J,IK,PLOT)                                            SMT
   36 CONTINUE                                                             SMT
C                                                                          SMT
C     FILL BARS ABOVE AND BELOW THE BOX                                    SMT
   56 DO 32 J=ILX,IQ1                                                      SMT
       IF(J.GT.50  .OR.  J.LT.1) GO TO 32                                  SMT
        PLOT(IX,J)=CBAR                                                    SMT
   32 CONTINUE                                                             SMT
      DO 33 J=IQ3,IHX                                                      SMT
       IF(J.GT.50  .OR.  J.LT.1) GO TO 33                                  SMT
       PLOT(IX,J)=CBAR                                                     SMT
   33 CONTINUE                                                             SMT
      IF(ILX.LE.50  .AND.  ILX.GE.1) PLOT(IX,ILX)=CROSS                    SMT
      IF(IHX.LE.50  .AND.  IHX.GE.1) PLOT(IX,IHX)=CROSS                    SMT
      IF(IQ1.LE.50  .AND.  IQ1.GE.1) PLOT(IX,IQ1)=DASH                     SMT
      IF(IQ2.LE.50  .AND.  IQ2.GE.1) PLOT(IX,IQ2)=CROSS                    SMT
      IF(IQ3.LE.50  .AND.  IQ3.GE.1) PLOT(IX,IQ3)=DASH                     SMT
      SUM=0.                                                               SMT
      DO 40 I=1,NY                                                         SMT
       SUM=SUM+Y(I)                                                        SMT
   40 CONTINUE                                                             SMT
      SUM=SUM/FLOAT(NY)                                                    SMT
      MEAN=(SUM-YMIN)*VSCALE+1                                             SMT
      IF(MEAN.LE.50  .AND.  MEAN.GE.1) PLOT(IX,MEAN)=CSTR                  SMT
   99 CONTINUE                                                             SMT
      RETURN                                                               SMT
      END                                                                  SMT

C*************************************************************************** SMT
      SUBROUTINE RREG(XS,YS,BS,M,N,IX1,IX2)                                SMT
C   ROBUST REGRESSION ON Y=X*B                                             SMT
C   X=M BY N MATRIX CONTAINED IN AN ARRAY OF DIM(IX1,IX2)                  SMT
C   Y=M-VECTOR CONTAINED IN AN ARRAY OF DIM(IX1)                           SMT
C   B=N-VECTOR CONTAINED IN AN ARRAY OF DIM(IX2)                           SMT
C   XX,XXI=WORK ARRAYS OF DIM(IX2,IX2)                                     SMT
C   WY=WORK ARRAY OF DIM(IX1)                                              SMT
C   WX=WORK MATRIX OF DIM(IX1,IX2)                                         SMT
C   XY=WORK ARRAY OF DIM(IX2)                                              SMT
C   WK=WORK ARRAY OF DIM(N**2 + 3*N)    OR LARGER                          SMT
```

```
C ***** +++++ +++++  MODIFICATION  USING CHOLESKY +++++                 SMT
C                                                                       SMT
      REAL*4 YS(8),XS(8,7),BS(7)                                        SMT
      REAL*8 Y(8),X(8,7),B(7),XTX(7,7),XTY(7)                           SMT
C ***** CONVERT  REAL*4 TO REAL*8 ***************************           SMT
      DO 10 I=1,IX1                                                     SMT
         Y(I)=DBLE(YS(I))                                               SMT
         DO 5 J=1,IX2                                                   SMT
            X(I,J)=DBLE(XS(I,J))                                        SMT
  5      CONTINUE                                                       SMT
 10   CONTINUE                                                          SMT
C                                                                       SMT
      CALL MATSQ (X,XTX,M,N)                                            SMT
      CALL MATMUL (X,Y,XTY,M,N)                                         SMT
      CALL CHOLES (XTX,XTY,B,N)                                         SMT
C                                                                       SMT
C ***** CONVERT  REAL*8 TO REAL*4 ***************************           SMT
C     DO 20 I=1,IX1                                                     SMT
C        YS(I)=SNGL(Y(I))                                               SMT
         DO 15 J=1,IX2                                                  SMT
            BS(J)=SNGL(B(J))                                            SMT
C           XS(I,J)=SNGL(X(I,J))                                        SMT
 15      CONTINUE                                                       SMT
 20   CONTINUE                                                          SMT
C                                                                       SMT
      RETURN                                                            SMT
      END                                                               SMT

C***********************************************************************SMT
      SUBROUTINE NUMPRT(IX,J,IK,PLOT)                                   SMT
C     NUMPRT PLOTS THE NUMBER IK IN THE 2-D ARRAY PLOT CENTERED ON      SMT
C     THE PLOT(IX,J) POSITION.                                          SMT
C     IX = COLUMN OF MATRIX PLOT WHERE NUMBER IS TO BE PRINTED.         SMT
C     J = ROW OF MATRIX WHERE MUMBER IS TO BE PRINTED.                  SMT
C     IK = NUMBER TO BE PRINTED                                         SMT
C     PLOT = 2-D ARRAY WHERE NUMBER IS TO BE PLOTED.                    SMT
C                                                                       SMT
      CHARACTER*1 NUM(10),PLOT(122,J)                                   SMT
      DATA NUM/'0','1','2','3','4','5','6','7','8','9'/                 SMT
      IF (IK.LT.10) GO TO 1                                             SMT
      IF (IK.LT.100) GO TO 2                                            SMT
      IF (IK.LT.1000) GO TO 3                                           SMT
      IF (IK.LT.10000) GO TO 4                                          SMT
      I10000 = IK/10000                                                 SMT
      PLOT(IX-2,J) = NUM(I10000+1)                                      SMT
      I1000 = (IK-I10000*10000)/1000                                    SMT
      PLOT(IX-1,J) = NUM(I1000+1)                                       SMT
      I100 = (IK-I10000*10000-I1000*1000)/100                           SMT
      PLOT(IX,J) = NUM(I100+1)                                          SMT
      I10 = (IK-I10000*10000-I1000*1000-I100*100)/10                    SMT
      PLOT(IX+1,J) = NUM(I10+1)                                         SMT
      I1 = (IK-I10000*10000-I1000*1000-I100*100-I10*10)                 SMT
      PLOT(IX+2,J) = NUM(I1+1)                                          SMT
      GO TO 22                                                          SMT
    4 I1000 = IK/1000                                                   SMT
      PLOT(IX-2,J) = NUM(I1000+1)                                       SMT
```

```
      I100 = (IK-I1000*1000)/100                                        SMT
      PLOT(IX-1,J) = NUM(I100+1)                                        SMT
      I10 = (IK-I1000*1000-I100*100)/10                                 SMT
      PLOT(IX,J) = NUM(I10+1)                                           SMT
      I1 = (IK-I1000*1000-I100*100-I10*10)                              SMT
      PLOT(IX+1,J) = NUM(I1+1)                                          SMT
      GO TO 22                                                          SMT
    3 I100 = IK/100                                                     SMT
      PLOT(IX-1,J) = NUM(I100+1)                                        SMT
      I10 = (IK-I100*100)/10                                            SMT
      PLOT(IX,J) = NUM(I10+1)                                           SMT
      I1 = (IK-I100*100-I10*10)                                         SMT
      PLOT(IX+1,J) = NUM(I1+1)                                          SMT
      GO TO 22                                                          SMT
    2 I10 = IK/10                                                       SMT
      PLOT(IX-1,J) = NUM(I10+1)                                         SMT
      I1 = (IK-I10*10)                                                  SMT
      PLOT(IX,J) = NUM(I1+1)                                            SMT
      GO TO 22                                                          SMT
    1 PLOT(IX,J) = NUM(IK+1)                                            SMT
   22 RETURN                                                            SMT
      END                                                               SMT

C***********************************************************************SMT
C                                                                       SMT
      SUBROUTINE SECEST(GENEST,IX,N,M,NEK,Y,KP)                         SMT
      REAL Y(1)                                                         SMT
      real*8  IX                                                        SMT
C     COMPUTE ESTIMATES "EST" FOR SECTION LENGTH NEK (LIMITED TO 1000)  SMT
      NBK=N/NEK                                                         SMT
      KP=0                                                              SMT
      DO 10 I=1,M                                                       SMT
       DO 15 J=1,NBK                                                    SMT
        KP=KP+1                                                         SMT
        CALL GENEST(IX,NEK,Y(KP))                                       SMT
   15  CONTINUE                                                         SMT
   10 CONTINUE                                                          SMT
      RETURN                                                            SMT
      END                                                               SMT

C***********************************************************************SMT
C                                                                       SMT
      SUBROUTINE MAXMIN(Y,N,YMAX,YMIN)                                  SMT
C     RETURNS MAX AND MIN VALUES OF VECTOR Y OF LENGTH N                SMT
      REAL Y(N)                                                         SMT
      YMAX=Y(1)                                                         SMT
      YMIN=Y(1)                                                         SMT
      DO 605 J=1,N                                                      SMT
       IF(Y(J).LT. YMIN) YMIN=Y(J)                                      SMT
       IF(Y(J).GT. YMAX) YMAX=Y(J)                                      SMT
  605 CONTINUE                                                          SMT
      RETURN                                                            SMT
      END                                                               SMT

C***********************************************************************SMT
C                                                                       SMT
```

```
      FUNCTION PCTL(Y,N,P,IC)                                          SMT
C     COMPUTES P PERCENTILE OF N VALUES IN Y.                          SMT
C     WHEN IC=1 DATA IS ALREADY SORTED                                 SMT
      REAL Y(N)                                                        SMT
      R=P*FLOAT(N+1)                                                   SMT
      IF(IC .NE. 1) CALL SORT(Y,N)                                     SMT
      I=MAX0(INT(R) , 1)                                               SMT
      I=MIN0(I,N)                                                      SMT
      J=MIN0(INT(R+1.) , N)                                            SMT
      R=R-INT(R)                                                       SMT
      PCTL=Y(I)+R*(Y(J)-Y(I))                                          SMT
      RETURN                                                           SMT
      END                                                              SMT

C********************************************************************** SMT
      SUBROUTINE DELETO(Y,KP,YMAX,YMIN)                                SMT
C     SUBROUTINE SCALES THE GRAPH TO UPPER (LOWER) QUARTILE + (-)      SMT
C     1.5 TIMES INTERQUARTILE DISTANCE OR TO FIRST POINT WITHIN        SMT
C     THESE LIMITS IF NO POINTS EXIST OUTSIDE.                         SMT
      REAL Y(KP)                                                       SMT
      P25 =PCTL(Y,KP,.25,0)                                            SMT
      P75 =PCTL(Y,KP,.75,1)                                            SMT
      P50 =PCTL(Y,KP,.50,1)                                            SMT
      YMIN=2.5*P25-1.5*P75                                             SMT
      YMAX=2.5*P75-1.5*P25                                             SMT
      IF(Y(1).GT.YMIN) YMIN=Y(1)                                       SMT
      IF(Y(KP).LT.YMAX) YMAX=Y(KP)                                     SMT
      RETURN                                                           SMT
      END                                                              SMT

C***********************************************************************SMT
C     CHOLESKI'S  METHODE  ***************************                 SMT
      SUBROUTINE CHOLES (XTX,XTY,BHAT,N)                               SMT
      REAL*8 L(7,7),SUM,LT(7,7),XTX(7,7),XTY(7),BHAT(7),WY(7).         SMT
      REAL*4 B(7)                                                      SMT
      INTEGER P                                                        SMT
C                                                                      SMT
C ***** INIT L ******************                                      SMT
      DO 100 I=1,N                                                     SMT
       BHAT(I)=0.0D0                                                   SMT
       DO 50 J=1,N                                                     SMT
        L(I,J)=0.0D0                                                   SMT
        LT(I,J)=0.0D0                                                  SMT
   50  CONTINUE                                                        SMT
  100 CONTINUE                                                         SMT
C ****  ALGORITHM  DECOMPOSITION  *********                            SMT
      L(1,1)=DSQRT(XTX(1,1))                                           SMT
      DO 500 K=2,N                                                     SMT
       KK=K-1                                                          SMT
       DO 200 J=1,KK                                                   SMT
        JJ=J-1                                                         SMT
        SUM=0.0D0                                                      SMT
        IF (J.EQ.1) GO TO 150                                          SMT
         DO 140 P=1,JJ                                                 SMT
          SUM=SUM+(L(K,P)*L(J,P))                                      SMT
  140    CONTINUE                                                      SMT
```

```
 150     CONTINUE                                                    SMT
         L(K,J)=(XTX(K,J)-SUM)/L(J,J)                                SMT
 200    CONTINUE                                                     SMT
        SUM=0.0D0                                                    SMT
        DO 300 P=1,KK                                                SMT
         SUM=SUM+(L(K,P)**2)                                         SMT
 300    CONTINUE                                                     SMT
        L(K,K)= DSQRT (XTX(K,K)-SUM)                                 SMT
 500   CONTINUE                                                      SMT
C  BUILD L-TRANSPOSE  IN  LT *********************                   SMT
       DO 540 I=1, N                                                 SMT
        DO 530 J=1, N                                                SMT
         LT(I,J)=L(J,I)                                              SMT
  530    CONTINUE                                                    SMT
  540 CONTINUE                                                       SMT
C                                                                    SMT
C *****  A L G O R I T H M   PART 1 A. 2 **********                  SMT
C ***     L * WY = XTY                                               SMT
       WY(1)=XTY(1)/L(1,1)                                           SMT
       DO 700 I=2,N                                                  SMT
          II=I-1                                                     SMT
          SUM=0.0D0                                                  SMT
          DO 600 J=1,II                                              SMT
             SUM=SUM+(WY(J)*L(I,J))                                  SMT
 600      CONTINUE                                                   SMT
          WY(I)=(XTY(I)-SUM)/L(I,I)                                  SMT
 700   CONTINUE                                                      SMT
C                                                                    SMT
C ***  LT * BHAT = WY  *******************                           SMT
       BHAT(N)=WY(N)/LT(N,N)                                         SMT
       DO 800 II=2,N                                                 SMT
          I=N-II+1                                                   SMT
          SUM=0.0D0                                                  SMT
          DO 750 J=I,N                                               SMT
             SUM=SUM+(BHAT(J)*LT(I,J))                               SMT
 750      CONTINUE                                                   SMT
          BHAT(I)=(WY(I)-SUM)/LT(I,I)                                SMT
 800   CONTINUE                                                      SMT
C  TTTTTTTTTTTTTTTTTTTTTTTTTTTTTTTTTT                                SMT
C                                                                    SMT
       DO 950 I=1,4                                                  SMT
          B(I)=SNGL(BHAT(I))                                         SMT
C         WRITE (6,900)BHAT(I),B(I)                                  SMT
 950   CONTINUE                                                      SMT
 900   FORMAT (1X,'BHAT ',F30.15,'BSNGL ',F30.15)                    SMT
C                                                                    SMT
       RETURN                                                        SMT
       END                                                           SMT

C                                                                    SMT
C *****   MATRIX MULTIPLICATION  XT * X = XRES   *******             SMT
C                                                                    SMT
       SUBROUTINE MATSQ ( X, XRES, M, N )                            SMT
       REAL*8 X(8,7),XT(7,8),XRES(7,7),SUM                           SMT
C                                                                    SMT
C ***  BUILD  X-TRANSPOSE IN LT  **********                          SMT
```

```
      DO 20 I=1,M                                                    SMT
         DO 10 J=1,N                                                 SMT
            XT(J,I)=X(I,J)                                           SMT
 10      CONTINUE                                                    SMT
 20   CONTINUE                                                       SMT
C                                                                    SMT
C ***   XT * X = XRES   ****                                         SMT
C                                                                    SMT
      DO 50 I=1,N                                                    SMT
         DO 40 J=1,N                                                 SMT
            SUM=0.0D0                                                SMT
            DO 30 K=1,M                                              SMT
                SUM=SUM+(XT(I,K)*X(K,J))                             SMT
 30             CONTINUE                                             SMT
                XRES (I,J) = SUM                                     SMT
 40      CONTINUE                                                    SMT
 50   CONTINUE                                                       SMT
      RETURN                                                         SMT
      END                                                            SMT

C                                                                    SMT
C  ****  MATRIX MULTIPLICATION  XT * Y = XTY  ********               SMT
C                                                                    SMT
      SUBROUTINE MATMUL ( X,Y,XTY,M,N )                              SMT
      REAL*8 Y(8),XT(7,8),X(8,7),XTY(7),SUM                          SMT
C TTTTT TEST  *****************************************              SMT
C                                                                    SMT
C     WRITE(6,102)                                                   SMT
  102 FORMAT (1X,'  Y  ',/)                                          SMT
C     WRITE (6,100)(Y(I),I=1,8)                                      SMT
  100 FORMAT (1X,6F20.10)                                            SMT
C     WRITE(6,101)                                                   SMT
  101 FORMAT (1X,'  X  ',/)                                          SMT
      DO 150 I=1,8                                                   SMT
C        WRITE (6,100)(X(I,J),J=1,7)                                 SMT
  150 CONTINUE                                                       SMT
C                                                                    SMT
C ???????????????????????????????????????????????                    SMT
C                                                                    SMT
C                                                                    SMT
C ****  BUILD XT  *******                                            SMT
      DO 20 I=1,M                                                    SMT
         DO 10 J=1,N                                                 SMT
            XT(J,I)=X(I,J)                                           SMT
 10      CONTINUE                                                    SMT
 20   CONTINUE                                                       SMT
C                                                                    SMT
C ****  XT * Y = XTY  *******                                        SMT
C                                                                    SMT
      DO 50 I=1,N                                                    SMT
         SUM=0.0D0                                                   SMT
         DO 40 J=1,M                                                 SMT
            SUM=SUM+(XT(I,J)*Y(J))                                   SMT
 40      CONTINUE                                                    SMT
         XTY(I)=SUM                                                  SMT
 50   CONTINUE                                                       SMT
```

```
      RETURN                                                            SMT
      END                                                               SMT

C ***********************************************************************SMT
      SUBROUTINE SORT (Y,N)                                             SMT
C    INPLACE SORT USING SHELL ALGORITHM *********                        SMT
      REAL Y(N),TEMP                                                    SMT
      INTEGER GAP                                                       SMT
      LOGICAL EXCH                                                      SMT
C                                                                       SMT
      GAP=(N/2)                                                         SMT
    5 IF (.NOT.(GAP.NE.0)) GO TO 500                                    SMT
   10    CONTINUE                                                       SMT
            EXCH=.TRUE.                                                 SMT
            K=N-GAP                                                     SMT
            DO 200 I=1,K                                                SMT
            KK=I+GAP                                                    SMT
            IF(.NOT.(Y(I).GT.Y(KK))) GO TO 100                          SMT
               TEMP=Y(I)                                                SMT
               Y(I)=Y(KK)                                               SMT
                  Y(KK)=TEMP                                            SMT
                  EXCH=.FALSE.                                          SMT
  100       CONTINUE                                                    SMT
  200       CONTINUE                                                    SMT
         IF (.NOT.(EXCH)) GO TO 10                                      SMT
         GAP=(GAP/2)                                                    SMT
      GO TO 5                                                           SMT
  500 CONTINUE                                                          SMT
      RETURN                                                            SMT
      END                                                               SMT
```

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